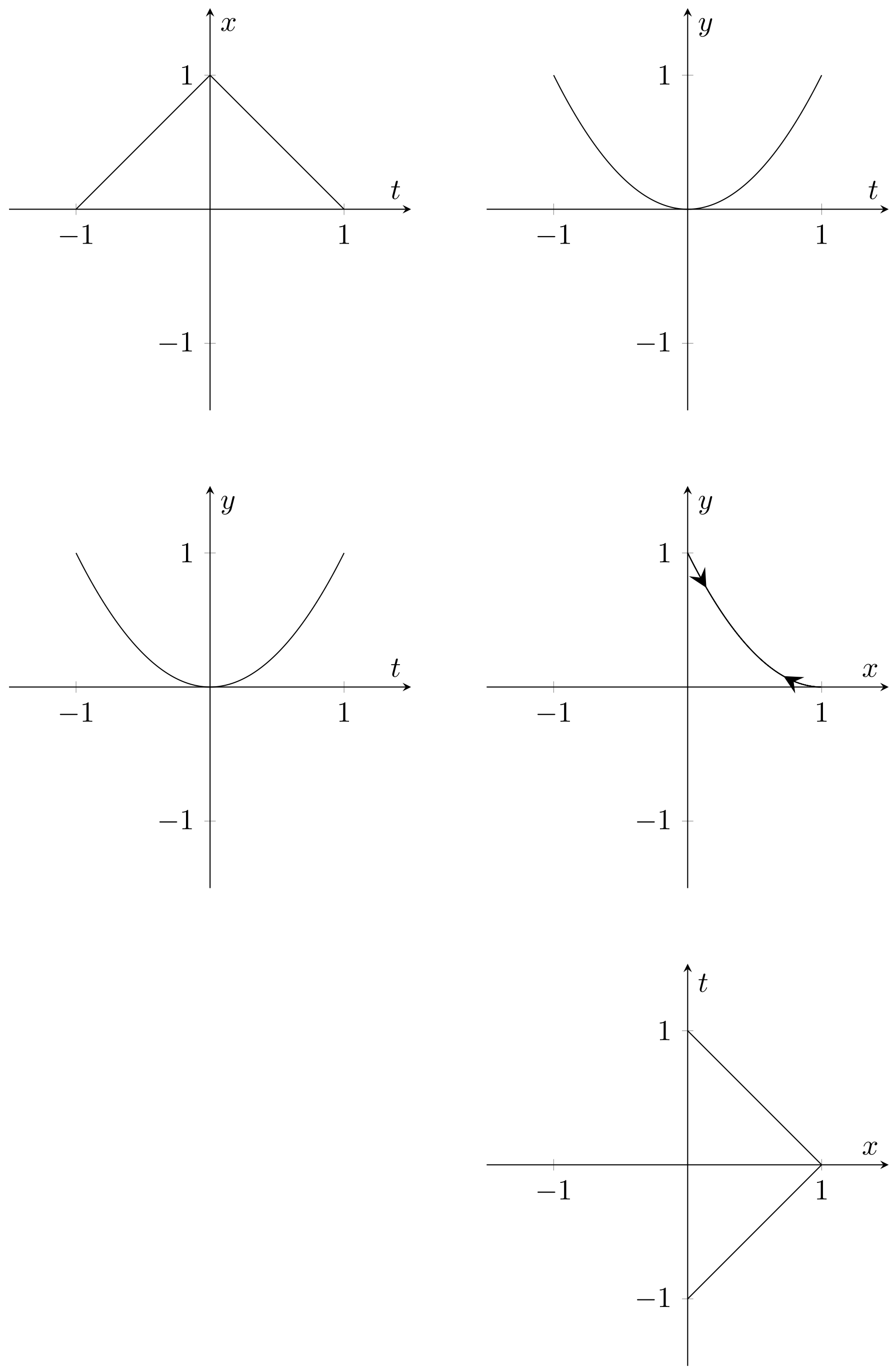
Name: ____________________ ID: ____________

1. Use the provided graphs of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve defined by these equations. Indicate with arrows the direction in which the curve is traced as t increases.



