DIFFERENTIAL EQUATIONS HOMEWORK 1, DUE SEPTEMBER 9

Instructions

(1) Write down the names of the people you worked with.
(2) Write down any resources you used other than ones that most of your classmates would be familiar with, such as Wikipedia or Wolfram Alpha.
(3) Write down the number of hours it took you to complete this assignment, including Webwork.
(4) Write your name, Math 217, and the homework number.
(5) Hand in your homework in class.

Problems

(1) Read the syllabus. Then write down that you read the syllabus.
(2) As you know, the derivative of e^x is e^x, so e^x satisfies the equation $f'(x) = f(x)$.
   (a) Does the constant function $f(x) = 0$ satisfy the equation $f'(x) = f(x)$? Show that your answer is correct. (Showing that your answer is correct means writing down an explanation that would convince a classmate that disagrees with you that you're right and they're wrong. Depending on the question, and the classmate, the explanation might be very long, or very short.)
   (b) Does the constant function $f(x) = 1$ satisfy the equation $f'(x) = f(x)$? Show that your answer is correct.
   (c) Write down two other functions that are their own derivatives, that is, they satisfy the equation $f'(x) = f(x)$.
   (d) For each of the functions you wrote down, what is $f(0)$?
   (e) Let C be a number. Write down a function that satisfies $f'(x) = f(x)$ and $f(0) = C$. Your answer should be in terms of C.
(3) (a) Verify that x^3 satisfies the equation $xf'(x) = 3f(x)$. (Again, to verify a claim, you should write down an explanation that would convince a classmate that disagrees with the claim that they're wrong and you're right.)
   (b) Write down two other functions that satisfy the equation $xf'(x) = 3f(x)$.
   (c) For each of the functions you wrote down, what is $f(0)$?
   (d) Write down two functions that satisfy the equation $xf'(x) = 5f(x)$.
   (e) Let k be a nonnegative whole number. Write down two functions that satisfy the equation $xf'(x) = kf(x)$. Your answer should be in terms of k.
(4) Consider a commute from home at $x = 0$ to work at $x = 5$ miles. Assume that the commute is in a straight line, and that there is one stop sign and one traffic light along the way. On the way to work, the traffic light was red, and on the way home, it was green.
   (a) Draw a graph of the displacement of the car as a function of time, from the time the driver puts the key in the ignition at the beginning of the day, until the end of the day when driver returns home and turns off the engine.
   (b) Directly below it, or on a new page, draw a second graph of the velocity of the car as a function of time.
   (c) Below that, or on a new page, draw a third graph of the acceleration of the car as a function of time.

Tips:

- Make your graphs large enough so that all features are clearly visible.
- Because for most of the day the car is sitting at work and the graphs would be boring, use scale breaks, also known as a broken axis, to omit that part of the graph.
- To decide how realistic and precise the graph needs to be, and how much detail to include in the labels of axes and other features, imagine that you hand your graphs to a friend who isn't in this class, but don't hand them the homework question. They should be able to answer all of the following questions based on your graph. In parentheses is the answer required by this assignment. For questions without an answer in parentheses, the answers should be
  (a) realistic, and
  (b) consistent with each other.

  Questions:
  1. What is this all about? (A commute.)
  2. A commute where? (From home to work and back.)
  3. What are these graphs of? (The displacement, velocity, and acceleration of the car.)
  4. What's this over here? (A stop sign.)
  5. And that? (A traffic light.)
  6. How far is work from home? (5.)
  7. 5 kilometers? (No, miles.)
  8. Was the traffic light red or green? (Red on the way there, green on the way back.)
  9. How far along the commute is the stop sign?
  10. How far along the commute is the traffic light?
  11. How long did the commute take on the way there?
  12. How long did it take on the way back?
  13. How long did the driver spend at work?
  14. What was the fastest speed of the car on the way there?
  15. What was the fastest speed on the way back?
  16. Did the driver roll through the stop sign?
  17. How long did the driver wait at the red light?
  18. During the course of the day, when was the velocity positive and when was it negative?
  19. How about the displacement?
  20. And the acceleration?

  Note: You do not need to write down the answers to these questions yourself, but the answers must be
  (a) something your aforementioned imaginary friend could read from your graph,
  (b) realistic, and
  (c) consistent with each other.