Stock Return Dynamics under Earnings Management∗

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Abstract

This paper explores how earnings management influences asset returns and return volatility via real economic activity. In the model, firms smooth earnings via the costly and economically suboptimal intertemporal transfer of assets and liabilities. As a result, the firm’s stock return follows a process that conforms to an EGARCH-like statistical model. The key idea is that real earnings management generates an unobservable cost, and the market has to infer the underlying wealth of the firm from the smoothed reported earnings series. This framework may help explain why asset returns underreact to good news and overreact to bad news, while no news is always good news to the market. Empirical evidence that earnings innovations impact future return volatility, in line with the model’s predictions, is found in the data.

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Introduction

The practice of earnings management is commonplace in the business world. The Duke survey and in-depth interviews on corporate financial reporting (Graham, Harvey and Rajgopal (2005)) find that, among other accounting information that firms disclose to the market, earnings, especially earnings per share, is the most important metric. Corporate managers are apparently willing to sacrifice real economic value for a stable reported earnings series. They also find that “managers make voluntary disclosures to reduce information risk associated with their stock but at the same time try to avoid setting a disclosure precedent that will be difficult to maintain.” A typical statement in favor of this practice is provided by Hepworth (1953): “Certainly the owners and creditors of an enterprise will feel more confident toward a corporate management which is able to report stable earnings than if considerable fluctuation of reported earnings exists.” Firms with stable reported earnings generally have more analysts following them and stocks with more analysts’ coverage are generally more liquid.\footnote{Lang and Lundholm (1996) document that firms with stable reported earnings streams and increased disclosure have greater analyst following. Brennan and Subrahmanyam (1995) and Roulston (2003) find a positive association between analyst following and liquidity. Mikhail, Walther, and Willis (2004) find that firms with repeated large earnings surprises, both positive and negative, experience a higher cost of equity capital after controlling for other determinants of the cost of capital. This is only part of the now lengthy literature on managerial preference for smoothed earnings reports.}

Hence seeing a volatile earnings process, managers will try to report a smoothed version. A large accounting literature documents the practice of real earnings management, which occurs when managers sacrifice firms’ present value to increase or decrease reported earnings via real economic activities.\footnote{Bange, De Bondt and Shrider (2005) find that U.S. corporations with large R&D budgets lower or boost R&D spending with an eye toward annual earnings targets. Among others, Levitt (1998), Graham, Harvey and Rajgopal (2005), Gunny (2005) and Kasznik and McNichols (2005) document that managers are willing to sacrificing real economic value for a smoothed reported earnings series.}

This paper develops a rational expectations earnings-smoothing model to analyze the implications of real earnings management for asset returns and return volatilities. This paper takes managers’ desire to smooth earnings as given instead of deriving it from first principles. Real earnings management is achieved through real economic activities, such as decelerating or accelerating sales, deferring maintenance, delaying desirable investment, estimating pension liabilities, and selling fixed assets. In contrast to accounting earnings management, real earn-\footnote{The past few decades have seen a voluminous research in leading accounting journals examining the relationship between financial statement information and the capital market (capital markets research) and investigating the relation between stock market values, or changes in values, and accounting numbers for the purpose of assessing the use of those numbers in an accounting standard (“value-relevance” literature). (See Kothari (2001) and Holthausen and Watts (2001).) However, the impact of accounting information on asset return volatility has not been thoroughly analyzed either theoretically or empirically by the capital markets research or the value-relevance literature. The current accounting research focuses more on the first moment of asset returns.}
ings management consumes real resources. Both increasing and decreasing the current period’s reported earnings decrease the firm’s present value, and this smoothing cost is unobservable to the market. The fact that earnings have both a persistent component and transient component, together with the unobservability of the smoothing cost, means that the market has to apply a Kalman filter to estimate the persistent component of earnings before it can rationally price the firm. Instead of exogenously assuming the firm’s cash flow follows some stochastic process, this paper endogenously derives the stochastic properties of the firm’s underlying cash flow under the practice of real earnings management. The analytic solutions for the return and the conditional volatility processes are obtained accordingly.

The past two decades have seen a widespread use of GARCH-type statistical models to characterize the conditional volatility process. It is well established that return volatility changes stochastically over time in the following manner: (1) volatility is predictable, (2) both good news and bad news lead to higher volatility (Engel (1982); Bollerslev (1986); and Bollerslev, Chou, and Kroner (1992)), and (3) return volatility exhibits an “asymmetric” response with bad news leading to more future volatility than good news. From this point on, this paper will refer to volatility’s asymmetric response to news as volatility smirk. The cited studies and numerous others in the GARCH literature have led to a situation in which statistical knowledge about conditional volatility of asset returns has greatly surpassed our theoretical understanding of the process. It is their practical utility that justifies the GARCH-type statistical models.

As Engle (2004) points out, “Volatility clustering is simply clustering of information arrivals. The fact that this is common to so many assets is simply a statement that news is typically clustered in time.” The income smoothing model proposed in this paper clearly demonstrates that it is the news about the fundamentals (i.e., earnings) that is clustered in time. This model theoretically proves that the conditional volatility of the return under income smoothing clusters in a way that is similar to the clustering of an EGARCH-type statistical model.

Moreover, this paper is the first in the literature empirically documenting that both positive and negative earnings news measured by the square of standardized unexpected earnings

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4 For example, Black and Scholes (1973), Merton (1974), Black (1976), Christie (1982) and many others label this well-documented phenomenon the “leverage” effect. As the stock price drops (typically this is what happens after a bad news announcement), the leverage of the firms increases (holding the total debt fixed), which in turn increases the future equity volatility. Another line of literature (Pindyck (1984), French, Schwert, and Stambaugh (1987) and Campbell and Hentschel (1992)) proposes the volatility feedback effect as an alternative explanation. An increase in the future stock market volatility raises the required return, which lowers the current stock prices accordingly. Although both the leverage effect and the volatility feedback effect seem plausible, Christie (1982) and Schwert (1989) find that the leverage effect is too small to account for the asymmetry in volatility, Figlewski and Wang (2004) find the leverage effect is not really a leverage effect, and Campbell and Hentschel (1992) find that the volatility feedback effect normally has little impact on returns.
increases future return volatility, while bad earnings news raises future volatility more than good earnings news does. It also finds that for firms with more asymmetric information regarding their earnings, measured by the dispersion of analysts’ forecast, earnings news increases future volatility more than it does for firms whose earnings are less opaque to the market. For firms with less dispersion in analysts’ forecasts, earnings news does not seem to affect future volatility much. The volatility smirk effect is only exhibited for stocks with the highest dispersion of analysts’ forecasts. Similar patterns are also displayed when stocks are divided into different groups based on the balance of power between shareholders and management. The impact of earnings news on conditional volatility monotonically increases as the management power increases. Earnings innovations do not move future volatility for firms with the strongest shareholder power. The volatility smirk effect only presents for firms with the strongest management power.

In the process of estimating the persistent part of a firm’s earnings, the model shows that the market may seem to overreact to bad news and underreact to good news. Under real earnings management, managers always try to hide bad news and spread out large positive shocks. A bad piece of earnings news thus reflects even worse prospects for the firm, since it implies that the managers really cannot find another “penny” to hide the bad news. On the other hand, good news indicates the possibility of managers’ wasting resources to spread out good realizations. The model also confirms one of the predictions of Campbell and Hentschel (1992) in which the volatility feedback effect is examined (i.e., no news is always good news to the market).

Rational Bayesian models attempt to explain the GARCH-type conditional volatility modeling through a gradual process of learning. However, those models predict lower volatility after good news which contradicts the stylized findings in the conditional volatility literature (see David (1997) and Veronesi (1999)). What generates volatility clustering in the model proposed here is the widely accepted assumption in the accounting literature that earnings have a permanent component and a transient component, and the commonplace fact that the market can only infer the income smoothing cost. Because of the unobservability of the persistent earnings and the smoothing cost, in order to price the asset rationally, the market has to use all available information to filter out the noise part of the earnings. When past and current reported earnings are used in the Kalman filter and the noise is screened out, what remains is an estimate of permanent earnings and the unobservable smoothing cost. The result of this process is a clustering of return volatility. Other than the behavioral model developed by Mc-

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5Standardized unexpected earnings is measured by the normalization of the difference of the actual reported earnings and the median of analysts’ forecast by the stock price.
7Report either more earnings or less will induce a cost. This paper assumes a quadratic cost for every unit of smoothed earnings.
Queen and Vorkink (2004), there is no other model in the current literature that can explain all the stylized facts about the conditional volatility process. The present model differs from the McQueen and Vorkink (2004) model in that it does not rely on behavioral assumptions but instead directly explores the impact of economic fundamentals on return volatility.

Managers solve a dynamic programming problem to come up with the reported earnings process. The market then uses that solution to determine the cash flow of the firm. The firm is priced according to the underlying cash flow process. The analytical solutions of the return process and the conditional volatility process are derived by the market. The empirical implications and evidence of this model are provided.

The paper is organized as follows: Section 1 lays out managers’ earnings smoothing problem and the firm’s cash flow is derived. Section 2 analyzes the distribution of unexpected earnings and asymmetric response of return to good news and bad news. Section 3 examines EGARCH-type behavior of conditional volatility. Section 4 provides empirical evidence. Section 5 concludes. Proofs are laid out in the Appendix.

1 The Model

Consider a finite horizon economy with a single consumption good. There is a single risky asset that serves as a claim to the firm’s assets. In the final period $T$, all the uncertainty is resolved and the firm is liquidated. All proceeds go to the firm’s equity holders. Figure (1) (on the next page) demonstrates how managers manipulate earnings through real economic activities. At the beginning of period 1, managers observe the persistent component of earnings $\eta_1$ and then decide to increase this period’s earnings by 0.5 unit by, for example, deferring maintenance. This will decrease the next period’s earnings since the machines cannot work as efficiently as they do with proper maintenance. Note that there is no real cash outflow since managers are in fact spending less on maintenance. Therefore this period’s cash increment (reported earnings in this model) is 8.5 instead of 8. However, this shifting decreases future earnings by the amount of the shift, 0.5, plus an extra cost to fix the machines later and any cost their malfunctions may impose, $(\frac{1}{2} \lambda s^2)$.\(^8\)

At the beginning of period 2, after observing the persistent part of earnings, managers decide to shift 3.5 units into this period by further deferring maintenance. Therefore this period’s cash increment is the cash generated from the process without smoothing minus what has been shifted to increase last period’s reported earnings plus managers’ further borrowing

\(^8\)Where $\lambda$ is the per unit quadratic smoothing cost. Note that for tractability, this paper assumes that managers pay a quadratic smoothing cost to smooth earnings.
Earnings without Smoothing

| Period | 8 | 6 | 14 | 10 |

Earnings Smoothing

| Period | 0.5 | 3.5 | -1 | 0 |

Reported Earnings (Underlying cash flow)

| Period | 8.5 | 9 | 9.5 | 10.325 |

Real wealth can not be created Smoothing cost is paid

\[
(10+1) - 0.5\lambda (0.5^2 + (3.5)^2 + (-1)^2)
\]

Where the cost of smoothing = 0.5 \* \(s_t^2\) = 0.675

\[
\lambda = 0.1
\]

Acumulative Cash flow in Period q without smoothing = 8 + 6 + 14 + 10 = 38

Acumulative Cash flow in Period q with smoothing = 8.5 + 9 + 9.5 + 11 - smoothing cost = 37.325

Figure 1: An Example of Earnings Smoothing: Intertemporal Shifting of Earnings.

from the future \((6 - 0.5 + 3.5 = 9)\). At the end of this period, managers will report 9 as earnings instead of 6.

