Differential Equations Homework 12

Due November 21

Instructions

1. Write down the names of the people you worked with.
2. Write down any resources you used other than ones that most of your classmates would be familiar with, such as Wikipedia or Wolfram Alpha.
3. Write down at the top of your submission for part 1, separately, the number of hours it took you to complete this hand-graded assignment, and the number of hours it took you to complete the corresponding Webwork.
4. Write your name, Math 217, and the homework number.
5. Hand in your homework in class.
6. You'll be handing in your solutions to parts 1, 2, and 3 to separate piles to go to separate graders. Make sure they're on separate sheets of paper.
7. Unless directed otherwise, show enough work to convince a classmate that disagrees with you that you're right and they're wrong. Answers alone will usually receive no credit.

Problems

Part 1

1. Do problem 3.6.24 on page 207.
2. Do problem 3.6.27 on page 207.
3. Do problem 3.6.28 on page 207.

Part 2

4. Do problem 3.7.1 on page 215.
5. Do problem 3.7.2 on page 215.
6. Do problem 3.7.3 on page 215.

7. Do problem 3.7.4 on page 215.

8. Do problem 3.7.5 on page 215.

9. Do problem 3.7.7 on page 215.

10. Do problem 3.7.8 on page 215.

11. Do problem 3.7.9 on page 215.

12. Do problem 3.7.10 on page 215.

Part 3

13. (a) Do problem 3.7.13 on page 215.

    (b) Do problem 3.7.21 on page 216.

14. Consider a standard resistor-inductor-capacitor (RLC) circuit. The charge on the two plates of the capacitor creates an electric field between them. Such an electric field has potential energy $U_C = \frac{1}{2}\frac{Q^2}{C}$, where Q is the charge on the capacitor. Meanwhile, current flowing through the coil of wire of the inductor creates a magnetic field inside the inductor. The magnetic field has potential energy $U_L = \frac{1}{2}LI^2$, where I is the current flowing through the inductor. The power supplied by the voltage source is $E(t)I$. (If $E(t)$ and I have opposite signs, then the voltage source removes energy from the circuit.) The power dissipated by the resistor is $V_R I = RI^2$. Use the differential equation
$$L\frac{dI}{dt} + RI + \frac{Q}{C} = E(t)$$
to show that the rate of change of the total energy $U_C + U_L$ is equal to the power supplied by the voltage source, $E(t)I$, minus the power dissipated by the resistor, $V_R I$.