# Math 243 Midterm 1

September 27, 2019

Name: _____ ID: _____

- Each page has a space at the top for the last 4 digits of your student ID. Make sure that you fill that out on at least one side of every sheet of paper.
- Show enough work that your solution would convince your peers that your answer is correct.
- The questions are ordered by topic, not by difficulty.
- Each question is worth the same number of points.
- You may not use any tools or resources other than writing implements. In particular, no calculators, phones, notes, and so forth.

1. Set up the integral that represents the length of the curve described by the equation

$$y = x - \ln x, \qquad 1 \leq x \leq 4.$$

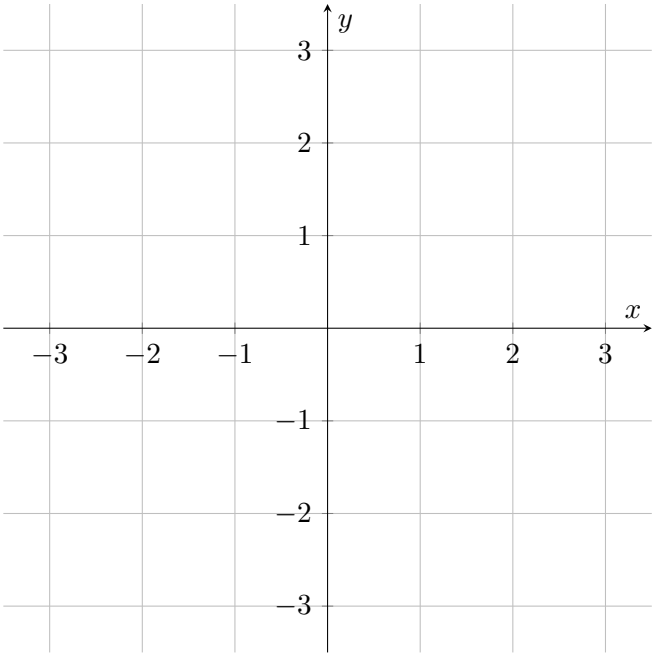
You do not need to evaluate the integral.

2. Consider the curve described by the parametric equations

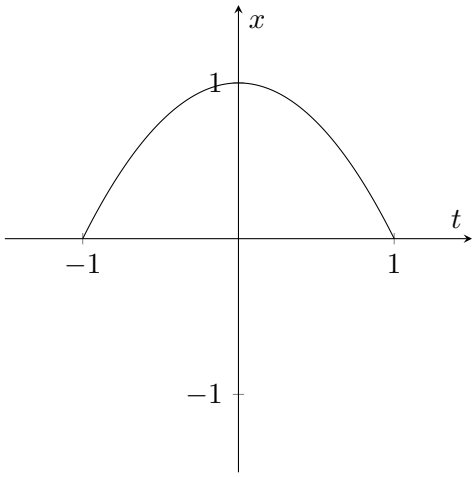
$$x = \sin t, \qquad y = \csc t, \qquad 0 < t < \tfrac{\pi}{2}.$$

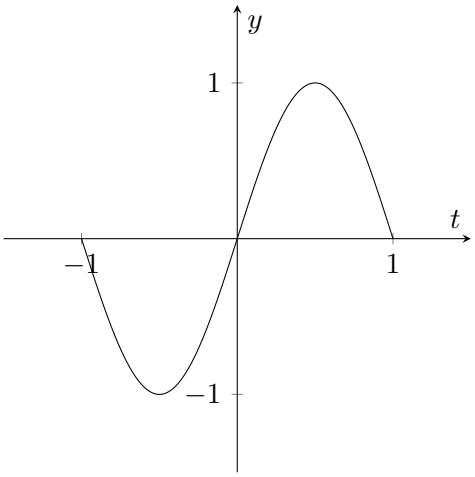
(a) Eliminate the parameter to find a Cartesian equation of the curve.

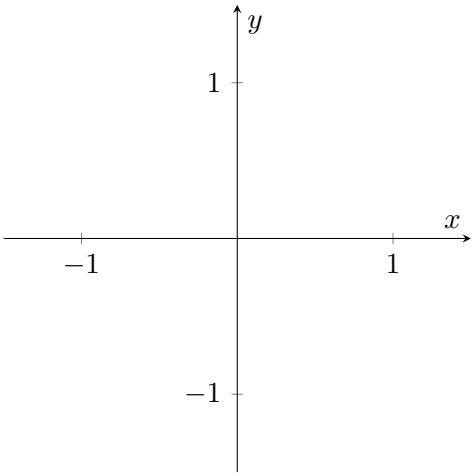
(b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.