At the beginning of period 3, managers observe a good signal for this period’s earnings and decide to save some of the earnings for the future, shifting 1 unit to the next period by, for example, delaying sales to the next period. Hence, this period’s reported earnings should be the cash generated from the process without shifting minus what has been shifted away to increase last period’s earnings and what has been saved for future \((14 - 3.5 - 1 = 9.5)\). Instead of reporting 14, managers report 9.5 at the end of period 3.

At the beginning of period 4, managers decide to do all the deferred maintenance and do not shift this period’s earnings \((s_t = 0)\). Therefore, this period’s cash increment is \(10 + 1 - 0.675 = 10.325\). At period 5, managers restart this practice of earnings management. By paying a smoothing cost \((0.675)\), managers can report a stably increasing reported earnings series 8.5, 9, 9.5, 10.325 instead of the volatile stream 8, 6, 14, 10.

Assumption 1: Earnings Generating Process without Income Smoothing. The
firm’s earnings generating process without income smoothing in period $t$, i.e., $A_t$, is defined as

$$A_t = \eta_t + M_t$$  

where $M_t$ are i.i.d. Gaussian random variables with mean 0, variance $\sigma^2_M$. The variable $M_t$ represents the transient component of earnings that cannot be perfectly predicted by the managers at the beginning of each period. Intuitively $M_t$ can be thought of as arising from the risk that is not fully predictable or controllable by the firm. However, managers have more power to predict and manipulate the firm level risk. The variable $\eta_t$ represents the firm specific risk that can be fully predicted by the managers at the beginning of period $t$. The $\eta_t$’s are assumed to following an AR(1) process with a drift $\mu$.

**Assumption 2: Costly Earnings Manipulations.** At the beginning of each period $t$, after seeing this period’s persistent component of earnings $\eta_t$, managers choose how to smooth this period’s reported earnings through real economic activities, such as deferring maintenance, stuffing the supply chain, and delaying real investment. They can smooth the earnings generating process without income manipulation ($A_t$), defined in Equation (1), by either increasing or decreasing the earnings of an amount ($s_t$), provided they pay a quadratic smoothing cost ($\frac{1}{2} \lambda s_t^2$). At the end of the period, after observing the realized transient component of earnings $M_t$, managers report earnings to the market.

**Assumption 3: Periodic Revelation.** Other than the intertemporal shifts of earnings and the quadratic smoothing cost, earnings management does not have any other real impact on the firm’s wealth. Clearly, corporate managers endogenously choose when to realize all the accumulated smoothing costs and do the deferred maintenance or R&D. For tractability, this paper assumes that after every $q$ periods’ smoothing, managers have to “clean house” and the fixed $q$ becomes known to the market. To simplify the math, this paper assumes the market knows the house-cleaning periodicity $q$.

Managers repeat the earnings smoothing and cleaning cycle until the last period $T = N*(q + 1)$. For each smoothing and cleaning cycle $j \in [1, 2, \ldots, N]$, they choose how much to smooth in period $t \in [0, 1, 2, \ldots, q - 1]$, in cycle $j$. In the last period of cycle $j$, they have to “clean house.” The accumulated wealth process for the firm in period $t$, cycle $j$, is given by

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9The reader should think of $A_t$ as the best the firm can do if it operates optimally to maximize its present value. All costs are thus reductions from this amount.

10All the conclusions are still valid if $\eta_t$ is assumed to follow an AR($n$) process where $n > 1$.

11Realistically, ex ante, the market can estimate the revelation date. However, it does not know this date for sure. All of the model’s conclusions still hold when managers endogenously choose $q$ and the market has to estimate the house-cleaning cycle, since assuming that the market does not know the exact time the managers do house-cleaning will make the market even more disadvantage over the earnings-smoothing firms. Detecting the timing of real earnings management is beyond the scope of this paper, hence, a parsimonious way of modeling is adopted.
the following

\[
W_{j,t+1} = W_{j,t} + A_{j,t} + s_{j,t}; \quad \forall \ 0 \leq t \leq q - 1, \ \forall 1 \leq j \leq N
\]

\[
W_{j,\text{End}} = W_{j,\text{Begin}} + \sum_{t=0}^{q} (\eta_{j,t} + M_{j,t}) - \frac{1}{2} \lambda \sum_{t=0}^{q-1} s_{j,t}^2; \quad \forall 1 \leq j \leq N
\]  \tag{2}

\[
W_{j+1,\text{Begin}} \equiv W_{j,\text{End}}, \quad 1 \leq j \leq N - 1
\]

\[
W_{0,\text{Begin}} \equiv W^*,
\]

where \(W^*\) is some positive constant serving as the initial wealth of the firm.

Equation (2) denotes the firm’s accumulated wealth process in period \(t \in [0, q - 1]\), cycle \(j\). It equals the accumulated wealth at the beginning of this period \(W_{j,t}\), plus the earnings without smoothing \(A_{j,t} = \eta_{j,t} + M_{j,t}\), and the amount managers smooth \(s_{j,t}\) for this period. Equation (2) denotes the accumulated wealth at the end of the cleaning cycle \(j\). As a practical matter, smoothing cannot create real wealth, the accumulated wealth at the end of each cleaning cycle \(j\) has to be equal to the accumulated wealth without income smoothing \(W_{j,\text{Begin}} + \sum_{t=0}^{q} A_{j,t}\), minus the accumulated smoothing cost \(\frac{1}{2} \lambda \sum_{t=0}^{q-1} s_{j,t}^2\) within this cleaning cycle \(j\).

Note that the earnings manipulation modeled in this paper is not about the managers’ playing with the book entries; instead they smooth earnings through real economic activities and truthfully report the earnings as the increment of the accumulated wealth. The reported earnings for period \(t\) in cycle \(j\) is given as

\[
R_{j,t} \equiv W_{j,t+1} - W_{j,t}, \quad 0 \leq t \leq q - 1; \quad 1 \leq j \leq N
\]  \tag{3}

\[
R_{j,\text{End}} \equiv W_{j,\text{End}} - W_{j,q}, \quad 1 \leq j \leq N
\]  \tag{4}

Equation (3) defines the reported earnings in period \(t \in [0, q - 1]\), cycle \(j\) when managers are engaging in smoothing. They report the earnings without smoothing \(A_t\), defined in Equation (1), plus the amount they smooth \(s_t\). At the end of cleaning cycle \(j\), managers pay the smoothing cost and report the increment of the accumulated wealth as Equation (4) demonstrates.

**Assumption 4: Utility Maximization.** At the beginning of each period \(t\), in cycle \(j\),
managers choose \( s_{j,t} \) to maximize

\[
\max_{\{S_{j,t}\}_{j=1}^{N}, \{s_{j,t}\}_{t=0}^{q-1}} \mathbb{E}_0 \left\{ \sum_{j=1}^{N} \left[ \sum_{t=0}^{q-1} \left( aR_{j,t} - \frac{1}{2}bR_{j,t}^2 \right) + cW_{j,\text{End}} \right] \right\} 
\]

subject to

\[
R_{j,t} = A_{j,t} + s_{j,t}; \quad \forall \ 0 \leq t \leq q - 1 \ \forall \ 1 \leq j \leq N
\]

\[
A_{j,t} = \eta_{j,t} + M_{j,t}; \quad \forall \ 0 \leq t \leq q \ \forall \ 1 \leq j \leq N
\]

\[
W_{j,t+1} = W_{j,t} + A_{j,t} + s_{j,t}; \quad \forall \ 0 \leq t \leq q - 1 \ \forall \ 1 \leq j \leq N
\]

\[
W_{j,\text{End}} = W_{j,\text{Begin}} + \sum_{t=0}^{q-1} A_{j,t} - \frac{1}{2} \lambda \sum_{t=0}^{q-1} s_{j,t}^2. \quad \forall \ 1 \leq j \leq N
\]

The control variable \( s_{j,t} \) is measurable with respect to \( \sigma \{ \eta^*, \eta_{k,\tau}, M_{k,\tau-1} : 0 \leq \tau \leq t; 1 \leq k \leq j \} \) (the \( \sigma \)-field generated by \( \{ \eta^*, \eta_{k,\tau}, M_{k,\tau-1} : 0 \leq \tau \leq t; 1 \leq k \leq j \} \)). The objective function of the earnings smoothing problem defined in Equation (5) states that managers would like to report high earnings if possible \( (aR_{j,t}) \), dislike reported earnings volatility \( (bR_{j,t}^2) \), and care about the underlying wealth of the firm at the end of each “house-cleaning period” \( (cW_{j,\text{End}}) \). Earnings without smoothing, the accumulated wealth process and the reported earnings for each period \( t \) in cycle \( j \) are defined in Assumptions 1, 2, and 3, respectively. The time discount parameter is set to be 1 to simplify notation.

**Assumption 5: Risk Neutrality.** Investors are risk neutral. The market prices the risky asset as the expectation of the discounted future cash flow of the firm. There is a risk free storage technology with a fixed return \( r \), which is set to 0. It is worth emphasizing that risk neutrality is assumed for the sake of simplification. Because the main goal of this paper is to analyze the implications of earnings smoothing on return volatility dynamics, adding risk aversion to the model would generate a risk premium to compensate investors’ aversion to risk. Even without this hedging motive, the model developed in this paper generates an asymmetric response to good news and bad news in the return process and the clustered conditional volatility process.

### 1.1 The Manager’s Problem

After managers solve the earnings manipulation problem defined in Equation (5), Assumption 4, the firm’s underlying wealth process can be determined. Knowing the cash flow of the firm, the market can rationally price this risky asset. Proposition 1 shows the earnings smoothing process, the smoothed reported earnings and the firm’s final wealth.

