Differential Equations Homework 11

Due November 14

Instructions

1. Write down the names of the people you worked with.
2. Write down any resources you used other than ones that most of your classmates would be familiar with, such as Wikipedia or Wolfram Alpha.
3. Write down at the top of your submission for part 1, separately, the number of hours it took you to complete this hand-graded assignment, and the number of hours it took you to complete the corresponding Webwork.
4. Write your name, Math 217, and the homework number.
5. Hand in your homework in class.
6. You'll be handing in your solutions to parts 1, 2, and 3 to separate piles to go to separate graders. Make sure they're on separate sheets of paper.
7. Unless directed otherwise, show enough work to convince a classmate that disagrees with you that you're right and they're wrong. Answers alone will usually receive no credit.

Problems

Part 1

1. (25 points) Do problem 3.5.43 on page 195.
2. (25 points) (a) (15 points) Do problem 3.5.55 on page 196 using the method of variation of parameters. Also, write down the general solution. Hint: The problem may be easier to simplify if you use a trig identity to rewrite $\sin^2 x$ in terms of $\cos 2x$ and 1, and then later recognize where you can use $\sin 4x$ and $\cos 4x$. Once you have your answer, you can find a way to simplify by rewriting all of the trig terms in terms of $\cos 2x$ and $\sin 2x$.
   (b) (10 points) Do the same problem using the method of undetermined coefficients, using the same trig identity to rewrite $\sin^2 x$ as a linear combination of $\cos 2x$ and 1.

Part 2

3. (20 points) Do problem 3.6.12 on page 206. Make sure to write the steady periodic response in the form required in terms of the amplitude and phase angle. The problem doesn't require that you do that for the transient solution, but it might be useful in order to check your work against the back of the book.

4. (10 points) Do problem 3.6.20 on page 206.

Part 3

5. (10 points) Do problem 3.6.21 on page 206. Hints: There are several ways to set up this problem. I recommend using the pendulum angle θ as the dependent variable, since it's best to deal with the more complicated part of the problem first. Start with the equation for the pendulum, but make sure your equation is in terms of force. That is, it should have a $mg\sin\theta$ term, the net force due to gravity. Approximate $\sin\theta \approx \theta$. It will be difficult to do the full problem if you just use $\ddot{\theta} + \frac{g}{L}\theta = 0$.

   Then, incorporate the force from the spring. Again, approximating $\sin\theta \approx \theta$, determine the horizontal displacement of the mass, and thus the force the spring provides.

6. (10 points) Do problem 3.6.23 on pages 206–207.