TOPOLOGY MIDTERM 1 STUDY GUIDE

Here is a list of things we've covered in class or on the homework, both of which are fair game. Obviously, not all of them will fit into a one-hour exam. Also, it's possible that I missed something we've covered, in which case it might still be on the exam. If there's something conspicuously absent from this list, let me know, and I'll tell you which of the following two categories it's in.

Know this stuff

I think that most mathematicians would be able to define most of the things on this list from memory and work with these concepts, so you should be able to do so, too.

- Basic set theory: subset, union, intersection, arbitrary union and intersection, power set, cartesian product, difference of sets, complement.
- Functions: domain, codomain, (direct) image, inverse image, composition, injective, surjective, bijection.
- Equivalence relation, equivalence class, set of equivalence classes.
- (Total) order, partial order, order-preserving map, order isomorphism, lexicographic order, least upper bound (supremum), greatest lower bound (infimum), least upper bound property, smallest (least) element, largest (greatest) element.
- Topology, topological space, open set, closed set.
- Continuous function, homeomorphism.
- The standard topology on $\mathbb{R}^n$.
- The discrete topology on X, the trivial topology on X, the cofinite topology on X.
- Closure, interior, boundary.
- Convergence of a sequence in a topological space.
- Open cover.
- Open map, closed map.
- Subspace topology, relatively open, relatively closed.
- Quotient topology, quotient map (projection).
- Hausdorff (T_2) space.
- Connected topological space.
- The circle S^1, the sphere S^2, the torus T^2, the Möbius strip, the Klein bottle, the real projective plane $\mathbb{RP}^2$, the real projective line $\mathbb{RP}^1$.

Be able to work with this stuff

Here are other topics we've covered, which many mathematicians will vaguely remember from when they took topology, so you should be able to work with them if reminded of the definitions.

- Pullback of an equivalence relation.
- Maximal element, minimal element.
- Immediate predecessor.
- T_0 (Kolmogorov), T_1 (Fréchet), T_3 (regular Hausdorff), T_4 (normal Hausdorff), T_6 (perfectly normal Hausdorff).
- Right order topology.
- Alexandrov topology, upper set, specialization order.

What to expect on the exam

Each question will be some three or four-part subset of the following megaquestion.

(1) (a) Define the thing t.
(b) Define another thing t'.
(c) Assuming assumption a, prove something about t and/or t'.
(d) What part of your proof fails if a does not hold?
(e) Provide a counterexample when a does not hold.
(f) Prove another thing about t and/or t'.
(g) Conclude something about concept c, where here is the definition of c.

Most of the questions will be comparable in difficulty to the easier questions on the homework. Some of the questions might be comparable to the medium questions on the homework. None of the questions will be comparable to the hard questions on the homework.