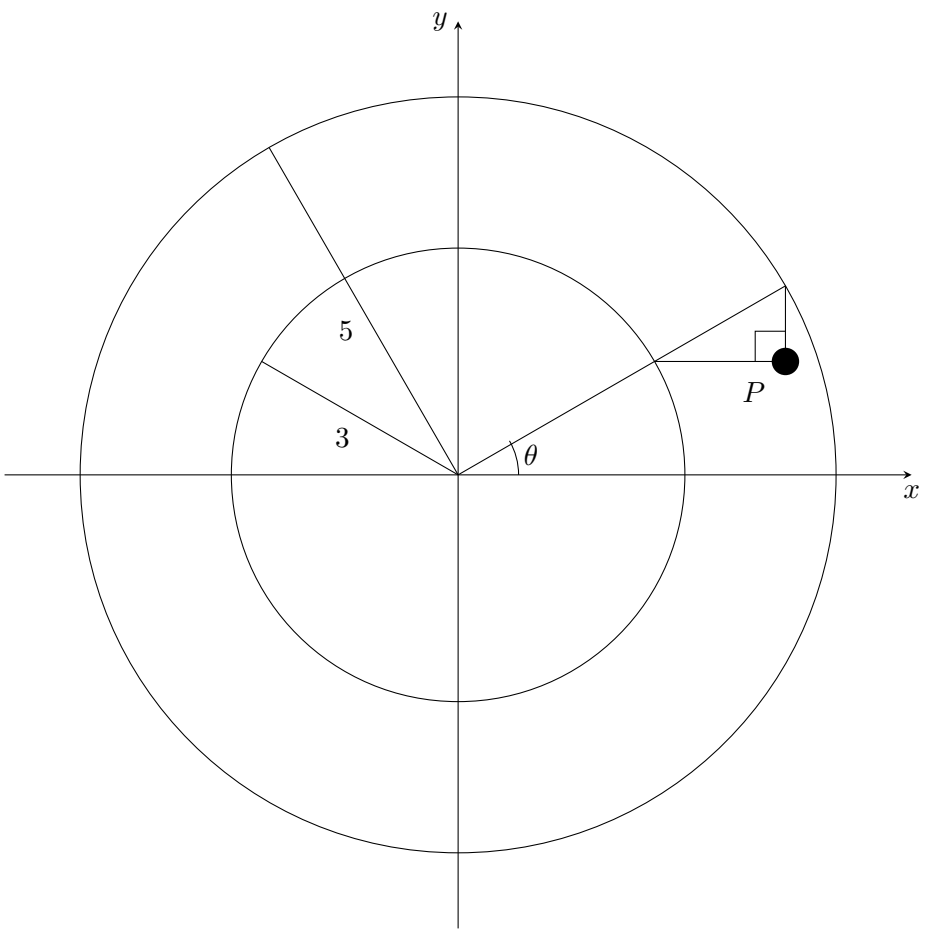
3. Use the provided graphs of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve defined by these equations. Indicate with arrows the direction in which the curve is traced as $t$ increases.



4. Find parametric equations that describe the curve traced out by the point $P$ as $\theta$ varies. Then add a sketch of the curve to the diagram.

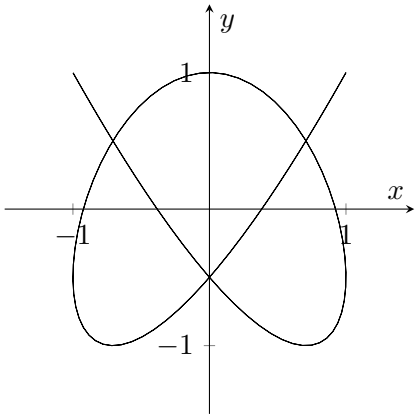


5. Consider the parametric curve defined by the equations

$$x = \cos 3\theta, \qquad y = \cos 4\theta.$$

(a) Compute $\frac{dy}{dx}$ in terms of $\theta$.

(b) The curve is plotted below, and you can see that there are three points where the tangent to the curve is horizontal and two points where the tangent to the curve is vertical. List the $(x, y)$ coordinates of these points.



