Math 243 Final

December 20, 2019

Name: _____ ID: _____

- Each page has a space at the top for the last 4 digits of your student ID. Make sure that you fill that out on at least one side of every sheet of paper.
- Show enough work that your solution would convince a skeptical peer that your answer is correct.
- The questions are ordered by topic, not by difficulty.
- Each question is worth the same number of points.
- You may not use any tools or resources other than writing implements and a 3 inch by 5 inch note card. In particular, no calculators, phones, additional notes, and so forth.

1. Set up an integral for the length of the curve described by the equations

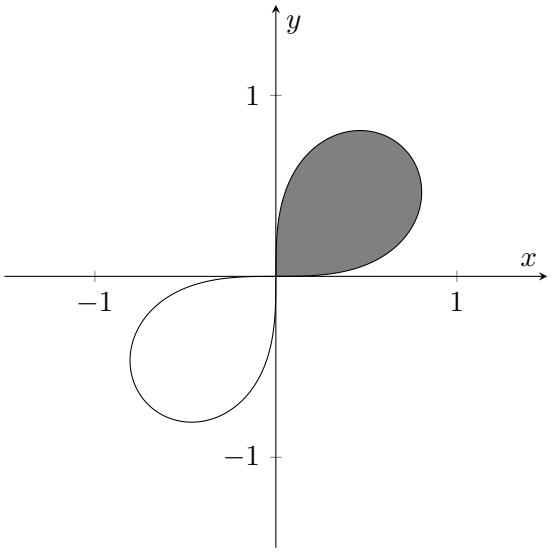
$$x = t - e^t, \qquad y = t + e^t$$

from $t = -6$ to $t = 6$. You do not need to evaluate the integral.

2. Consider the point A with Cartesian coordinates $(-4, 4)$, and the point B with Cartesian coordinates $(3, 3\sqrt{3})$.

   For each point, find *two* polar coordinates of the point.

3. Find the area of the shaded region.



$r^2 = \sin 2\theta$

4. Let $\mathbf{a} = \langle 4, 1, \frac{1}{4} \rangle$ and $\mathbf{b} = \langle 6, -3, -8 \rangle$. Compute $\mathbf{a} \cdot \mathbf{b}$.

5. Consider the space curve

$$\mathbf{r}(t) = \sin^2 t\,\mathbf{i} + \cos^2 t\,\mathbf{j} + \tan^2 t\,\mathbf{k}.$$

Find the unit tangent tangent vector at $t = \frac{\pi}{4}$.

6. Find the curvature of the graph of the function $y = xe^x$.

7. Find the velocity, acceleration, and speed of a particle with the position function

$$\mathbf{r}(t) = t\,\mathbf{i} + 2\cos t\,\mathbf{j} + \sin t\,\mathbf{k}.$$

8. Find and sketch the domain of the function

$$f(x, y) = \sqrt[4]{x - 3y}.$$

9. Use polar coordinates to find the limit

$$\lim_{(x,y)\to(0,0)} \left(x^2+y^2\right) \ln\left(x^2+y^2\right).$$

10. Let

$$f(x, y) = x^2 y - 3y^4.$$

Find f_x and f_y.

11. Find an equation of the tangent plane to the surface

$$z = e^{x-y}$$

at the point $(2, 2, 1)$.

12. Let

$$z = x^4 + x^2y, \qquad x = s - 2t - u, \qquad y = stu^2.$$

Compute $\frac{\partial z}{\partial s}$, $\frac{\partial z}{\partial t}$, and $\frac{\partial z}{\partial u}$ when $s = 4$, $t = 2$, and $u = 1$.

13. Find the equation of the tangent plane to the hyperboloid

$$xy + yz + zx = 5$$

at the point $(1, 2, 1)$.

14. Find the points on the hyperboloid $y^2 = 9 + xz$ that are closest to the origin.