

TOPOLOGY HOMEWORK 12, DUE NOVEMBER 21

- (1) Do problem 20.9 on page 128. The hint is best interpreted as, use $\|a\mathbf{x} \pm b\mathbf{y}\|^2 \geq 0$. It's easier to compute things if you square the norm.
- (2) Do problem 20.10 on page 128.
- (3) Do problem 21.1 on page 133. For this and subsequent problems, Lemma 13.2 is a good tool to use. When using it, don't forget to check that the sets in your collection are actually open.
- (4) Do problem 30.1 on page 194. Make sure your proof fails without the T_1 axiom.
- (5) Do problem 30.2 on page 194. Be careful that "contains" means a subcollection of sets in \mathcal{C} , not a collection of subsets of sets in \mathcal{C} .
- (6) Do problem 30.5(a) on page 194. A subset A of a topological space X is *dense* if $\overline{A} = X$. The rationals in \mathbb{R} are an example, as are the irrationals. I'd recommend drawing a picture to get a sense for how small the balls need to be.
- (7) Do problem 30.6 on page 194 for \mathbb{R}_l only, which is defined on pages 81–82. Hint: Use the previous exercise.
- (8) Do problem 30.10 on page 194. Warning: A countable product of countable sets is not countable. A trick we saw in class will help.
- (9) Next homework: Do problem 30.12 on page 194. Make sure to notice that f is assumed to be open, that is, the image of open sets is open.
- (10) Next homework: Do problem 30.13 on page 194.