**Proposition 1.** *The solution for the managers’ earnings manipulation problem defined in*
Equation (5) is as follows:

\[ s_{j,t} = \frac{a - b_\eta_{j,t}}{b + c_\lambda}; \quad \forall \ 0 \leq t \leq q - 1; \quad \forall \ 1 \leq j \leq N \]  

The reported earnings process is

\[ R_{j,t} = \frac{c_\lambda}{b + c_\lambda} \eta_{j,t} + M_t + \frac{a}{b + c_\lambda}; \quad \forall \ 0 \leq t \leq q - 1; \quad \forall \ 1 \leq j \leq N \]

The final wealth of this firm is

\[ W_{N, \text{End}} = W^* + \sum_{j=1}^{N} \sum_{t=0}^{q} M_t + \sum_{j=1}^{N} \sum_{t=0}^{q} \eta_{j,t} - \frac{1}{2} \lambda \sum_{j=1}^{N} \sum_{t=0}^{q-1} s_{j,t}^2. \]

**Proof.** See Appendix A.1.

The shifted earnings process in Equation (6) is actually quite intuitive. Since managers like high firm values, they tend to report higher earnings by the amount of \( \frac{a}{b + c_\lambda} \) each period. They dislike the variance of the reported earnings; therefore, after seeing a good piece of information at the beginning of each period \( t \) in cycle \( j \) (\( \eta_{j,t} > 0 \)), managers will try to decrease that period’s reported earnings by shifting away \( \frac{b_\eta_{j,t}}{b + c_\lambda} \) from the cash flow. After seeing a bad piece of information about earnings (\( \eta_{j,t} < 0 \)), they will increase the reported earnings by the amount \( \frac{b_\eta_{j,t}}{b + c_\lambda} \).

**Proposition 2.** The smoothing of earnings (\( s_{j,t} \)) increases as the managers’ preference for the mean (\( a \)) increases, and decreases as the cost of smoothing (\( \lambda \)) and the weight they put on the underlying wealth of the firm (\( c \)) increases.

**Proof.** Differentiate Equation (6) with respect to \( a, \lambda \) and \( c \).

Managers intertemporally shift earnings by paying the quadratic smoothing cost, hoping the next period’s positive earnings shock can offset this cost. At the same time, they care about the firm’s underlying wealth. They are optimally balancing the cost and benefit each period by solving the dynamic programming problem laid out in Proposition 1. The intuition for Proposition 2 is straightforward. As the managers’ preference for the mean of the reported earnings increases (\( a \)), they would like to increase their shifted earnings (\( s_{j,t} \)). If the cost of income smoothing is very high (\( \lambda \)) or they care more about the present value of the firm (\( c \)), managers will engage less in earnings smoothing.
1.2 The Market’s Problem

Having firms’ practicing earnings management in mind, and after observing the reported earnings together with the history of all the past reported earnings, the market applies a Kalman filter to filter out the noise \( M_t \) before it can rationally price the cash flow of this firm. At the beginning of each period \( t \), in cycle \( j \), the market observes the reported earnings \( R_{j,t-1} \), which is reported by the manager at the end of period \( t-1 \) in cycle \( j \). The market updates its estimation of the firm’s underlying wealth, aware of the managers’ “cleaning house” periodicity \((q + 1 \text{ period for each cycle})\). Lemma 1 and Lemma 2 present the Kalman filter estimate and the fixed-point smoother\(^{12}\) of the unobservable hidden-state variable, or, in other words, the persistent component of earnings \( \eta_{j,t} \). Proposition 3 characterizes the market price and the dollar return process for this firm under earnings management.

**Lemma 1.** After observing the reported earnings \( R_{j,t} \) or (equivalently) \( y_{j,t} \) defined below at the beginning of period \( t+1 \) in cycle \( j \), the market applies a Kalman filter to the following state space model:

\[
\begin{align*}
y_{j,t} &\equiv R_{j,t} - \frac{a}{b + c\lambda}, \quad \forall \; 0 \leq t \leq q-1; \; \forall \; 1 \leq j \leq N \\
y_{j,t} &\equiv \phi\eta_{j,t} + M_{j,t}, \quad \forall \; 0 \leq t \leq q-1; \; \forall \; 1 \leq j \leq N \\
\eta_{j,t} &\equiv \eta_{j,t-1} + \epsilon_{j,t}, \quad \forall \; 0 \leq t \leq q-1; \; \forall \; 1 \leq j \leq N
\end{align*}
\]

where \( \phi = \frac{\alpha}{b + c\lambda} \), \( \epsilon_{j,t} \) are i.i.d. Gaussian random noise and \( \text{Cov}(\epsilon_{j,t}, M_{k,t_2}) = 0 \) \( \forall \; j, k, t_1, t_2 \). For all \( 0 \leq t \leq q-1 \) and \( 1 \leq j \leq N \), let \( a_{j,t-1} \) denote the optimal estimator of \( \eta_{j,t-1} \) based on the observations up to and including \( y_{j,t-1} \). Let \( Q_{j,t-1} \) denote the variance of the estimation error up to and including observations in period \( t-1 \), cycle \( j \), or, \( Q_{j,t-1} \equiv \mathbb{E}(\eta_{j,t-1} - a_{j,t-1})^2 \). Normalize the standard deviation of the noise \( \sigma^2_M \) to 1, and let \( \kappa \) denote the signal-to-noise ratio \( \frac{\sigma^2_{\epsilon}}{\sigma^2_M} \). The Kalman filter estimator for \( \eta_{j,t} \) based on observations up to and including \( y_{j,t} \) is

\[
a_{j,t} = a_{j,t-1} + \frac{\phi Q_{j,t-1}(y_{j,t} - \phi a_{j,t-1})}{1 + \phi^2 Q_{j,t-1}} \quad \forall \; 0 \leq t \leq q-1; \; \forall \; 1 \leq j \leq N
\]

where \( Q_{j,t-1} = Q_{j,t-1} + \kappa \). The recursion for the error covariance matrix, i.e., the Riccati

\(^{12}\)A smoother for a hidden-state variable at time \( t \) under a Kalman filter is defined as the estimate of this hidden-state variable taking account of the information made available after time \( t \).
Equation (11) is given by

\[
Q_{j,t+1|t} = Q_{j,t} + \kappa \\
Q_{j,t} = \frac{\phi^2 Q_{j,t|t-1}}{1 + \phi^2 Q_{j,t|t-1}} \\
Q_{j,t+1|t} = \frac{\phi^2 Q_{j,t|t-1}}{1 + \phi^2 Q_{j,t|t-1}} + \kappa
\]

**Proof.** See Appendix A.2.

The Kalman filter derived in Lemma 1 is a recursive algorithm that optimally computes the estimate of the unobservable hidden-state variable, or, the persistent component of earnings \((\eta_{j,t})\). As the market observes an earnings announcement, it updates its expectations on the persistent component of earnings by updating the error covariance matrix \(Q_{j,t|t-1}\) given in Equation (11). The updating of this error covariance matrix in the Riccati Equation is the main part of the calculation in the Kalman filtering process. Given the special structure of the state space model defined in Equation (9), the Riccati Equation given in Equation (11) has a unique solution. The speed of convergence to this solution, which is the time-invariant error covariance matrix, is exponential. This paper focuses on and analyzes this time-invariant Kalman filter which is given in the following Lemma.

**Lemma 2.** The time-invariant error covariance matrix \(\bar{p}\) is given by

\[
\bar{p} = \frac{1}{2} \left( \kappa + \sqrt{\kappa^2 + 4 \kappa \phi^2} \right)
\]

Let the initial value for \(\eta_{1,t}\) process of cycle 1 be \(\eta^* = 0\) (i.e., \(a^* = 0\)). The initial value for \(\eta_{j,t}\) process of cycle \(j\), where \(j \geq 2\), is \(a_{j-1,\text{End}}\). The time-invariant Kalman filter estimate up to and including observation \(y_{j,t}\) (i.e., \(a_{j,t} = \mathbb{E}(y_{j,t}|T_{j,t}^{\text{End}})\)), for the persistent earnings \(\eta_{j,t}\) is

\[
a_{j,t} = a_{j,t-1} + \frac{\gamma}{\phi} (y_{j,t} - \phi a_{j,t-1}) \quad \forall \; 0 \leq t \leq q - 1; \; \forall \; 1 \leq j \leq N
\]

Define the innovation of this filter as:

\[
v_{j,t} = y_{j,t} - \phi a_{j,t-1},
\]

where \(\gamma = \frac{\phi^2 \bar{p}}{1 + \phi^2 \bar{p}}\). For all \(0 \leq t \leq q - 1\) and \(1 \leq j \leq N\), the innovation process in period \(t\) and cycle \(j\), \(v_{j,t}\), is i.i.d. Gaussian with mean 0 and variance \(F \equiv 1 + \phi^2 \bar{p}\). The fixed-point
smoother denoted as \( m_{j,\tau|t} = E(y_{j,\tau}|T_{j,t}^{\text{End}}) \), \( \forall \tau = 0, 1, \ldots, t - 1 \) and \( \forall 1 \leq j \leq N \) for \( \eta_{j,\tau} \) is

\[
m_{j,\tau|t} = m_{j,\tau|t-1} + \frac{\gamma}{\phi} (1 - \gamma)^{t-\tau} v_{j,t}, \quad \forall 0 \leq \tau \leq t - 1; \quad \forall 1 \leq j \leq N
\]

(13)

where \( \gamma = \frac{\phi^2 \bar{p}}{1 + \phi^2 \bar{p}} \) and \( \phi \) is defined in Lemma 1.

Proof. See Appendix A.3.

Lemma 2 illustrates the structure of the time-invariant Kalman filter estimate and the fixed-point smoother of the hidden-state variable \( (\eta_{j,\tau}) \) in Equation (12) and Equation (13), respectively. Clearly, both the Kalman filter estimate \( (a_{j,t} = y_{j,t}|t) \) and the fixed-point smoother \( (m_{j,\tau}, \forall 0 \leq \tau \leq t - 1) \) are martingales. They inherit this persistence from the unobservable persistent component of earnings and the nature of the filtering procedure. After the market solves the filtering problem, it can rationally price the cash flow of the firm. The market price and the dollar return process are characterized in the following Proposition.