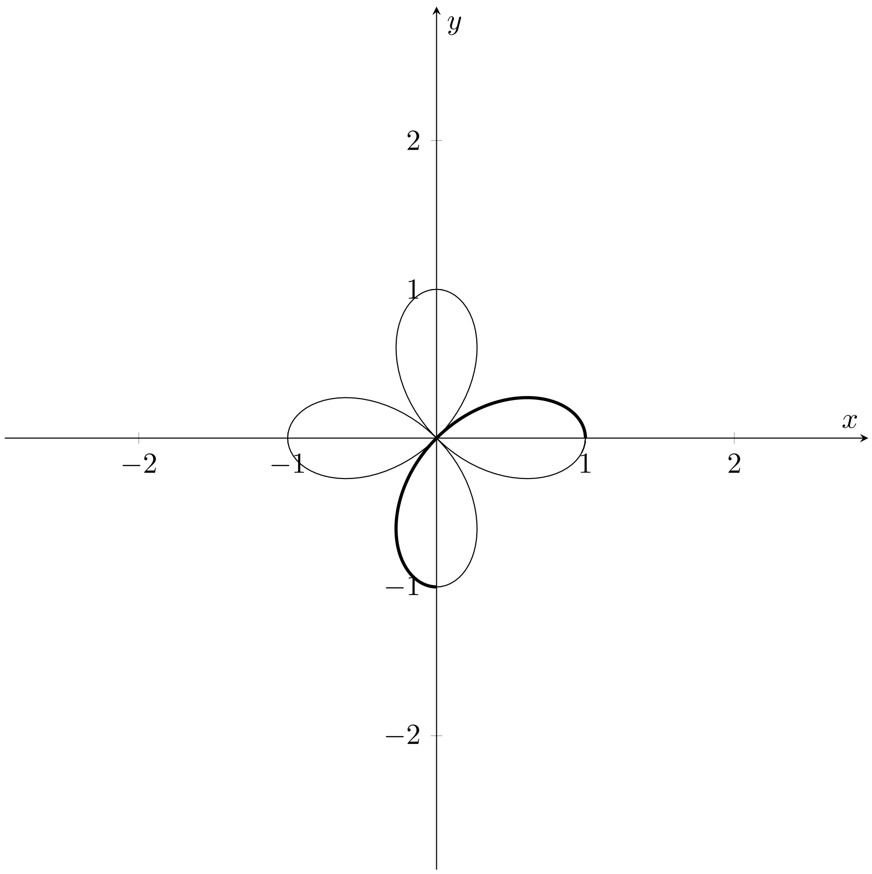
Solution: Following the method from class, we redraw the given plots to line up the y-axis of the ty plot with the y-axis of the xy plot, and line up the x-axis of the tx plot with the x-axis of the xy plot.

From these two plots, we see that, at $t = -1$, the curve is at $x = 0$ and $y = 1$, and from there it moves down and to the right until it reaches $x = 1$ and $y = 0$ at $t = 0$. At this point the curve goes up and to the left, going back exactly the way it came, reaching $x = 0$ and $y = 1$ at $t = 1$. Because the curve is traced out in both directions, we need to place arrows going both ways.

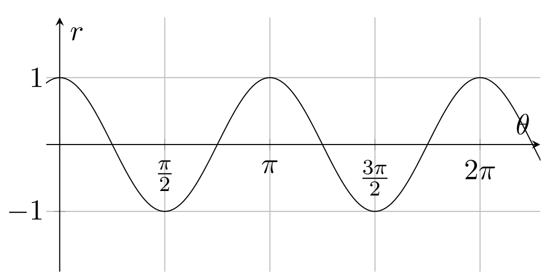
2. Sketch the curve described by the polar equation

$$r = \cos 2\theta.$$

You may find it helpful to first make a separate graph of r as a function of θ in Cartesian coordinates.



Solution: As suggested, we sketch a Cartesian plot of r versus θ in Cartesian coordinates. It's a cosine graph compressed horizontally by a factor of 2.



When $\theta = 0$, we have $r = 1$, so we start our plot at 1 on the x-axis. We see that the radius decreases and eventually reaches 0 at $\frac{\pi}{4}$, so we have our curve move closer to the origin, crossing it at a diagonal angle of $\frac{\pi}{4}$.

Thereafter, r is negative, so instead of plotting in the first quadrant, we plot in the third quadrant. As θ goes from $\frac{\pi}{4}$ to $\frac{\pi}{2}$, the value of r goes from 0 to -1, so our curve moves away from the origin and is at a distance of 1 from the origin when it crosses the negative y axis.

I have emphasized this portion of the plot in the diagram with a thicker curve. We could continue in this manner, but we don't need to if we understand symmetry. Since cosine is even, we have that $\cos(2(-\theta)) = \cos(-2\theta) = \cos 2\theta$, which means that our curve is symmetric about the x-axis. Since cosine has period 2π, we we see that $\cos(2(\theta + \pi)) = \cos(2\theta + 2\pi) = \cos 2\theta$, which means that our curve has 180° rotational symmetry. So, taking the thick part of the curve that we've drawn so far, we can reflect it across the x-axis and rotate it 180°, or both, and in doing so, we obtain the rest of the curve.