Math 308 Midterm 2

April 4, 2018

Name: ____

- Show your work. If you solve a problem with anything other than a straightforward computation, write one complete sentence explaining what you're doing.
  - For example, if you're computing a cross product using the standard method, just show your computation.
  - But if, for example, you find that a line integral is zero without actually computing the line integral, you need to write one complete sentence convincing an imaginary peer that that's true.
- Use the back of the previous page for scratchwork. By default, I won't grade the scratchwork, so you can write wrong things there without penalty.
- If you run out of space on the printed page and need more space, then use the back of the previous page, but make sure to:
  - Make a note on the printed page that your work continues on the back of the previous page.
  - On the back of the previous page, put a box around the work that you want graded.
- There are four questions, worth between 15 and 35 points each.
  - The problems are ordered by topic, not by difficulty.

1. Consider the ball of radius 3 centered around the origin. The electric displacement field is

$$\mathbf{D} = x\mathbf{i} + y\mathbf{j} + \left(z + x^2 + y^2 + z^2 - 9\right)\mathbf{k}.$$

The units of x, y, and z are in meters, and the units of $\mathbf{D}$ are Coulombs/meter^2. In this problem, you will compute the total charge inside the ball in two different ways.

(a) (10 points) Compute the total charge inside the ball by evaluating an appropriate surface integral on the sphere of radius 3. Specify the units of your answer.

(b) (10 points) Compute the total charge inside the ball by evaluating an appropriate volume integral on the ball of radius 3. Specify the units of your answer.

2. Consider the surface S of the horizontal cylinder of radius 3 and length 10, described by the equations $y^2 + z^2 = 9$ and $-5 \le x \le 5$. Let

$$\mathbf{A} = -xz\mathbf{j} + xy\mathbf{k}.$$

You can compute that

$$\operatorname{curl} \mathbf{A} = 2x\mathbf{i} - y\mathbf{j} - z\mathbf{k}.$$

The two ends of the cylinder are closed circular curves. Call them γ_- and γ_+, oriented as shown. Each curve is the boundary of a disk. Call those disks D_- and D_+, and orient them using unit normals as shown. Call the round surface of the cylinder S, and orient it as shown.

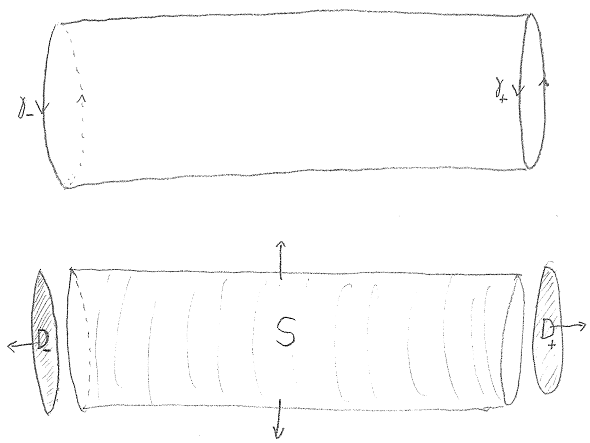

Figure 1: Please ask me if you have questions about interpreting this schematic diagram.

Be careful about orientations and signs in this problem.

(a) (15 points) Compute $\int_{D_-} \operatorname{curl} \mathbf{A} \cdot \mathbf{n}\, d\sigma$ and $\int_{D_+} \operatorname{curl} \mathbf{A} \cdot \mathbf{n}\, d\sigma$ and then explain what that tells you about $\oint_{\gamma_-} \mathbf{A} \cdot d\mathbf{r}$ and $\oint_{\gamma_+} \mathbf{A} \cdot d\mathbf{r}$.

(b) (15 points) Compute $\int_S \operatorname{curl} \mathbf{A} \cdot \mathbf{n}\, d\sigma$ and then explain what that tells you about $\oint_{\gamma_-} \mathbf{A} \cdot d\mathbf{r}$ and $\oint_{\gamma_+} \mathbf{A} \cdot d\mathbf{r}$.

(c) (0 points) For up to 5 bonus points, check your work by computing the line integrals $\oint_{\gamma_-} \mathbf{A} \cdot d\mathbf{r}$ and $\oint_{\gamma_+} \mathbf{A} \cdot d\mathbf{r}$ directly.

3. In the previous problem, you worked with the vector fields

$$\mathbf{A} = -xz\mathbf{j} + xy\mathbf{k},$$
$$\mathbf{V} = 2x\mathbf{i} - y\mathbf{j} - z\mathbf{k},$$

satisfying $\operatorname{curl} \mathbf{A} = \mathbf{V}$.

(a) (10 points) Find a vector field $\mathbf{A}'$ such that $\operatorname{curl} \mathbf{A}' = \mathbf{V}$ and such that $A'_y = e^x$.

(b) (5 points) If γ is a closed curve, do you expect $\oint_\gamma \mathbf{A} \cdot d\mathbf{r}$ and $\oint_\gamma \mathbf{A}' \cdot d\mathbf{r}$ to be equal? Why or why not?

4. (a) (25 points) Let $f(x)$ be the 2π-periodic function such that

$$f(x) = \begin{cases} 1 & \text{for } 0 < x < \frac{2\pi}{3}, \\ 0 & \text{for } \frac{2\pi}{3} < x < 2\pi. \end{cases}$$

Compute the Fourier series for f.

Guidelines for acceptable answers by way of analogy:

- Acceptable: $e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$.
- Acceptable: $e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots$. Include enough terms so the pattern is clear.
- Unacceptable: $c_n = \frac{1}{n!}$.
- Acceptable: $e^x = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$, where $c_n = \frac{1}{n!}$.

(b) (10 points) Compute the sum of the series that you found in part (a) for each of the following five values of x: $0, \frac{\pi}{3}, \frac{2\pi}{3}, 2\pi$, and 3π.

Your answer for each value of x should be a number. It is neither sufficient nor required to write down a series.

Question	Points	Score
1	20	
2	30	
3	15	
4	35	
Total:	100	