Proposition 3. Under risk neutrality, the price of this risky asset at the beginning of each period \( t \), cycle \( j \), is given as follows:

\[
P_{j,t} = E\left[W_{N,\text{End}}|T_{j,t}^{\text{End}}\right], \quad \forall 0 \leq t \leq q - 1; \quad \forall 1 \leq j \leq N
\]

(14)

Where \( \Lambda_1 = \left(\frac{b}{b+c\lambda} + \frac{\lambda ab}{(b+c\lambda)^2}\right)^2 \), \( \Lambda_2 = \frac{1}{2} \left(\frac{b}{b+c\lambda}\right)^2 \), \( \Lambda_3 = \frac{ab\lambda}{(b+c\lambda)^2} \) and

\[
A_{j,t,0} = W_{j-1,\text{End}} - \frac{at}{b+c\lambda} - \frac{1 + \lambda (N - j)}{2(b+c\lambda)^2} - \Lambda_2 \bar{p} \left( (N - j) q + (q - t - 1) + \sum_{t=0}^{t-2} (1 - \gamma)^{t-\tau-1} \right) - \Lambda_2 \bar{p} \left( \frac{(q - t + 1)(q - t)}{2} + \left[(N - 1)(N - 2j + 2) - (N - j)(t - 1)\right] q \right),
\]

\[
A_{j,t,1} = (q - t + 1) + \frac{b}{b+c\lambda} + \Lambda_3 \left[(q - t - 1) + q(N - j) + (N - j)(q + 1)\right],
\]

\[
A_{j,t,2} = \Lambda_2 [(q - 1 - t) + (N - j)].
\]

Let the dollar return process \( r_{j,t} \) denote the return from the beginning of period \( t \) in cycle \( j \) to
the beginning of period \( t + 1 \) in cycle \( j \):

\[
r_{j,t} = P_{j,t+1} - P_{j,t} \quad \forall \ 0 \leq t \leq q - 1; \quad \forall 1 \leq j \leq N
\]

\[
= B_{j,t,0} + B_{j,t,1}v_{j,t} - \Lambda_2 \Psi_{j,t-1}(v) v_{j,t} - \Lambda_2 B_{j,t,2} v_{j,t}^2 - \frac{2\gamma}{\phi} \Lambda_2 B_{j,t,3} a_{j,t-1} v_{j,t}
\]

where \( B_{j,t,0}, B_{j,t,1}, B_{j,t,2}, B_{j,t,3} \) and \( \Psi_{j,t-1}(v) \) are given as follows:

\[
B_{j,t,0} = \Lambda_1 \left\{ \bar{p} \left[ \gamma + 1 - (1 - \gamma)^t \right] + (q - t) + (N - j) q \right\} \sigma_r^2,
\]

\[
B_{j,t,1} = 1 + \gamma (q - t) + \frac{\gamma}{\phi} \Lambda_1 \left[ \sum_{\tau = 0}^{t-2} (1 - \gamma)^{t-\tau} + (1 - \gamma) \right] + \frac{\gamma}{\phi} \Lambda_3 [(q - t) + (N - j) q] + \ldots \frac{\gamma}{\phi} \left[ 1 + (N - j) (q + 1) + \frac{b}{b + c\lambda} (q - t) \right],
\]

\[
B_{j,t,2} = \sum_{\tau = 0}^{t-2} (1 - \gamma)^2 (t-\tau) + \frac{\gamma^2}{\phi^2} \left[ (1 - \gamma)^2 + (q - t) + (N - j) \right],
\]

\[
B_{j,t,3} = (1 - \gamma) + (q - t) + (N - j) q,
\]

\[
\Psi_{j,t-1}(v) = \frac{\gamma^2}{\phi^2} \sum_{\tau = 0}^{t-2} 2 (1 - \gamma)^{t-\tau} \left( \sum_{k=0}^{\tau} v_{j,k} + \sum_{k=1}^{t-1-\tau} (1 - \gamma)^k v_{j,\tau+k} \right) v_{j,t}.
\]

The Kalman filter estimate \( a_{j,t} \), the fixed-point smoother \( m_{j,\tau|t} \), and the innovation of this filter \( v_{j,t} \) are derived in Lemma 1 and Lemma 2 (see Equation (12) and Equation (13)).

**Proof.** See Appendix A.4.

Lemma 1 and Lemma 2 derive the Kalman filter that the market applies to the reported earnings series to estimate the hidden-state variable, persistent earnings \((\eta_{j,t})\). The special structure of the state space model defined in Equation (9) Lemma 1 determines the way the market updates its estimation process. The Kalman filter estimate and the fixed-point smoother of the hidden-state variable (persistent earnings) given in Equation (12) and (13) respectively are weighted sums of all the available earnings innovations. Applying the time-invariant Kalman filter derived in Lemma 2, the market prices the firm accordingly.

Proposition 3 characterizes the market price of the firm and the dollar return process. Clearly, both the price process (Equation (14)) and the return process (Equation (15)) are quadratic functions of the available reported earnings to date and the history of news. The driving forces of this special structure are as follows: (1) The earnings process has a persistent component \((\eta_{j,t})\), (2) the cost of earnings smoothing is convex (i.e., managers have to pay a cost regardless of whether they would like to increase or decrease this period’s reported earnings), and (3) managers do not report the persistent component of earnings and the transient com-
ponent of earnings separately, nor do they truthfully report how much they spend on earnings smoothing. The market has to apply a filter to estimate the persistent part of earnings and the smoothing cost. This special structure makes the conditional volatility process follow an ARMA process. Section 2 and 3 are devoted to the detailed analysis of the implications of this special structure on the return and conditional volatility process respectively.

2 Asymmetric Response to Good News and Bad News and Concentration of Earnings Innovations to Zero

Because of their mean variance preference, managers tend to report higher earnings if possible. The reported earnings given in Equation (7) have a positive constant component \( \left( \frac{a}{b + c \lambda} \right) \). After seeing \( \eta_{j,t} > 0 \) at the beginning of period \( t \) in cycle \( j \), managers know the firm will do relatively well, and they reduce that period’s reported earnings by \( \frac{b \eta_{j,t}}{b + c \lambda} \) at a cost of \( \frac{1}{2} \lambda \sigma^2_{j,t} \). After seeing \( \eta_{j,t} < 0 \) at the beginning of each period \( t \) in cycle \( j \), managers will try to increase that period’s reported earnings by \( \frac{|b \eta_{j,t}|}{b + c \lambda} \) again at a cost of \( \frac{1}{2} \lambda \sigma^2_{j,t} \). Therefore, a good piece of news at the end of period \( t \) in cycle \( j \) (i.e., a positive realization of innovation \( v_{j,t} = R_{j,t} - E(R_{j,t}|I_{t}^{Begin}) > 0 \)), implies that the firm does receive a positive shock that period. However, the final wealth of the firm is not as large as what the good news would indicate, since managers incur a cost to spread out the good news into the future.\(^{13}\) A good earnings announcement now may also be achieved at the cost of firms’ future performance. On the other hand, a bad piece of news (i.e., a negative realization of innovation \( v_t = R_t - E(R_t|I_t^{Begin}) < 0 \)) implies that, even after managers attempt to report more for that period, they still cannot manage to avoid the negative information release. Moreover, they have to pay a smoothing cost to report this “not so bad” number. A bad news release thus implies that the prospect of the underlying wealth can be even worse than what has been reported. Therefore both good news and bad news are in their own ways “bad” news to the market. This model endogenously generates what Campbell and Hentschel (1992) call the “no news is good news” effect in their volatility feedback model, where volatility feedback is exogenously assumed.

Proposition 4. No News Is Good News. If the earnings innovation is zero, the dollar return in period \( t \), cycle \( j \), \( (r_{j,t} \equiv P_{j,t+1} - P_{j,t}) \), rises by \( \Lambda_1 \left\{ \bar{p} \left[ \frac{\gamma}{1 - \gamma} \right] + (q - t + N - j) q \right\} \sigma^2 \).

Proof. Take the return process defined in Equation (15), and set \( v_{j,t} = 0 \), \( r_{j,t} = B_{j,t,0} > 0 \).

Unlike Campbell and Hentschel (1992), whose model generates “no news is good news”\(^{13}\)Gunny (2005) empirically documents that real earnings management impairs firms’ future performance in an economically significant way.
through the volatility feedback effect. Proposition 4 implies no news is good news to the market through the earnings smoothing cost (i.e., both good news and bad news are costly to the underlying wealth of the firm). Gunny (2005) confirms the crucial assumption of this model (i.e., real earnings management is costly) by empirically examining the consequences of real earnings management. The results of Gunny (2005) provide strong evidence that real earnings management has an economically significant negative impact on future performance. Hence, if the market receives no news at all, this is indeed good news to the future performance of the firm.

**Proposition 5. Asymmetric Response to Good News and Bad News.** After a series of nonpositive accumulated earnings news, the return drops much more than another piece of bad news would suggest. After a series of nonnegative accumulated earnings news, the return goes up much less than another piece of good news would imply. The return drops more after a series bad news releases than it rises following a series of good news releases. In mathematical terms, \[ |\Delta r_{j,t}|\{|\sum_{\tau=0}^{t-1} v_{j,\tau} \leq 0, v_{j,t} < 0\}| > |\Delta r_{j,t}|\{|\sum_{\tau=0}^{t-1} v_{j,\tau} \geq 0, v_{j,t} > 0\}|. \] Conditional on the accumulated earnings news being nonpositive, return drops more than it rises after a nonnegative accumulated earnings news release.

**Proof.** See Appendix A.5.

After seeing a series of nonpositive accumulated news releases \( \sum_{\tau=0}^{t-1} v_{j,\tau} \leq 0 \), the market figures out that the future fundamental of this firm must be much worse than what has been released, since managers do not have any good earnings shocks saved and they will realize all the smoothing cost in the forthcoming house-cleaning period. Another piece of bad news will make the price drop much more radically, which may seem to be overreaction to bad news if the impact of earnings management is ignored. On the other hand, after a series nonnegative news releases \( \sum_{\tau=0}^{t-1} v_{j,\tau} \geq 0 \), the market figures that the firm must have been wasting money on spreading out good earnings news. Therefore, another piece of good news will not raise the price as much as it “should”. Unlike other models in the existing literature, this model generates “overreaction” to bad news and “underreaction” to good news even when investors are risk neutral. It does not require the investors to become more sensitive to news after a series loss from their portfolios as McQueen and Vorkink (2004) do, nor does it need investors’ risk aversion as in Veronesi (1999). Bad news has to move the future return more than good news and the market has to “overreact” to bad news, since a bad news release implies that managers really cannot find another penny to hide the bad realizations of earnings. Good news implies managers are wasting resources to spread out good news.

The market is fully aware of managers’ earnings management, applies a Kalman filter to
estimate the persistent component of earnings and the unobservable smoothing cost, and forms rational expectations for next period’s reported earnings. Since any deviation from the market expected earnings will ultimately increase the volatility of the reported earnings, the more managers would like to smooth, the closer the reported earnings will be to the market’s expectation. The following proposition describes how the distribution of the unexpected earnings should look under earnings management.

**Proposition 6.** The unexpected earnings series are the innovations under the Kalman filter, which are i.i.d. Gaussian random variables with mean 0 and variance $\bar{F} = 1 + \phi^2 \bar{p}$, where $\phi = \frac{c\lambda}{b + c\lambda}$. The cheaper the smoothing cost ($\lambda$), the more managers care about smoothing ($b$) and the less they care about the underlying wealth ($c$), the more concentrated the unexpected earnings will be at 0.

