Differential Equations Homework 6

Due October 10

Instructions

1. Write down the names of the people you worked with.
2. Write down any resources you used other than ones that most of your classmates would be familiar with, such as Wikipedia or Wolfram Alpha.
3. Write down at the top of your submission for part 1, separately, the number of hours it took you to complete this hand-graded assignment, and the number of hours it took you to complete the corresponding Webwork.
4. Write your name, Math 217, and the homework number.
5. Hand in your homework in class.
6. You'll be handing in your solutions to parts 1, 2, and 3 to separate piles to go to separate graders. Make sure they're on separate sheets of paper.
7. Unless directed otherwise, show enough work to convince a classmate that disagrees with you that you're right and they're wrong. Answers alone will usually receive no credit.

Problems

Part 1

1. Do problem 1.5.45 on page 55 of the textbook. There appears to be an erratum in this question. The question at the end should instead ask for how long it takes for the pollutant *amount* to reach 10 million liters.
2. Do problem 1.5.46 on page 55 of the textbook. Again, the question at the end should instead ask for how long it takes for the pollutant amount to reach 10 million liters.

Part 2

3. Do problem 1.5.42 on page 54 of the textbook. You may need to recall the formula for the volume of a sphere and the definition of density. Because this problem ignores units, you can, too.
4. Do Problem 1.6.49 on page 69 of the textbook.

Part 3

5. (a) Find an implicit general solution to the separable differential equation

$$y' = 6x^2 \sec y.$$

   (b) Rewrite the above differential equation in the form $M\,dx + N\,dy = 0$, where M only depends on x, and N only depends on y, and check that the differential equation you wrote down is exact.

   (c) Solve your differential equation using our method for solving exact equations. Check that your answer matches the one from part a. As a result, we can think of separable equations as a special case of exact equations.

6. (a) Find a general solution to the differential equation

$$dy + 2xy\,dx = e^{-x^2} \sin x\,dx.$$

   using our method for solving linear equations.

   (b) Write down the above equation in the form $M\,dx + N\,dy = 0$, and multiply through by the same integrating factor. Check that the resulting equation is exact.

   (c) Solve this equation using our method for solving exact equations, and check that you got the same answer as in part a.

7. As we saw in the previous question, sometimes we can find an integrating factor to turn an equation that is not exact into an equation that is exact. We can't do that for an arbitrary equation of the form $M\,dx + N\,dy = 0$, but there are two general cases where we can find such an integrating factor.

   Let

$$D = \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x},$$

   which is the difference of the "mixed partials." You might also recognize this expression as the curl of the vector field $\left(\begin{smallmatrix} M \\ N \end{smallmatrix}\right)$.

   As we know from class, if $D = 0$, then our equation is already exact. However, even if $D \neq 0$, if it so happens that the expression $\frac{D}{N}$ is a function of x only and does not depend on y, then we can use

$$e^{\int \frac{D}{N}\,dx}$$

   as our integrating factor. That is, if we multiply our differential equation by that integrating factor, the equation will become exact. Likewise, if $\frac{D}{M}$ is a function of y only and does not depend on x, then we can use

$$e^{-\int \frac{D}{M}\,dy}$$

   as our integrating factor. (Notice the negative sign.)

   (a) Consider the differential equation

$$dy + \left(2xy - e^{-x^2} \sin x\right)\,dx = 0,$$

which is the same as the differential equation from problem 6. Use the method outlined in this question to find the integrating factor, and make sure it matches with the integrating factor you used in problem 6. As a result, we can also think of linear equations as a special case of differential equations that can be made exact using an integrating factor.

(b) Use an integrating factor to find an implicit solution of the differential equation

$$(3x^2y + y^2)\,dx + (3x^3 + y^2 + 4xy)\,dy = 0.$$

Remember that you can use our test for exactness to check that you've multiplied by the integrating factor correctly before going on the rest of the problem.