Differential Equations Homework 14

Due December 5

Instructions

1. Write down the names of the people you worked with.
2. Write down any resources you used other than ones that most of your classmates would be familiar with, such as Wikipedia or Wolfram Alpha.
3. Write down at the top of your submission for part 1, separately, the number of hours it took you to complete this hand-graded assignment, and the number of hours it took you to complete the corresponding Webwork.
4. Write your name, Math 217, and the homework number.
5. Hand in your homework in class.
6. You'll be handing in your solutions to parts 1, 2, and 3 to separate piles to go to separate graders. Make sure they're on separate sheets of paper.
7. Unless directed otherwise, show enough work to convince a classmate that disagrees with you that you're right and they're wrong. Answers alone will usually receive no credit.

Problems

Part 1

1. Do problem 3.8.2 on page 226.
2. Do problem 3.8.6 on page 226.

Part 2

3. Do problem 3.8.13 on page 227.
4. Do problem 3.8.14 on page 227. The book has an error and does not list all of the eigenvalues. You should find all of the eigenvalues.

Part 3

5. (a) Do problem 5.2.29 on page 293.

   (b) The book asks you to plot graphs of x_1 and x_2 with respect to time. In addition, provide a parametric phase plot of your solution in the x_1–x_2 plane. Label the solution to the initial value problem, and then draw a few additional solutions to the differential equation. Make sure to include arrows specifying the direction of motion. The solution curves should look special, because for any trajectory, the total amount of salt, $x_1 + x_2$, is constant through time.

   In addition, there is a family of constant solutions. The solutions correspond to initial conditions where the two tanks have equal concentrations, and so the amount of salt in each tank does not change. Draw this family of constant solutions. Clearly label it, so that there is no confusion about which of your curves are trajectories and which curve is the family of constant solutions.

   The phase plotter links on the course website may deceive you, so make your sketch by hand using the general solution you found. Some things to think about to help you draw the solution to the initial value problem: Where does the solution start at $t = 0$? In which direction does $\mathbf{x}(t)$ move? What value does it approach as t becomes large?

6. Do problem 5.2.38 on page 294.