*Proof:* See Appendix A.6.

The reported earnings can be written as the sum of last period’s expectation of current earnings and the earnings shock. If the market is fully rational in the sense that it uses up all the information to form expectations of future reported earnings, earnings shocks have to be i.i.d. white noise. Otherwise, the market is either consistently underestimating or consistently overestimating the next period’s earnings. Hence, larger earnings shocks imply more volatile reported earnings.

Those managers who care less about the underlying wealth (small $c$) and more about the volatility in reported earnings (large $b$) will try to stay as close to the market’s expected earnings as they can (a small variance $\bar{F}$ for earnings shocks). If it is cheap for some managers to smooth (small $\lambda$), their reported earnings will also be very close to the market expectation. For those firms whose persistent earnings component is transparent to the market (big signal-to-noise ratio with a small $\sigma_M^2$), it is almost impossible to hide the smoothing cost from the market. Hence, those firms are less likely to engage in earnings smoothing. Proposition 6 implies that if managers dislike the variance of the reported earnings they will try to stay close to the analysts’ forecasts. This confirms the findings in numerous empirical and theoretical papers that managers would like to exhaust all the available resources to meet analysts’ forecasts.\(^{16}\)

Empirical evidence of this concentration of the unexpected earnings to 0 is provided in Section 4.2. The widely accepted empirical measure of “differences of opinions”, dispersion of analysts’ forecasts, is taken as a proxy for the transparency of the persistent component of earnings. The “Governance Index” (Gompers, Isshii, and Metrick (2003)) which measures the level of shareholder rights is taken as a proxy for how much the managers care about the underlying wealth of the firm ($c$).

\(^{15}\)Note $\sigma_M^2$ is normalized to 1 in all the propositions and Lemmas.

\(^{16}\)See Degeorge, Patel, and Zeckhaust (1999), Graham, Harvey, and Rajgopal(2005), among many others.
3 Whence EGARCH?

Earnings have an unobservable persistent component and a transient component. Through the smoothing cost, which will never be reported by the managers, the persistent component of earnings carries over into the conditional volatility process. The implication of the persistent component of earnings is that firms moving a large absolute valued cash flow last period have to move a large absolute amount this period as well. Since smoothing earnings induces a convex cost, this cost carries the persistence in earnings into the second moment of the return process. Proposition 7 characterizes the stylized effect of conditional volatility: (1) Conditional volatility follows an ARMA process (GARCH), (2) both good news and bad news increase future volatility, and (3) bad news increases future volatility more than good news (EGARCH).

The return process \( r_{j,t} \) is defined so as to denote the asset’s return from the beginning of period \( t \) to the beginning of period \( t + 1 \) in cycle \( j \). The conditional volatility \( \sigma^2_{j,t} \equiv \text{Var}(r_{j,t} | I_{\text{Begin}j,t}) \) is defined to be the volatility of return \( r_{j,t} \) based on all the information available to the market up to the beginning of period \( t \) in cycle \( j \), i.e., the end of period \( t - 1 \) in cycle \( j \). The following proposition illustrates the analytical solution of the conditional volatility process under earnings management for all the periods \( t \in [0, q - 1] \) in cycle \( j \in [1, N] \).

Proposition 7. Volatility Clustering and Volatility Smirk The conditional volatility process has two components: the clustering component, which follows an ARMA process, and the smirk component, which makes bad news increase future conditional volatility more than good news. The conditional volatility process is given as follows:

\[
\sigma^2_{j,t} = \text{VolCon}_{j,t} + \text{Clust}_{j,t} + \text{Smirk}_{j,t},
\]

where \( \text{VolCon}_{j,t} \), \( \text{Clust}_{j,t} \), and \( \text{Smirk}_{j,t} \) denote the constant component, volatility clustering component, and the smirk component respectively:

\[
\text{VolCon}_{j,t} = B_{j,t,1}^2 \bar{F} + 3 \bar{A}_2 B_{j,t,2},
\]

\[
\text{Clust}_{j,t} = \bar{A}_2 \left[ \frac{4 \gamma \phi}{\phi^2 B_{j,t,3}^2 a_{j,t-1}^2} + \left| \Psi_{j,t-1}(v) \right|^2 + \frac{2 \gamma \psi}{\phi^2} B_{j,t,3} \Psi_{j,t-1}(v) a_{j,t-1} \right],
\]

\[
\text{Smirk}_{j,t} = -\Lambda_2 B_{j,t,1} \bar{F} \left[ \Psi_{j,t-1}(v) + \frac{2 \gamma \phi}{\phi^2} B_{j,t,3} \left( a_{j,t-2} + \frac{\gamma v_{j,t-1}}{\phi} \right) \right],
\]

where constant \( \bar{F} \) is the variance of the news \( v_{j,t} \) defined in Lemma 2 and \( \Lambda_2 \), \( B_{j,t,1} \), and \( B_{j,t,2} \) are defined in Proposition 3. \( \Psi_{j,t-1}(v) \) denotes a linear combination of all the available news up to the end of period \( t - 1 \) in cycle \( j \) with positive coefficients, which is also defined in Proposition 3. \( \gamma = \frac{\phi \bar{p}}{1 + \phi^2 \bar{p}} \) and \( \bar{p} \), the time-invariant error covariance matrix, are derived in
Lemma 2. $\phi = \frac{c}{\sigma + \alpha}$ is given in Lemma 1.

Proof. See Appendix A.7.

Proposition 7 states that an EGARCH statistical model can successfully estimate the conditional volatility process for those firms engaging in earnings management. Equation (17), Equation (18) and Equation (19) illustrate the constant component, the clustering component that follows an ARMA process, and a volatility smirk component of the conditional volatility process respectively. What makes the conditional volatility cluster is the clustering of the earnings news—earnings have a persistent component ($\eta_{j,t}$)—and the unobservable smoothing cost. The persistence of earnings carries over to the second moment of the return process through the earnings management cost, which turns out to be the quadratic terms of the news’ history ($\{v_{k,\tau}\}_{j=k, \tau=0}^{j, t-1}$) and the Kalman filter estimate of the persistent part of earnings ($a_{j,t-1}$) at the end of period $t-1$ in cycle $j$. The news shocks ($\{v_{j,t}\}_{j=1, t=0}^{N, q-1}$) are i.i.d. Gaussian random variables and the Kalman filter estimate of the persistent part of earnings follows an AR (1) process (i.e., $a_{j,t} = a_{j,t-1} + \gamma \phi v_{j,t}$). The quadratic function of the news and the Kalman filter estimate of the persistent earnings constitute the clustering component of the conditional volatility process.

The intuition behind the math is straightforward. The market has to use the reported earnings history to form rational expectations of the future price. Through the convex (quadratic in this paper) earnings smoothing cost, the persistent earnings carry into the second moment of the return process. Managers pay a convex cost to achieve their goal of a smoothed reported earnings series. If they have a large cash flow (in the absolute-level sense) to smooth last period, they will have a relatively large cash flow (in the absolute-level sense) to smooth this period. Either direction increases the cost of the underlying cash flow, which in turn makes the firm’s true accumulated wealth more volatile next period. Therefore, ARMA models can capture earnings clustering in the conditional volatility process. Unlike the rational learning models (David (1996) and Veronesi (1999)), which also generate stochastic volatility and the volatility smirk effect, earnings smoothing implies that both good news and bad news increase future volatility, whereas the learning models predict that future volatility decreases after good news releases. What is worth pointing out here is that the volatility clustering effect does not depend on the quadratic cost assumption used in this paper. The conditional volatility process will cluster provided earnings have a persistent part and there is a convex cost associated with the income smoothing. The market has to apply a filter to estimate the hidden-state variable ($\eta_{j,t}$). The quadratic cost is assumed for tractability and is not necessary for the conclusion of this Proposition 7.

The smirk component of the conditional volatility process is given in Equation (19).
coefficient in front of the earnings news at the end of period $t - 1$ or the beginning of period $t$ in cycle $j$, $(v_{j,t-1})$, is negative. This means bad news increases future volatility more than good news. The asymmetric response to good news and bad news also shows up in the second moment of the return process through the earnings management cost. Since bad news indicates an even worse and more volatile future accumulated wealth process than good news indicates, volatility increases more dramatically after a bad news announcement than after a good news announcement. Empirical evidence of earnings news moves future volatility asymmetrically is provided in Section 4.1. The next section is devoted to the simulation evidence.

3.1 Simulations

To investigate further the impact of earnings management on the conditional volatility process, simulations are provided to illustrate how the conditional return volatility under real earnings management behaves in a manner similar to that described by an EGARCH statistical model. The simulation exercise is conducted with use of the monthly Enron stock price and earnings data obtained from the CRSP and the Compustat data sets. Historical return and the simulated return are compared to illustrate the validity of the model.

The sample period starts in June 1947 and ends in January 2002. The parameters such as $a$, $b$, $c$ and $\lambda$ are set to match the mean and the variance of the historical Enron return series. Each month, after seeing the reported earnings that can be obtained from the Compustat tape, the market applies the Kalman filter developed in Lemma 1 section 1 to update its estimation for the persistent part of earnings ($\eta_t$). The simulated price series can be obtained by plugging the Kalman filter estimate and the fixed point smoother into the pricing Equation (14). With this simulated price series for Enron, the percentage return can be calculated. Table 1 contains the sample summary statistics for the monthly return series of Enron. Although the time-series average of Enron’s monthly return for both the historical and the simulated series are quite similar, the variance for the simulated return is much larger than that of the historical return. This excess volatility of the simulated return is driven by the unrealistic assumption of the number of the cleaning-house period $N$ explained in Footnote 18. However, this simulation exercise does shed light on a plausible explanation for the famous excess volatility puzzle. Stock prices seem to be too volatile relative to dividends data under the discounted cash flow model. Since the market does not know the periodicity of the house-cleaning cycle, it has to assume

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17 The managers of Enron, without doubt, engaged actively in earnings management.
18 To simplify the coding, I assume that the market takes the periodicity of the cleaning period $q + 1$ to be the last period $N \times (q + 1)$ by setting $N$ to 1. The market assumes that managers keep on smoothing and do not “clean their house” until the last period. A more realistic way to simulate the price series would be for the market endogenously to estimate the unobservable cleaning-house periodicity $(q + 1)$. 

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there is more variance when it forms the rational expectations for the asset prices.

