

HOMEWORK 5

MATH 4121

- (1) (Exercise II.4.A, page 67 of textbook.) Show that, in the class L , if $f \leq g$, then $\int_a^b f(x) dx \leq \int_a^b g(x) dx$.
- (2) (Exercise II.4.B, page 67 of textbook.) Show that for Riemann integrable functions the converse of Proposition 4.5 is not valid. (That is, find a function f such that $|f|$ is Riemann integrable but f is not Riemann integrable.)
- (3) (Exercise II.4.C, page 67 of textbook.) Let $f \in L$ and $\epsilon > 0$. Show that there exists a step function φ on $[a, b]$ such that

$$\int_a^b |f(x) - \varphi(x)| dx < \epsilon.$$

- (4) (Exercise II.5.A, page 73 of textbook.) Let (f_n) be a monotone sequence of functions in the class L which converges to a function in L . Show that

$$\int_a^b \left[\lim_{n \rightarrow \infty} f_n(x) \right] dx = \lim_{n \rightarrow \infty} \int_a^b f_n(x) dx.$$

- (5) (Exercise II.5.C, page 73 of textbook.) Let $f \in L$ and $0 \leq f < 1$. Show that

$$\lim_{n \rightarrow \infty} \int_a^b f^n(x) dx = 0.$$

- (6) (Exercise II.5.H, page 73 of textbook.) Let (f_n) be a monotone decreasing sequence of functions in the class L such that $f_n \geq 0$. If $\lim_{n \rightarrow \infty} \int_a^b f_n(x) dx = 0$, show that $\lim_{n \rightarrow \infty} f_n = 0$ almost everywhere.