Name: _____ ID: _____

1. Describe in words the region of three-dimensional space represented by the equation $y^2 = 9$. Be specific.

Solution: The equation $y^2 = 9$ is true if and only if $y = 3$ or $y = -3$. The points where $y = 3$ form a plane parallel to the xz-plane three units away from it in the positive y direction. The points where $y = -3$ form a plane parallel to the xz-plane three units away from it in the negative y direction.

Thus, the region represented by the equation $y^2 = 9$ is made up of two parallel planes six units apart on either side of the xz-plane.

2. Let $\mathbf{a} = 8\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = 4\mathbf{j} + 2\mathbf{k}$. Find the angle between $\mathbf{a}$ and $\mathbf{b}$. Is the angle acute or obtuse?

   You'll find that you won't be able to completely simplify the expression for the angle, but get as far as you can without a calculator.

Solution: We compute that

$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} &= 0 - 4 + 8 = 4, \\ |\mathbf{a}| &= \sqrt{64 + 1 + 16} = 9, \\ |\mathbf{b}| &= \sqrt{16 + 4} = 2\sqrt{5}. \end{aligned}$$

Thus, letting θ be the angle between $\mathbf{a}$ and $\mathbf{b}$, we have

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|} = \frac{4}{(9)(2\sqrt{5})} = \frac{2}{9\sqrt{5}} = \frac{2\sqrt{5}}{45}.$$

Because this expression is positive (or because $\mathbf{a} \cdot \mathbf{b}$ is positive), we know that θ is acute. More precisely,

$$\theta = \arccos\left(\frac{2\sqrt{5}}{45}\right).$$