Figures 2 and 3 depict the news impact curves for Enron’s historical return and the simulated return, respectively. The news impact curve plots the conditional return volatility measured by an EGARCH model against the past return shocks, which are defined as the difference between the realized return and last period’s expectation of this realized return. A clear asymmetry in responding to negative and positive shocks is exhibited in both the historical return series and the simulated return series. While both good news and bad news increase the future volatility, volatility following a bad news release rises significantly more than volatility after a good news release.

Table 2 reports the EGARCH (1, 1) estimates of the conditional volatility of Enron’s historical return and the simulated return.

\[
\begin{align*}
    \epsilon_t & = r_t - E(r_t|I_{t-1}), \\
    \epsilon_t & \equiv \sqrt{h_t} \times v_t, \quad C \equiv E(r_t), \\
    \ln (h_t) & = K + GARCH \ln (h_{t-1}) + ARCH [|v_{t-1}| - E(|v_{t-1}|)] + Smirk v_{t-1}.
\end{align*}
\]

The variable \( h_t \) denotes the conditional volatility of the return series \( r_t \), parameters \( C \) and \( K \) denote the unconditional mean of the return \( (r_t) \), and the unconditional mean of \( \ln (h_t) \), respectively. Parameters \( GARCH \), \( ARCH \) and \( Smirk \) denote the coefficients that capture the GARCH effect, ARCH effect, and volatility smirk effect, respectively. For comparison purposes, panel A of Table 2 reports the EGARCH estimates for the historical return. Table 2 clearly shows that both the historical data and the simulated data exhibit the following stylized facts: (1) Volatility estimated by using the monthly data clusters. Both the GARCH and the ARCH coefficients are statistically significant. (2) Bad news increases future volatility more than good news. The volatility smirk coefficient is also statistically significant.

The evidence presented in Table 2 demonstrates the low-frequency persistence in the conditional volatility process at the monthly level instead of the high frequency clustering of volatility at the daily or intraday levels. This confirms the well-documented empirical findings in the conditional volatility literature that volatility has a long-run component that is persistent and a short-run component that is transient. The impact of earnings management on return volatility is a long-run effect. If volatility clustering is, at least in part, due to the clustering of earnings, then this impact has to contribute to the long-run component of conditional volatility.

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19 See Pagan and Schwert (1990) and Engle and Ng (1993) for a detailed illustration of the news impact curve.

4 Empirical Evidence

The empirical implications of the model are tested on U.S. equity market data. Firm-level analysis presented in this section suggests that earnings innovations influence future return volatility, in line with the model’s predictions. Negative earnings shocks measured by the square of the negative standardized unexpected earnings (SSUEN) are followed by significantly higher conditional volatility. Firms are sorted into groups based on different measures capturing the likelihood of firms’ practicing real earnings management. A number of tests are conducted for firms within different groups. As will be seen, earnings innovations’ impact on future return volatility is much less for firms that seem likely to engage in less real earnings management. Conversely this impact is both economically and statistically significant for firms that are more likely to smooth earnings.

From the CRSP monthly stock return file, data was obtained for the period covering January 1962 to December 2003. This data was supplemented with the standardized unexpected earnings (SUE) measure obtained from Gao (2005) whose sample period starts in January 1985 and ends in December 2003, and Gompers, Ishii and Metric (2003)’s “Governance Index” measure which is available from year 1990 to year 2004. A stock is included in a particular month only if CRSP provides return and price data in that month and the SUE measure is available in that month. Gao’s sample period is constrained by the I/B/E/S data set. Year 1985 is taken as the starting year of his sample, since it is the first year the number of firms with available quarterly EPS forecast data exceeds 1000. Following Doyle, Lundholm and Soliman (2003), Gao (2005) measures $SUE$ as the standardized difference of the actual reported earnings and the median of the analysts’ forecast from the I/B/E/S data set. \(^{21}\) This paper uses the square of the standardized unexpected earnings to measure earnings innovations. Table 3 contains the sample summary statistics for the sample used in this section’s empirical test. Panel A of table 3 report the summary statistics for the joint sample of the CRSP monthly stock return file and I/B/E/S summary history file. Panel B of table 3 reports the summary statistics for the joint sample of the CRSP monthly stock return file and the Investors Responsibility Research Center’s (IRRC) corporate governance provision data.

4.1 Earnings News Moves Future Conditional Volatility Asymmetrically

If earnings news did not have any impact on conditional volatility, the validation of the model proposed in this paper would be seriously challenged. If return volatility clustering is due to the clustering of earnings news, then firms with either a positive or a negative earnings shock

\[ SU_{E_i,t} = \frac{(Actual_{i,t} - Forecast_{i,t})}{Price_{i,t}} \]

\(^{21}\)In mathematical terms, standardized unexpected earnings is defined as: $SU_{E_i,t} = \frac{(Actual_{i,t} - Forecast_{i,t})}{Price_{i,t}}$
will have higher expected future volatility. Equation (16) in Section 3 states that the square of news \(v_{t-1}^2\) increases future conditional volatility;\(^{22}\) the coefficient in front of \(v_{t-1}^2\) is positive. A necessary empirical test for this real earnings management model would be to analyze whether earnings news does increase future conditional volatility and whether bad news does increase future volatility more than good news. This section provides empirical evidence supporting the predictions of Proposition 7 and Proposition 5.

Table 4’s Panel A displays the regression of the conditional volatility on earnings innovations measured by the square of the standardized unexpected earnings (SSUE) after controlling for other firm characteristics, such as size, dollar volume, and liquidity. The conditional volatility is measured by an EGARCH model for each individual stock. In line with the theoretical prediction of the model demonstrated in Proposition 7, firms with high SSUE this month have higher conditional volatility next month. This is true with or without controlling for firms’ size, dollar volume, and liquidity. The cross-sectional regression analysis indicates that earnings news does move stocks’ future conditional volatility. It is also interesting to note that large size indicates lower conditional volatility, high volume implies higher future total volatility, and less liquidity predicts higher conditional volatility. The Newey-West t-statistics indicates that all the firm-level characteristics are statistically significant.

Having established the link between earnings news and future conditional volatilities, Panel B of Table 4 further displays the conditional volatility smirk effect. The smaller coefficient on the square of the positive earnings news (SSUEP) relative to the square of the negative earnings news (SSUEN) reflects the asymmetry derived in Proposition 5. With and without controlling for firm size, dollar volume and liquidity, negative earnings news increases conditional volatility more than positive earnings news does.

### 4.2 Controlling for Different Proxies

This subsection further explores the impact of earnings innovations on conditional volatility after controlling for different proxies for likelihood of firms’ practicing real earnings management. The empirical evidence provided in Table 4 demonstrates that earnings innovation is associated with conditional volatility in an asymmetric fashion. These are necessary tests for the model to have empirical content. However, there may be other explanations for the findings in Table 4. To further establish the relevance of real earnings management to conditional volatility, Tables 5, 7 and 8 explore how earnings innovations impact conditional volatility among different firms.

\(^{22}\)Conditional volatility is defined as \(\sigma_t^2 \equiv \text{Var}\left(r_t|\mathcal{F}_t^{\text{Regen}}\right)\)
4.2.1 Controlling for Governance Index

Proposition 2 indicates that the more managers care about the underlying wealth, the less they will engage in income smoothing. The balance of power between shareholders and managers can serve as a proxy for managers’ concern for the underlying wealth. If shareholders have stronger rights over the managers, it is easy for them to get rid of the managers wasting their resources, which consequently makes the managers more concerned about the firms’ ultimate value. On the other hand, if the managers are entrenched, they have more power managing earnings for their own interest at the cost of the shareholders’ interest. Using the publications of the Investor Responsibility Research Center (IRRC), Gompers, Ishii, and Metrick (2003) construct a measure for this balance of power between shareholders and managers. The details of the construction of this “Governance Index” ($G_{index}$) can be found in their paper and will not be elaborated here. For every firm, they “add one point for every provision that reduces shareholder rights.” Therefore, the smaller this $G_{index}$ is, the stronger the power shareholders have over management.

A natural test of earnings management’s impact on return volatility is to examine whether earnings shocks affect firms that are actively smoothing their earnings different from those that are not. Managers for firms with strong shareholder power over management are less likely to practice earnings smoothing actively than those with stronger rights over the shareholders. Table 5 reports the set of tests conducted within each “Governance Index”-sorted group. Each year, stocks are sorted into one of the four groups in accordance with that year’s $G_{index}$. Group 1 contains stocks with the smallest $G_{index}$, representing the strongest shareholder right (i.e., $G_{index}$ ranges from 2 to 5). Group 5 contains firms with strongest management right (i.e., $G_{index}$ ranges from 14 to 17). Pooled OLS regression of conditional volatility on the squared earnings shock after controlling for firm characteristics is reported in Table 5 Panel A. The squared earnings news does not move the conditional volatility for stocks in the lowest group of the index (strongest shareholder right) with an insignificant coefficient in front of $SSU_{E}$. From group 2 to 5, every unit of earnings news increases future monthly conditional volatility by 4.00%, 5.20%, and 15.84% respectively. As the shareholder right increases, the impact of earnings news on conditional volatility decays monotonically. The Spearman rank correlation is trivially 1 in this case. The joint F-test rejects the null hypothesis that the impact of earnings shock on conditional volatility is the same across four $G_{index}$-based groups. The joint F-test also rejects the possibility that this impact is equal for firms with the strongest shareholder rights and those with the weakest shareholder rights.

Panel B of Table 5 reports the volatility smirk effect among different $G_{index}$-based groups. Group 1 and 2 contain firms with the stronger shareholder rights and group 3 and 4 con-
stitute firms with stronger management rights. Both good earnings news and bad earnings news increase future volatility only for those firms in group 4 (i.e., firms with the strongest management rights). In contrast, earnings news does not move future volatility for firms with the strongest shareholder rights (i.e. firms in group 1). Bad earnings announcement increases future volatility for firms in group 3 and 4 while good earnings release does not affect future volatility for these firms. The volatility smirk effect only exhibits for firms with stronger management power (i.e., firms in group 3 and 4). The joint F-test cannot reject the null hypothesis that good news and bad news move future conditional volatility in a symmetric fashion for firms with stronger shareholder power (i.e., firms in group 1 and 2).

4.2.2 Controlling for “Meeting the Analysts’ Forecasts” and “Big-Bath” Effect

Proposition 6 states that the unexpected earnings for firms engaging more in earnings management are more concentrated around 0. Cross-sectionally, the unexpected earnings should accumulate an unexceptionally large mass around 0 provided that there are enough firms actively engaged in earnings management. Since the goal of this paper is to examine whether earnings innovation does move future return volatility via earnings management, a quarterly measure for earnings news is estimated by taking the difference of the actual reported earnings and the average of the median of analysts’ forecasts for a given quarter. Figure 4 and 5 plot the histogram for 227051 firm-quarter standardized unexpected earnings calculated from I/B/E/S actual earnings and analysts’ forecasts earnings data from January 1985 to December 2003. Table 6 reports the distributions of earnings forecast errors measured in two different ways.

