Differential Equations Homework 7

Due October 19

Instructions

1. Write down the names of the people you worked with.
2. Write down any resources you used other than ones that most of your classmates would be familiar with, such as Wikipedia or Wolfram Alpha.
3. Write down at the top of your submission for part 1, separately, the number of hours it took you to complete this hand-graded assignment, and the number of hours it took you to complete the corresponding Webwork.
4. Write your name, Math 217, and the homework number.
5. Hand in your homework in class.
6. You'll be handing in your solutions to parts 1, 2, and 3 to separate piles to go to separate graders. Make sure they're on separate sheets of paper.
7. Unless directed otherwise, show enough work to convince a classmate that disagrees with you that you're right and they're wrong. Answers alone will usually receive no credit.

Problems

Part 1

1. (10 points) (a) (5 points) Write down an exact equation whose general solution on the domain $x > 0$ is
$$y = \frac{1}{x}\ln(x^3 + C).$$
If needed, check your answer by computing $\frac{dy}{dx}$ for the above solution and plugging it in to your differential equation.

   (b) (5 points) Check that your differential equation is exact using the criterion for exactness.

2. (15 points) (a) (3 points) Write down a fourth-order polynomial with integer coefficients so that $\sqrt{2} + \sqrt{5}$ is one of its roots. Hint: Let $x = \sqrt{2} + \sqrt{5}$. Compute the powers of x and combine them to get rid of the square roots.

(b) (3 points) Compute the derivatives of $y = x\cos 2x$ up to the fourth order. Group terms, and combine appropriate multiples of your expressions to cancel all terms involving x. Based on your work, write down a fourth-order homogeneous linear equation with constant coefficients so that $y = x\cos 2x$ is one of its solutions.

(c) (3 points) Write down a first-order homogeneous linear equation so that $y = x\cos 2x$ is one of its solutions, at least on the domain $0 < x < \frac{\pi}{4}$. Write your answer in the standard form of a first-order linear equation, and simplify it as much as possible. Hint: One of the requirements of the previous part is not present here.

(d) (3 points) Write down the general solution of your equation in part c on the domain $0 < x < \frac{\pi}{4}$.

(e) (3 points) Write down a second-order homogeneous linear equation with constant coefficients so that $y = 42e^{3x} - 60e^{-5x}$ is one of its solutions. Hint: Don't use the method of part b. Instead, use your understanding of solutions to second-order homogeneous linear equations.

Part 2

3. (15 points) (a) (5 points) Do problem 3.1.17 on page 147 of the textbook.

   (b) (5 points) Do problem 3.1.18 on page 147 of the textbook.

   (c) (5 points) Do problem 3.1.19 on page 147 of the textbook.

4. (15 points) Do problem 3.1.30 on page 147 of the textbook.

5. (15 points) Do problem 3.1.32 on page 147 of the textbook.

Part 3

6. (15 points) (a) (8 points) Do problem 3.1.49 on page 148 of the textbook.

   (b) (7 points) Do problem 3.1.50 on page 148 of the textbook.

7. (15 points) (a) (5 points) Do problem 3.1.51 on page 148 of the textbook. Hint: Carefully apply the chain rule and the product rule. Those rules only apply to single derivatives, not second derivatives, so when you get to $\frac{d^2y}{dx^2}$, think of $\frac{dy}{dx}$ as a function, and write $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$. It sounds trite, but if you avoid writing false things and write down true things regardless of whether they seem helpful or not, you will eventually write down enough true things to let you solve the problem.

   (b) (5 points) Do problem 3.1.54 on page 148 of the textbook.

   (c) (5 points) Do problem 3.1.56 on page 148 of the textbook.