

HOMEWORK 3

MATH 4121

- (1) **Textbook, I.6.A, page 48.** Find a sequence $f_n(x)$ of Riemann integrable functions on $[0, 1]$ such that the limit $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx$ does not exist (or is $+\infty$) but $\lim_{n \rightarrow \infty} f_n = f$ a. e. for a bounded function f . (Suggestion: Try a variation on the sequence of functions of Exercise I.6.B, Figure 1.6, page 49 of the textbook.)
- (2) **Textbook, I.6.C, page 49.** Let $f : [0, 1] \rightarrow \mathbb{R}$ be such that

$$f(x) = \begin{cases} x^2 \sin(1/x^2) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0. \end{cases}$$

Then f is continuous and differentiable. Show that the derivative f' is not Riemann integrable.

- (3) **Exercise II.2.D, page 58** Let f be continuous on $[a, b]$. Find a monotone increasing sequence (φ_n) of step functions on $[a, b]$ which converges to f almost everywhere on $[a, b]$ and such that $\lim \int_a^b \varphi_n(x) dx < \infty$.