For each firm quarter, following Abarbanell and Lehavy (2003), this paper calculates earnings forecast errors as the actual earnings per share (as reported in I/B/E/S) minus the average of the median of analysts’ forecasts for the given quarter, scaled by the stock price at the end of this quarter and multiplied by 100. Figure 5 and Panel A of Table 6 replicate the finding in Abarbanell and Lehavy (2003) that earnings news clearly concentrates around 0 with more positive earnings shocks than with negative ones. Although this asymmetry is not implied by Proposition 6, the result that managers actively engaging in smoothing always try to stay close to analysts’ forecasts is a clear implication of the model.\footnote{Since the managers in this model do not have a preference of reporting a positive unexpected earnings over reporting a negative one and all they care about is a smoothed earnings stream, Proposition 6 does not imply the asymmetry of the concentration around 0. Note that the efforts of reporting higher mean are well expected by the market; hence, managers’ effort in pushing the mean of the reported earnings up will not show up in the unexpected earnings.} Figure 4 and Panel B of Table 6 present a similar pattern by using Doyle, Lundolm and Soliman (2003)’s measure of unexpected earnings, which is basically Abarbanell and Lehavy (2003)’s measure without multiplying by 100. This model unrealistically assumes the market knows, \textit{ex ante}, the house-cleaning date,
which makes the model silent on the well-known “big-bath” phenomenon in which the reported earnings are significantly below the market expectation. Firms taking big bath must be engaging actively in earnings management, or else they would not have accumulated a huge smoothing cost to clean.

Tests are conducted to examine whether earnings shocks differently affect firms that are actively smoothing and those that are not. Firms are divided into two groups based on their standardized unexpected earnings ($SUE$) and dispersion of analysts forecasts. Dispersion of analysts forecast is a widely accepted empirical measure for differences of opinions among investors in the market. The market will naturally have less disagreement about firms’ future earnings for those firms whose earnings are less opaque. In the most extreme case, if a firm’s earnings process is totally transparent to the market, it is impossible for the manager to hide the smoothing cost. Therefore, this firm can not practice income smoothing. Group 2, referring to those firms actively practicing earnings smoothing, contains firms whose $SUE$ falls in the intervals $[-0.01, 0.01]$ and $(-\infty, -0.431]$, where $-0.431$ is the one percentile of the $SUE$ measure and the dispersion of analysts’ forecasts is greater than 0.007, where 0.007 is the 20 percentile of the dispersion of analysts’ forecasts. The rest of the firms belong to group 1, referring to those firms not actively practicing smoothing. Firm-level cross-sectional tests are conducted for these two different groups.

Panel A of Table 7 reports the impact of an earnings innovation on future return volatility among the two groups. In group 2 (firms actively engaging in smoothing) earnings news increases future return volatility much beyond that of firms in group 2. One unit of earnings shock increases monthly conditional volatility for 5.10% of firms in group 2, whereas only 2.24% increment of monthly conditional volatility is found for every unit of earnings shock for stocks within group 2. While the significance level of $SSUE$ is at 1% level for group 1, it is only at 10% level for group 2. The joint F-test also rejects the possibility of the equality of the impact of earnings news for these two different groups at 1% level. It is safe to conclude that earnings news moves future volatility, both economically and statistically, more for firms that appear to actively manage their earnings than for those that appear not to.

After confirming the differential impact of earnings news between the two control groups, Panel B of Table 7 examines whether these two groups are indeed the actively managed firm group and the non-actively managed firm group. “Governance Index” is calculated for each firm in the sample and two nonparametric tests are conducted to test the null hypothesis that the two $G\text{index}$ distributions of the two groups are the same. Both the Mann-Whitney

\[24\]5, 10 and 15 percentiles of the dispersion of analysts’ forecasts and 2, 3 and 5 percentiles of the $SUE$ are also chosen to split the data. They all produce qualitatively similar results to those presented here and thus are not discussed in the text or included in the table for the sake of brevity.

25
test and the Kolmogorov-Smirnov test reject the null hypothesis at 1% level. Panel B of Table 7 suggests that the actively-smoothing group has a significantly higher $G_{index}$ than the non-actively smoothing group. Firms with stronger management power engage in earnings management more than those with stronger shareholder power.

### 4.2.3 Controlling for Dispersion of Analysts’ Forecasts

Although detecting real earnings management is beyond the scope of this paper, the model developed here does predict that managers are less likely to engage in earnings smoothing if the persistent component of earnings is less opaque to the market. Clearly, it is impossible for the managers of firms whose earnings are transparent to the market to hide the smoothing cost, hence their $c$ is set to infinity, which implies that they will never smooth earnings. The dispersion of analysts’ forecast is a widely used measure for differences of opinions on firms’ earnings. Therefore, it can serve as a proxy for the transparency of firms’ persistent earnings to the market. If every analyst following the firm makes exactly the same forecast for earnings, then it may be safe to conclude that there is little difference in opinion regarding the firm’s earnings. For those firms whose analysts’ forecasts vary enormously, their managers have much more room to hide the smoothing cost.

Table 8 reports a set of pooled ordinary least squares (OLS) regressions of conditional volatilities on earnings news, controlling for firm size, dollar volume, and liquidity for different dispersion-based quintiles of firms. Each quarter, every stock is assigned to one of the five quintiles sorted by the dispersion of analysts’ forecasts. This dispersion increases from quintile 1 to quintile 5. The goal of this table is to demonstrate how earnings news affects conditional volatilities within different dispersion quintiles. Although the impact of earnings news on conditional volatility does not exhibit a monotonic pattern as has been seen in the “Governance Index”-sorted groups, Panel A of Table 8 still shows that earnings news moves volatility more in quintile 5 (highest dispersion of analysts’ forecasts) than in quintile 1 (lowest dispersion of analysts’ forecasts). The joint F-test rejects the possibility that the impact of earnings news on volatility is the same in quintile 1 and 5 at 1% level. For stocks with relatively low dispersion of analysts’ forecast, (i.e., quintile 2) earnings news does not move future conditional volatility. For stocks in quintile 3 and above, earnings shocks are followed by significantly higher conditional volatility.

Panel B of Table 8 reports the volatility smirk effect among different dispersion-sorted quintiles. For firms with low dispersion of analysts’ forecasts, bad earnings news does not move

$^{25}$The transparency of the persistent earnings makes the shareholders aware of the earnings management – they can boot those managers out immediately. This forces the managers to put infinite weight on firms’ underlying wealth.
future volatility more than good news does. Moreover, for stocks in quintiles 1 to 3, good news moves volatility more than bad news does. The volatility smirk effect is only observable in quintile 5 which contains stocks with the highest dispersion of analysts’ forecasts.

5 Conclusion

This paper proposes a rational expectations earnings-smoothing model to explain the return dynamics under real earnings management. The analytical solutions for the return and the conditional volatility process are derived. The model makes specific predictions about the intertemporal dynamics of conditional volatility and return process under real earnings management. Unexpected earnings shocks move future volatility and there exists a clear asymmetric impact of positive earnings shocks versus negative earnings shocks. The impact of earnings news on conditional volatility is almost negligible for firms with the least capability to practice real earnings management.

GARCH have been used for decades to model the time series behavior of conditional volatility. The theoretical justification of it is to address the economic questions. This paper takes the well-known practice of earnings management and demonstrates how it leads to a GARCH-type behavior in asset returns. This leads to predictions about what types of assets and in what economics, GARCH should be more pronounced. Empirical evidence using U.S. equity market data provides support for the model.

Empirical tests demonstrate the cross sectional differences in the relationship between earnings surprises and conditional volatility. This evidence should have implications for predicting and hedging risk, as well as providing guidance on earnings information dissemination process in the capital markets.

Appendix

Appendix A.1 Proof of Proposition 1

Proof. Clearly, the manager’s problem is stationary for each cleaning-cycle. For any admissible policy \( \pi = \{s_{j,0}, s_{j,1}, \ldots, s_{j,q-1}\} \) and each \( t = 0, 1, \ldots, q-1 \), denote \( \pi^t = \{s_{j,t}, s_{j,t+1}, \ldots, s_{j,q-1}\} \). The HJB equation of
this problem is
\[ V_{j,\text{End}} (X_{j,\text{End}}) = c W_{j,\text{End}} \quad \forall j \in [1, N] \]
\[ = c \left[ W_{j,\text{Begin}} + \sum_{k=0}^{q} (\eta_{j,k} + M_{j,k}) - \frac{1}{2} \lambda \sum_{k=0}^{q-1} s_{j,k}^2 \right], \]  
(A-1)
\[ V_{j,t} (X_{j,t}) = \max_{s_{j,t}} a (s_{j,t} + \eta_{j,t}) - \frac{1}{2} \left[(s_{j,t} + \eta_{j,t})^2 + \sigma^2 M_{j,t}^2 \right] + \mathbf{E} \phi_{j,t+\tau+\lambda+1} (V_{j,t+\tau+\lambda}) \quad \forall t \in [0, q-1]. \]

where \( X_{j,t} = \{ R_{j,t} W_{j,t} \} \forall j \in [1, N], \forall t \in [0, q] \). By backward induction, the first order condition gives out that \( s_{j,t} = \frac{a - h_{j,t}}{b - c}. \) Plugging the optimal control variable back into the reported earnings and the final wealth, we obtain the reported earnings (Equation (7)) and the expression for final wealth (Equation (8)). \( \square \)

**Appendix A.2  Proof of Lemma 1**

**Proof.** Again, the Kalman filtering problem is the same for each cleaning-cycle. As Appendix A.1 shows, \( R_{j,t} = \eta_{j,t} + s_{j,t} + M_{j,t} + \frac{\lambda}{\lambda + \phi} \eta_{j,t} + M_{j,t}. \) Define \( y_{j,t} \equiv R_{j,t} - \frac{\lambda}{\lambda + \phi} \eta_{j,t} \) and \( \phi = \frac{\sigma}{\lambda}, \) the state space model that the Kalman filter is applied to is
\[ y_{j,t} = \phi \eta_{j,t} + M_{j,t}, \forall 0 \leq t \leq q-1; \forall 1 \leq j \leq N \]  
(A-2)
\[ \eta_{j,t} = \eta_{j,t-1} + \epsilon_{j,t}, \forall 0 \leq t \leq q-1; \forall 1 \leq j \leq N \]

