Name: ______________________ ID: ______________

1. Find a vector that is perpendicular to both $\mathbf{a} = 3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$.

Solution: One way to find a vector perpendicular to two given vectors is using the cross product. We know that $\mathbf{a} \times \mathbf{b}$ is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$, and we compute that

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & -3 \\ 3 & -3 & 3 \end{vmatrix} = (9-9)\mathbf{i} + (-9-9)\mathbf{j} + (-9-9)\mathbf{k} = -18\mathbf{j} - 18\mathbf{k}.$$

We can check our work by verifying that

$$\mathbf{a} \cdot (-18\mathbf{j} - 18\mathbf{k}) = (3)(-18) + (-3)(-18) = 0,$$
$$\mathbf{b} \cdot (-18\mathbf{j} - 18\mathbf{k}) = (-3)(-18) + (3)(-18) = 0.$$

2. Find the point at which the line with parametric equations

$$x = 2 + 3t, \qquad y = -4t, \qquad z = 5 + t$$

intersects the plane

$$4x + 5y - 2z = 18.$$

Solution: We plug in the equations of the line into the equation of the plane, obtaining

$$\begin{aligned} 4(2 + 3t) + 5(-4t) - 2(5 + t) &= 18, \\ 8 + 12t - 20t - 10 - 2t &= 18. \\ -10t - 2 &= 18, \\ -10t &= 20, \\ t &= -2. \end{aligned}$$

We plug this value of t into the equations of the line to find that

$$x = 2 + 3(-2) = -4, \qquad y = -4(-2) = 8, \qquad z = 5 + (-2) = 3.$$

Thus, the point of intersection is $(-4, 8, 3)$.

We can check our work by verifying that this point satisfies the equation of the plane.

$$4(-4) + 5(8) - 2(3) = -16 + 40 - 6 = 18.$$