FINAL STUDY GUIDE

MATH 4121, SPRING 2019

Material. I'm going to roughly aim to have about a third of the final cover material from the first two midterms, and to have the remaining two thirds cover new material: chapter III sections 3–7, chapter IV sections 1–3 and 5, and chapter V sections 1 and 4–6. Here is a **non-exhaustive** list of topics:

- Section III.3: algebras, σ-algebras, σ-algebras generated by a collection, the Borel σ-algebra, the σ-algebra of measurable sets.
- Section III.4: constructing a non-measurable set, consequences.
- Section III.5: various results about approximating measurable sets by finite unions of intervals, by open sets, and by closed sets.
- Section III.6: a simpler characterization of when a function is measurable.
- Section III.7: almost uniform convergence, Egoroff's theorem, convergence in measure, Lusin's theorem, other relationships between modes of convergence of functions on $[a, b]$.
- Section IV.1: integration on measurable subsets, extending our earlier results about integration to this context, the mean value theorem, countable additivity of the integral, other results on integration over a union, relationship with improper Riemann integration, absolute continuity of the integral.
- Section IV.2: step functions and integration on the real line, relationship with improper Riemann integration.
- Section IV.3: measurable functions on the real line, Lebesgue measure on the real line, extending our earlier results to this context, additional hypotheses needed to extend some of our earlier results.
- Section IV.5: measure zero, step functions, and integration in two dimensions, the Fubini Theorem, the Tonelli Theorem.
- Section V.1: a continuous nowhere differentiable function.
- Section V.4: total variation, functions of bounded variation, relationship with monotone functions, a.e. differentiability, integrability of the derivative, the Lebesgue singular function (more commonly known as the Cantor function, see Wikipedia).
- Section V.5: absolute continuity, relationship to uniform continuity and bounded variation, image of measurable sets and sets of measure zero.

- Section V.6: Fundamental Theorems of Calculus I and II (we stated them but only proved the claim that the indefinite integral is absolutely continuous).

Study suggestions. My recommendation is to study by writing down a problem you think would be a reasonable problem for the exam, closing your book, solving it, checking your work, and fixing any mistakes. If you solve it easily, set that topic aside; if you have trouble, do additional problems on that topic, or let some time pass and then try the same problem again.

Problem sources. Here are some good sources of problems that you should use while studying (and I'll use while writing the exam for the most part).

- Stating definitions, theorems, etc.
- Examples and counterexamples. Counterexamples are in some sense a mnemonic device for remembering when something works and when it doesn't.
- Proving things we proved in class. Keep in mind length: If a claim takes more than 15 minutes to prove, it's probably too complex for the exam. Maybe there's a lemma or a portion of the proof that's short enough and self-contained.
- Homework problems. Again, keep in mind length. If it's too long, it won't be on the exam, unless there's a self-contained piece that's shorter.
- Other textbook problems or problems from Apostol. If a topic is causing a lot of trouble, doing the same homework problem over again will be limited in its usefulness; you might want to try something new for extra practice.

Resources.

- I have office hours scheduled this Thursday; if those times don't work for you, send me an email and we can schedule something, or send me an email with a question and I can try to answer it.
- Your classmates.
- The posted homework solutions.
- The textbook.
- The supplementary textbook (Apostol).