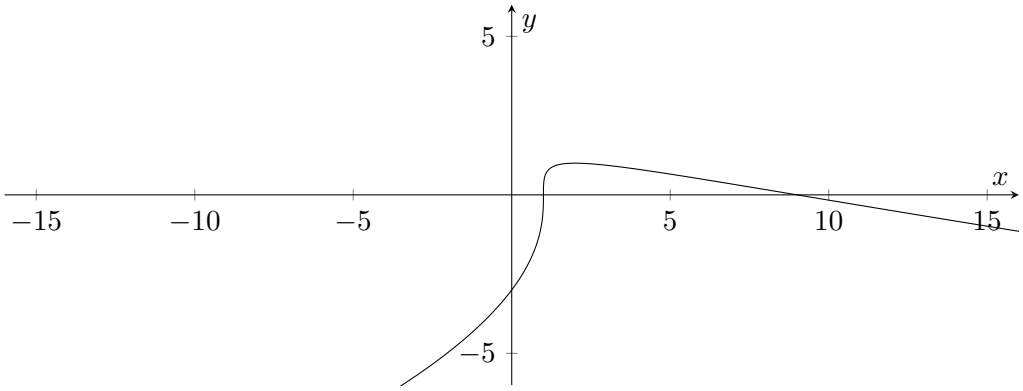

Points with horizontal tangents: _____

Points with vertical tangents: _____

6. Find the area enclosed by the $x$-axis and the curve defined by the parametric equations

$$x = t^3 + 1, \qquad y = 2t - t^2.$$

The curve is plotted below for reference.



7. Set up an integral that represents the length of the curve defined by the parametric equations

$$x = t + e^{-t}, \qquad y = t - e^{-t}, \qquad 0 \leq t \leq 2.$$

You do not need to evaluate the integral.

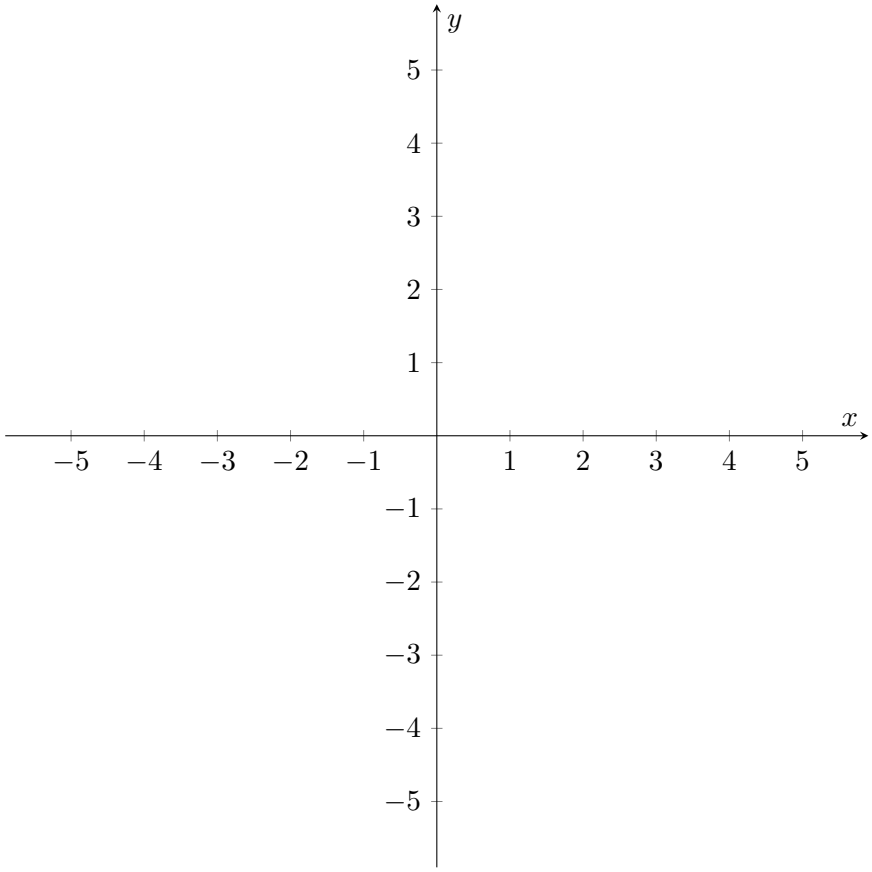
8. Consider the point $A$ with Cartesian coordinates $(-4, 4)$, and the point $B$ with Cartesian coordinates $(3, 3\sqrt{3})$.

   For each point, find *two* polar coordinates of the point.

9. Sketch the curve described by the polar equation

$$r = 1 + 3\cos\theta.$$

You may find it helpful to first make a separate graph of $r$ as a function of $\theta$ in Cartesian coordinates.



10. Find the slope of the tangent line to the curve described by the polar equation $r = 2\cos\theta$ at the point specified by $\theta = \pi/3$.