1. Find the topological entropy for each of the maps below. Justify your answer.
   
   (a) An expanding map of the circle of degree $m \geq 2$.
   
   (b) A translation $T_{\alpha, \beta}(x, y) = (x + \alpha, y + \beta) \pmod{1}$ on the torus $\mathbb{T}^2$.
   
   (c) The map $f : [0, \pi] \to [0, \pi]$ given by $f(x) = \frac{1}{3} \sin(x)^2$.

2. Let $X$ and $Y$ be compact metric spaces and let $f : X \to X$ and $g : Y \to Y$ be continuous maps. Show that if $g$ is a factor of $f$, then $h(g) \leq h(f)$.

   Recall that $g$ is a factor of $f$ if there exists a continuous surjective map $H : X \to Y$ such that $H \circ f = g \circ H$. Hint: use an argument similar to the one we used in class to show that the entropy is the same for two metrics generating the same topology.

3. Let $X$ be a compact metric space and let $f, g : X \to X$ be continuous maps.

   (a) Show that $h(f^2) = 2 h(f)$, where $f^2 = f \circ f$.

   (b) Give an example of $f$ and $g$ such that $h(f \circ g) < h(f) + h(g)$.

4. Show that the topological entropy of the full two-sided shift $\sigma : \Omega_m \to \Omega_m$, $m \geq 2$, equals $\log m$. Use the metric $d_\lambda(\omega, \omega') = \sum_{i=\infty}^\infty \frac{\delta(\omega_i, \omega'_i)}{\lambda^{|i|}}$ with $\lambda > 3$.

Extra credit problem

5*. Consider the space of all two-sided sequences of numbers in the interval $[0, 1]$:

$$\Omega_{[0,1]} = \{(\omega_i)_{i=-\infty}^\infty \mid \omega_i \in [0, 1]\}$$

with the metric $d_\lambda(\omega, \omega') = \sum_{i=-\infty}^\infty \frac{|\omega_i - \omega'_i|}{\lambda^{|i|}}$.

Show that the topological entropy of the shift $\sigma : \Omega_{[0,1]} \to \Omega_{[0,1]}$ is infinite.