HOMEWORK 4

MATH 4121

(1) (Exercise II.2.E, textbook page 58) Construct a sequence (φ_n) of step functions on $[0,1]$ such that $\varphi_n \leq \varphi_{n+1}$ for all n, (φ_n) diverges on the Cantor ternary set and the sequence $n \mapsto \int_0^1 \varphi_n(x)\,dx$ converges.
(2) (Exercise II.3.A, textbook page 62) If $f \in L^+$ and g is such that $g = f$ almost everywhere on $[a,b]$, show that $g \in L^+$.
(3) (Exercise II.3.B, textbook page 62) Find a non-Riemann-integrable function which is equal to a Riemann integrable function almost everywhere.
(4) (Exercise II.3.E, textbook page 63) If $f \in L^+$ and $-f \in L^+$, show that there exists a Riemann integrable function g such that $f = g$ almost everywhere. (f need not be Riemann integrable.)
(5) (Exercise II.3.G, page 63 of texbook) If $f, g \in L^+$, show that $\min\{f,g\} \in L^+$. Note: essentially the same proof also gives $\max\{f,g\} \in L^+$.
(6) (Exercise II.3.H, textbook page 63) Let F be a closed set in $[a,b]$. Show that the characteristic function of $[a,b] \setminus F$ belongs to the class L^+. Hint: Use Theorem 5.3 in chapter 0.