Math 243 Midterm 3

November 22, 2019

Name: ______________________________ ID: ______________

- Each page has a space at the top for the last 4 digits of your student ID. Make sure that you fill that out on at least one side of every sheet of paper.
- Show enough work that your solution would convince your peers that your answer is correct.
- The questions are ordered by topic, not by difficulty.
- Each question is worth the same number of points.
- You may not use any tools or resources other than writing implements. In particular, no calculators, phones, notes, and so forth.

1. Find the volume of the parallelepiped determined by the vectors $\mathbf{i}+\mathbf{j}$, $\mathbf{j}+\mathbf{k}$, and $\mathbf{i}+\mathbf{j}+\mathbf{k}$.

2. The line L_1 has equation

$$x = y = z.$$

The line L_2 has equation

$$x + 1 = \frac{y}{2} = \frac{z}{3}.$$

Show that the two lines are skew.

3. Consider the surface

$$3x^2 + y + 3z^2 = 0.$$

(a) Identify the horizontal traces of the surface in the planes $z = k$. If the answer depends on the value of k, be sure to specify which values of k give which answer.

(b) Identify the vertical traces of the surface in the planes $x = k$. If the answer depends on the value of k, be sure to specify which values of k give which answer.

(c) Identify the vertical traces of the surface in the planes $y = k$. If the answer depends on the value of k, be sure to specify which values of k give which answer.

(d) Identify the surface.

(e) Sketch the surface. Your sketch does not need to be quantitatively correct, but it should show the correct type of surface in the correct orientation. If you feel like you need to, feel free to write a sentence to clarify the orientation.

4. Consider the circular cylinder $x^2+y^2=4$ and the parabolic cylinder $z=x^2$. These two surfaces intersect in a curve. Find parametric equations for this curve.

5. Let $\mathbf{u}$ be a space curve that has $\mathbf{u}(2) = \langle 1, 2, -1\rangle$ and $\mathbf{u}'(2) = \langle 3, 0, 4\rangle$. Let $\mathbf{v}$ be the space curve defined by $\mathbf{v}(t) = \langle t, t^2, t^3\rangle$. Let $f(t) = \mathbf{u}(t) \cdot \mathbf{v}(t)$. Compute $f'(2)$.

6. Set up an integral for the length of the curve defined by

$$\mathbf{r}(t) = \langle \cos(\pi t), 2t, \sin(2\pi t) \rangle$$

from $(1, 0, 0)$ to $(1, 4, 0)$. You do not need to evaluate the integral.