Math 308 Midterm 1

February 21, 2018

Name: _____

- Show your work. If you solve a problem with anything other than a straightforward computation, write one complete sentence explaining what you're doing.
  - For example, if you're computing a cross product using the standard method, just show your computation.
  - But if, for example, you find that a line integral is zero without actually computing the line integral, you need to write one complete sentence convincing an imaginary peer that that's true.
- Use the back of the previous page for scratchwork. By default, I won't grade the scratchwork, so you can write wrong things there without penalty.
- If you run out of space on the printed page and need more space, then use the back of the previous page, but make sure to:
  - Make a note on the printed page that your work continues on the back of the previous page.
  - On the back of the previous page, put a box around the work that you want graded.
- There are five questions, worth between 10 and 40 points each.
  - Problems worth more points are likely to take more time, but won't necessarily be more difficult.
  - The problems are ordered by topic.

1. Let $\mathbf{A} = -3\mathbf{i} - 2\mathbf{k}$. Let $\mathbf{B} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

   (a) (10 points) Compute $\mathbf{A} \cdot \mathbf{B}$ and $\mathbf{A} \times \mathbf{B}$.

   (b) (10 points) Let $\mathbf{A}'$ and $\mathbf{B}'$ be the reflections of $\mathbf{A}$ and $\mathbf{B}$ across the yz-plane. Compute $\mathbf{A}' \cdot \mathbf{B}'$ and $\mathbf{A}' \times \mathbf{B}'$.

   Don't forget to justify your answer with one complete sentence if needed.

2. (15 points) The comet 67P completes a full rotation every 12.4 hours. For a safe landing, the velocity vector of the lander should be close to the velocity vector of the point on the comet that it's trying to land on as the comet rotates. In this problem, you'll determine that velocity vector.

   For the purposes of the problem, approximate this rotation speed as 0.5 radians/hour, and set up the coordinate system so that the axis of rotation is the z-axis and the rotation direction is counterclockwise in the xy-plane. For the purposes of this problem, the landing site will be at $2\mathbf{i} - 4\mathbf{j} + \mathbf{k}$, with units in kilometers.

   What is the velocity vector of the landing site due to the comet's rotation? Specify the units of your answer.

3. (10 points) Let $\mathbf{A} = 3\mathbf{i} + 3\mathbf{j}$, let $\mathbf{B} = -2\mathbf{i} + \mathbf{k}$, and let $\mathbf{C} = -3\mathbf{j} - 3\mathbf{k}$. Compute $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$. Don't forget to justify your answer with one complete sentence if needed.

4. Let $\mathbf{V}$ be the vector field $-xy\,\mathbf{i} + y^2\,\mathbf{j} - xy\,\mathbf{k}$.

   (a) (10 points) Compute $\operatorname{div}\mathbf{V}$.

   (b) (5 points) Interpreting $\mathbf{V}$ as the velocity vector field of a gas, describe the region, if any, where the gas is expanding. Describe the region, if any, where the gas is contracting.

5. Consider the two force fields

$$F_1 = y\mathbf{i} + x\mathbf{j} + \mathbf{k},$$
$$F_2 = \mathbf{i} + y\mathbf{j} + x\mathbf{k}.$$

(a) (10 points) Exactly one of the two force fields is conservative. Which one is it?

(b) (15 points) For the conservative force field, compute a scalar potential. That is, find a scalar field ϕ such that $\mathbf{F} = -\nabla\phi$.

Note the negative sign. This ϕ is the same as the U we talked about in class, and the negative of the W we talked about.

(c) (15 points) For the other force field, compute

$$\oint_\gamma \mathbf{F} \cdot d\mathbf{r},$$

where γ is the circle of radius 2 in the xy-plane centered at $(x, y, z) = (2, 2, 0)$, oriented counterclockwise.

Question	Points	Score
1	20	
2	15	
3	10	
4	15	
5	40	
Total:	100	