Name: ______________________________ ID: ________________

1. Consider a particle with position function

$$\mathbf{r}(t) = 3\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j}.$$

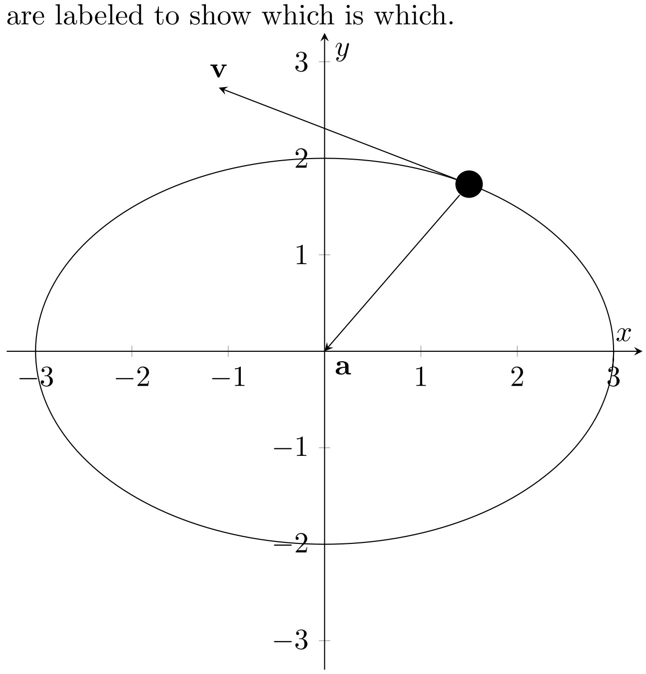
(a) Compute the position, velocity, acceleration, and speed of the particle at time $t = \frac{\pi}{3}$.

Solution: We compute that

$$\begin{aligned}
\mathbf{r}(\tfrac{\pi}{3}) &= 3(\tfrac{1}{2})\mathbf{i} + 2(\tfrac{\sqrt{3}}{2})\mathbf{j} = \tfrac{3}{2}\mathbf{i} + \sqrt{3}\mathbf{j},\\
\mathbf{v}(t) &= -3\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j}\\
\mathbf{v}(\tfrac{\pi}{3}) &= -3(\tfrac{\sqrt{3}}{2})\mathbf{i} + 2(\tfrac{1}{2})\mathbf{j} = -\tfrac{3\sqrt{3}}{2}\mathbf{i} + \mathbf{j},\\
v(\tfrac{\pi}{3}) &= \sqrt{\tfrac{27}{4} + 1} = \sqrt{\tfrac{31}{4}} = \tfrac{\sqrt{31}}{2}.\\
\mathbf{a}(t) &= -3\cos t\,\mathbf{i} - 2\sin t\,\mathbf{j},\\
\mathbf{a}(\tfrac{\pi}{3}) &= -3(\tfrac{1}{2})\mathbf{i} - 2(\tfrac{\sqrt{3}}{2})\mathbf{j} = -\tfrac{3}{2}\mathbf{i} - \sqrt{3}\mathbf{j}.
\end{aligned}$$

Thus, the position is $\frac{3}{2}\mathbf{i} + \sqrt{3}\mathbf{j}$, the velocity is $-\frac{3\sqrt{3}}{2}\mathbf{i} + \mathbf{j}$, the acceleration is $-\frac{3}{2}\mathbf{i} - \sqrt{3}\mathbf{j}$, and the speed is $\frac{\sqrt{31}}{2}$.

(b) The path of the particle is shown below. Draw a point at the position of the particle at $t = \frac{\pi}{3}$. Starting from that point, draw the velocity and acceleration vectors at $t = \frac{\pi}{3}$. Make sure that your vectors point in the right direction, are roughly the right size, and are labeled to show which is which.



Solution: The point has coordinates $(\frac{3}{2}, \sqrt{3})$. We know that $\frac{3}{2} = 1.5$, so we find 1.5 on the x-axis and draw our point above it on the ellipse.

The velocity vector is $\langle -\frac{3\sqrt{3}}{2}, 1\rangle$. We know that the velocity vector is in the direction of travel, so it must be tangent to the ellipse. Because $-\frac{3\sqrt{3}}{2}$ is negative and 1 is positive, we draw the velocity vector going left and up. To draw it the right length, we go until we have gone up 1 from the point. Alternatively, we go a distance of $\sqrt{31}/2$, which is between $\frac{5}{2}$ and $\frac{6}{2}$, so just under 3 units.
The acceleration vector is the negative of the position vector, so we draw a vector from the point back to the origin.

2. Find the domain and range of the function $f(x,y) = \sqrt{9 - x^2 - y^2}$. Be specific: shape, location, size, filled in or not, etc.

Solution: This function is defined as long as $9 - x^2 - y^2 \geq 0$, which is equivalent to $x^2 + y^2 \leq 3^2$. This is the equation of a filled-in disk of radius 3, centered at the origin.

The range of f is $[0, 3]$. To determine the smallest value of f, we note that the square root is nonnegative, and we see that $f(x, y)$ does actually attain the value of 0 at $(x, y) = (3, 0)$. Meanwhile, the largest value of f corresponds to the largest value of $9 - x^2 - y^2$, which corresponds to the smallest value of $x^2 + y^2$, which is zero when $(x, y) = (0, 0)$. At this point, the value of f is $\sqrt{9 - 0 - 0} = 3$.