HOMEWORK 7

MATH 4121

(1) (Text, Exercise III.1.C, page 90, 10 points.) Suppose that f is measurable and g and h are integrable on $[a, b]$. Then $\max\{h, \min\{f, g\}\}$ is integrable on $[a, b]$.

(2) (Text, Exercise III.1.E, page 90, 10 points.) If g and h are integrable on $[a, b]$, if f is measurable, and if

$$g(x) \leq f(x) \leq h(x)$$

almost everywhere, then f is integrable on $[a, b]$.

(3) (Text, Exercise III.2.C, D, E, and F, page 94, 20 points.) Let A and B be measurable sets in $[a, b]$. Denote

$$A \Delta B := (A \setminus B) \cup (B \setminus A).$$

This set is called the *symmetric difference* of A and B.

(a) Prove that $m(A\Delta B) = 0$ if and only if $m(A \setminus B) = 0$ and $m(B \setminus A) = 0$.

(b) Prove that if $m(A\Delta B) = 0$, then $m(A) = m(A \cap B) = m(B)$.

(c) Let A, B, and C be measurable sets in $[a, b]$. Show that if $m(A\Delta B) = 0$ and $(B\Delta C) = 0$, then $m(A\Delta C) = 0$. *Hint:* $A \setminus C \subset (A \setminus B) \cup (B \setminus C)$.

(d) Let $\mathcal{M}$ be the family of all measurable sets in $[a, b]$. Let $A \sim B$ mean that $m(A\Delta B) = 0$ for A and B in $\mathcal{M}$. Prove that $\sim$ is an equivalence relation on $\mathcal{M}$.

(4) (Text, Exercise III.2.L, page 94, 10 points.) Show that open and closed sets in $[a, b]$ are measurable. Hint: Use a result from Section 0.5.