Math 603 Midterm

November 8, 2019

1. State the mean-value property and an associated theorem.

2. (a) State the maximum principle.

   (b) State the strong maximum principle.

3. Let U be an open bounded domain, and assume that there is a continuous function $K\colon U \times \partial U \to \mathbb{R}$ with the following property:

   - For all $g \in C(\partial U)$, we can define a function $u\colon U \to \mathbb{R}$ by

     $$u(x) = \int_{\partial U} K(x, y) g(y)\, dy.$$

     This function u is harmonic on U, and u can be extended to a continuous function on $\overline{U}$ that is equal to g on ∂U.

   Use a result from class to show that $K(x, y) \geq 0$ for all $x \in U$ and $y \in \partial U$.

4. Let $f \in C(\overline{U})$. For any function $w \in C^2(\overline{U})$, define

   $$L(w) = \int_U \left(\tfrac{1}{2} |\nabla w|^2 - f w \right) dx.$$

   Let $u \in C^2(\overline{U})$, and assume that $L(u) \leq L(w)$ for all $w \in C^2(\overline{U})$ such that $w = u$ on ∂U. Prove that $-\Delta u = f$.