Name: ______________________ ID: ______________

1. Consider the vector-valued function

$$\mathbf{r}(t) = (1 + t^3)\mathbf{i} + te^{-t}\mathbf{j} + \sin(2t)\mathbf{k}.$$

(a) Find the derivative of $\mathbf{r}(t)$.

Solution: Taking the derivative of each component, we compute that

$$\mathbf{r}'(t) = 3t^2\mathbf{i} + (e^{-t} - te^{-t})\mathbf{j} + 2\cos(2t)\mathbf{k}.$$

(b) Find the unit tangent vector at the point where $t = 0$.

Solution: Evaluating the above expression at $t = 0$, we find that

$$\mathbf{r}'(0) = 0\mathbf{i} + (1 - 0)\mathbf{j} + 2\mathbf{k} = \mathbf{j} + 2\mathbf{k},$$
$$\left|\mathbf{r}'(0)\right| = \sqrt{1 + 2^2} = \sqrt{5}.$$

The unit tangent vector $\mathbf{T}(0)$ is the unit vector pointing in the direction $\mathbf{r}'(0)$. Thus, we have

$$\mathbf{T}(0) = \frac{\mathbf{r}'(0)}{|\mathbf{r}'(0)|} = \frac{1}{\sqrt{5}}(\mathbf{j} + 2\mathbf{k}).$$

2. Consider the helix with vector equation

$$\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}.$$

Find the length of the helix from the point $(1, 0, 0)$ to the point $(1, 0, 2\pi)$.

Solution: We compute that the velocity is

$$\mathbf{r}'(t) = -\sin t\mathbf{i} + \cos t\mathbf{k} + \mathbf{k},$$

and thus the speed is

$$\left|\mathbf{r}'(t)\right| = \sqrt{\sin^2 t + \cos^2 t + 1^2} = \sqrt{1+1} = \sqrt{2}.$$

The equation of the curve tells us that $z = t$, so we are at the point $(1, 0, 0)$ when $t = 0$ and at the point $(1, 0, 2\pi)$ when $t = 2\pi$. Travelling at a speed of $\sqrt{2}$ for 2π units of time, we cover a distance of $2\sqrt{2}\pi$.