

Math 243 Midterm 1

February 20, 2020

Name: _____ ID: _____

- Each page has a space at the top for the last 4 digits of your student ID. Make sure that you fill that out on at least one side of every sheet of paper.
- Show enough work that your solution would convince a skeptical peer that your answer is correct.
- Simplify your answers as much as possible.
- The questions are ordered by topic, not by difficulty.
- Each question is worth the same number of points.
- You may not use any tools or resources other than writing implements. In particular, no calculators, phones, notes, and so forth.

1. Set up an integral that computes the arc length of the curve defined by

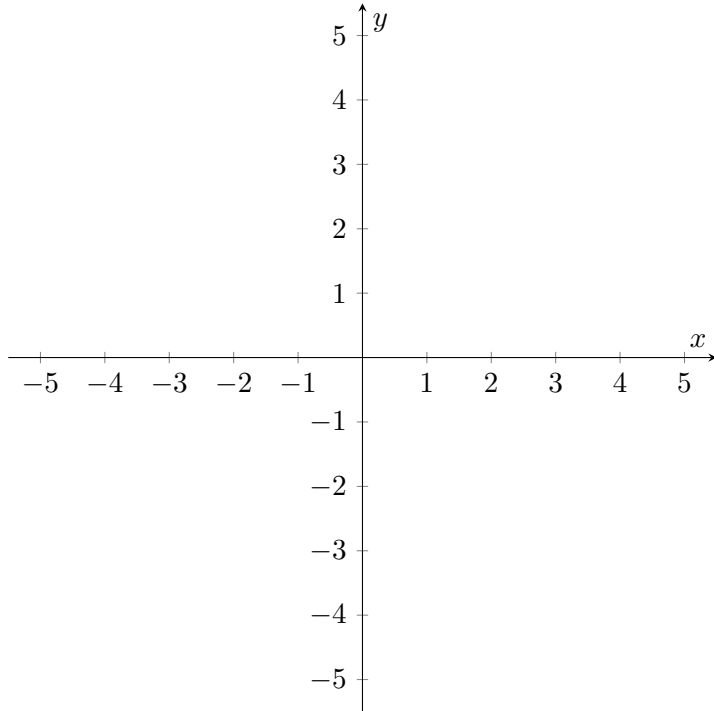
$$y = \sqrt[3]{x}, \quad 1 \leq x \leq 6.$$

You do not need to evaluate the integral, but be sure to simplify the integrand.

2. Suppose a curve is given by the parametric equations

$$x = f(t), \quad y = g(t),$$

where the range of f is $[1, 4]$ and the range of g is $[2, 3]$. What can you say about the curve?



3. Consider the curve parametrized by the equations

$$x = \frac{t}{1+t}, \quad y = \sqrt{1+t}.$$

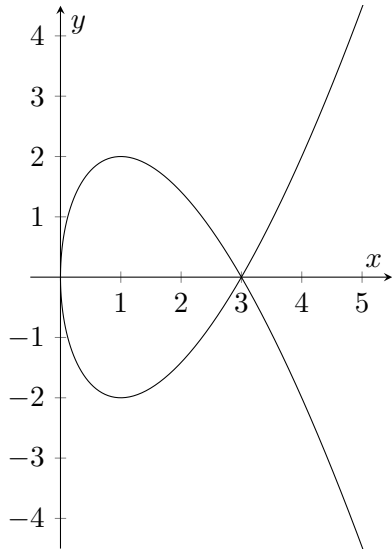
Compute $\frac{dy}{dx}$.

4. Consider the curve parametrized by the equations

$$x = t^2,$$

$$y = t^3 - 3t.$$

Find the area inside the loop.



5. Find the distance bewteen the points with polar coordinates $(4, 4\pi/3)$ and $(6, 5\pi/3)$.

6. Find a polar equation for the curve given by the Cartesian equation $4y^2 = x$. Your answer should be of the form $r = f(\theta)$.

7. Consider the polar curve with equation

$$r = \frac{1}{\theta}, \quad \frac{\pi}{2} \leq \theta \leq 2\pi.$$

Find the area of the region that is bounded by this curve and lies in the sector $\frac{\pi}{2} \leq \theta \leq 2\pi$.

8. Sketch the hyperbola with vertices at $(\pm 3, 0)$ and asymptotes $y = \pm 2x$. Find the equation of the hyperbola. Find the foci of the hyperbola and plot them.