Let \( a_{j,t-1} \) denote the optimal estimator of \( \eta_{j,t-1} \) based on the observations up to and including \( y_{j,t-1} \) and \( Q_{j,t-1} \) denote the covariance matrix of the estimation error, \( Q_{j,t-1} = \mathbf{E} (\eta_{j,t-1} - a_{j,t-1})^2. \) Given \( a_{j,t-1}, Q_{j,t-1}, \) the fact that the hidden-state variable \( \eta_t \) follows a unit root process, and the variable \( M_t \) is the independent Gaussian noise, the prediction equations are given
\[ a_{j,t|t-1} = a_{j,t-1}, \]  
(A-3)
\[ Q_{j,t|t-1} = Q_{j,t-1} + \frac{\sigma^2}{\sigma_M^2}. \]  
(A-4)

The updating equations are
\[ a_{j,t} = a_{j,t-1} + \frac{\phi Q_{j,t|t-1}}{1 + \phi^2 Q_{j,t|t-1}} (y_{j,t} - \phi a_{j,t-1}), \]  
(A-5)
\[ Q_{j,t} = Q_{j,t|t-1} \left( 1 - \frac{\phi^2 Q_{j,t|t-1}}{1 + \phi^2 Q_{j,t|t-1}} \right). \]  
(A-6)

Equation (A-3), (A-4), (A-5) and (A-6) make up the Kalman filter. Equation (A-6) can also be written as a single set of recursions going directly from \( Q_{j,t|t,t-1} \) to \( Q_{j,t+\tau|t,t-1} \), which gives the Riccati equation
\[ Q_{j,t+\tau|t,t} = Q_{j,t|t,t-1} \left( 1 - \frac{\phi^2 Q_{j,t|t,t-1}}{1 + \phi^2 Q_{j,t|t,t-1}} \right) + \frac{\sigma^2}{\sigma_M^2}. \]  
(A-7)

The details of the derivation of the prediction equations and updating equations can be found in Harvey [22]. \( \square \)
Appendix A.3 Proof of Lemma 2

Proof. The time-invariance error variance (\( \bar{p} \)) is the solution of the Riccati equation defined in Equation (A-7). Substituting \( \bar{p} \) for \( Q_{j,t|j,t-1} \) and \( Q_{j,t+1|j,t} \), the Riccati equation turns into the following quadratic equation. Let \( \kappa \) denote the signal-to-noise ratio \( \frac{\sigma^2}{\sigma_M^2} \) and set \( \sigma_M^2 = 1 \).

\[
\bar{p} = \bar{p} \left( 1 - \frac{\phi^2 \bar{p}}{1 + \phi^2 \bar{p}} \right) + \kappa \tag{A-8}
\]

Clearly, Equation (A-8) has a positive root and a negative one. The error variance has to be positive, therefore, \( \bar{p} = \frac{1}{2} \left( \kappa + \sqrt{\kappa^2 + \frac{4}{\phi^2}} \right) \). Plugging the time-invariant filter into Equation (A-5), we obtain Equation (12) in Lemma 2. The prediction error, or, in other words, the Kalman filter innovation (\( v_{j,t} = y_{j,t} - y_{j,t|j,t-1} \)) is i.i.d. Gaussian with mean 0 and variance \( F = 1 + \phi^2 \bar{p} \). This follows from the definition of the Kalman filter estimate \( y_{j,t|j,t-1} \) and the assumption of the Gaussian i.i.d. noise \( M_t \). The details of the derivation of the fixed-point smoother for \( \eta_{j,\tau} \) at time \( \tau < t < q \) can be found in Harvey [22]. The basic idea is that adding \( a_{j,\tau} \) to the state vector at times \( t \geq \tau \) gives an augmented state space model and applying the Kalman filter to this augmented model yields \( m_{j,\tau|t} \) at time \( \tau < t < q \). After some mechanical calculations, we obtain Equation (13). \( \square \)

Appendix A.4 Proof of Proposition 3

Proof. Under risk neutrality, the price of this risky asset is equal to the conditional expectation of the final wealth. Plugging in the Kalman filter estimate and the fixed-point smoother for the hidden-state variable \( \eta_{j,t} \), after some tedious but straightforward calculations, we obtain the price of the firm (Equation (14)). Plugging the expression of the price (\( P_{j,t} \)) into the dollar return process \( r_{j,t} \), we obtain Equation (15). \( \square \)

Appendix A.5 Proof of Proposition 5

Proof. Differentiate the dollar return process (see Equation (15) derived in Proposition 3 with respect to period \( t \) cycle \( j \)'s news innovation \( v_{j,t} \), conditional on \( \sum_{\tau=0}^{t-1} v_{j,\tau} < 0 \), one unit of bad news reduces return by \( \Delta (RNeg) \equiv B_{j,t+1} + \Lambda_2 \Psi_{j,t-1} (v) \mid + 2 \Lambda_2 B_{j,t+2} |v_{j,t}| + 2 \Lambda_2 B_{j,t+3} |a_{j,t-1}| \) units. One unit of good news increases return by \( \Delta (RPos) \equiv B_{j,t+1} - \Lambda_2 \Psi_{j,t-1} (v) \mid - 2 \Lambda_2 B_{j,t+2} |v_{j,t}| - 2 \Lambda_2 B_{j,t+3} |a_{j,t-1}| \) units, if \( \sum_{\tau=0}^{t-1} v_{j,\tau} < 0 \). Clearly, \( \Delta (RNeg) > \Delta (RPos) \) and \( \Delta (RNeg) > 1 \) which implies one unit of bad news reduces return by more than one unit. \( \square \)

Appendix A.6 Proof of Proposition 6

Proof. The unexpected earnings is \( R_{j,t} - E \left( R_{j,t} | T_{j,t}^{\text{Regen}} \right) = y_{j,t} - \phi a_{t-1} \), the innovation \( v_{j,t} \) under the Kalman filter is \( y_{j,t} - \phi a_{t-1} \). Appendix A.3 proves that \( v_{j,t} \) are i.i.d. Gaussian with mean 0 and variance \( \bar{F} \). Differentiate \( \bar{F} \) with respect to \( c, \lambda \) and \( b \), and it is clear to see that as \( c \) and \( \lambda \) decrease, \( \bar{F} \) decreases. As \( b \) increases, \( \bar{F} \) decreases. A smaller variance implies more concentration to the mean 0. \( \square \)
Appendix A.7  Proof of Proposition 7

Proof. As proved in Appendix A.3, the time-invariant Kalman filter estimate and the fixed-point smoother follow a martingale. The first and the second moment of $a_{1,t}$ and $m_{1,t}$ can be obtained from Equation (A-3) and (13). Appendix A.4 shows how to obtain the price and the dollar return process. Plugging the first and the second moment of the Kalman filter estimate and the fixed-point smoother into Equation (15), after some tedious algebra, we obtain the conditional return volatility process $\sigma^2_{j,t}$.

References


Table 1: Summary Statistics for the Enron Return Series

The summary statistics represent the time-series averages and the standard deviation for each variable. The monthly earnings in real dollars are used as the reported earnings and a Kalman filter is applied to estimate the hidden state variable ($\eta_t$). The “cleaning house period” ($q + 1$) is assumed to be the final period of the sample by setting the number of the cleaning cycles ($N$) equal to 1. The price of the S&P Composite Index is simulated following the pricing Equation 14 in section 1. Annualized percentage returns are calculated.

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<th>Mean</th>
<th>Std Dev</th>
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Figure 2: Enron Historical Return

Figure 3: Enron Simulated Return

Figure 4: \( \frac{\text{Actual EPS} - \text{Forecast EPS}}{\text{Price}} \times 100 \)

Figure 5: \( \frac{\text{Actual EPS} - \text{Forecast EPS}}{\text{Price}} \)
This table reports the EGARCH estimates for the monthly Enron return series available from the CRSP data set. It also reports the EGARCH estimates of the simulated percentage return for Enron following the pricing Equation 14. The monthly earnings per share data is obtained from the Compustat tape. The sample period starts in June 1947 and ends in January 2002. A Kalman filter is applied to estimate the hidden state variable \( \eta_t \). The “cleaning house period” \((q + 1)\) is assumed to be the final period of the sample by setting the number of the cleaning cycles \((N)\) to 1. The price of Enron is simulated following the pricing Equation (14) in section 1. Monthly returns are used in the EGARCH estimation.

\[
\begin{align*}
\epsilon_t &= r_t - \mathbb{E}(r_t | I_{t-1}), \\
\epsilon_t &\equiv \sqrt{h_t} \times v_t, \ C \equiv \mathbb{E}(r_t), \\
\ln(h_t) &= K + GARCH \ln(h_{t-1}) + ARCH [\|v_{t-1}\| - \mathbb{E}(\|v_{t-1}\|)] + Smirk v_{t-1}.
\end{align*}
\]

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<th>Parameters</th>
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Table 3: Summary Statistics for CRSP and I/B/E/S Data Sets

The summary statistics represent the time-series averages of the cross-sectional means for each variable. The CRSP monthly return data file and the I/B/E/S data are used to form the sample. There is an average of 2313 stocks in each quarter over the 76 quarter from January 1985 to December 2003.

**EVol:** EGARCH estimation of the conditional volatility of stock return.


\[ SUE_{i,t} = \left( \frac{Actual_{i,t} - Forecast_{i,t}}{Price_{i,t}} \right) \]

**Dispersion:** I/B/E/S statistical measure of dispersion of the estimates of analyst’s forecast for the fiscal period indicated.

**Dvol:** natural logarithm of the dollar volume of trading in the security.

**Firm Size:** natural logarithm of the stock price times the number of shares outstanding.


### Panel A: Summary Statistics for CRSP and I/B/E/S Merged Data Sets

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</tr>
<tr>
<td>Firm Size</td>
<td>13.2161</td>
<td>1.7018</td>
<td>13.1051</td>
</tr>
<tr>
<td>Gibbs</td>
<td>0.0079</td>
<td>0.0075</td>
<td>0.0049</td>
</tr>
</tbody>
</table>

### Panel B: Summary Statistics for CRSP and Corporate Governance Index Merged Data

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>EVol</td>
<td>0.0190</td>
<td>0.3850</td>
<td>0.0112</td>
</tr>
<tr>
<td>Dvol</td>
<td>10.5303</td>
<td>2.6911</td>
<td>10.4316</td>
</tr>
<tr>
<td>Firm Size</td>
<td>11.8520</td>
<td>2.1355</td>
<td>11.7559</td>
</tr>
<tr>
<td>Gibbs</td>
<td>0.0082</td>
<td>0.0105</td>
<td>0.0046</td>
</tr>
</tbody>
</table>
Table 4: Earnings News and Conditional Return Volatility

This table reports the pooled OLS regression of the conditional volatility on stock characteristics. Conditional volatility (EVol) is measured as the EGARCH estimates of the conditional volatility of the stock returns. Panel A reports the pooled OLS regression of EVol on SUE, controlling for other firm-level characteristics. A stock is included in the sample if the SUE measure is available for a given month. The sample starts in April 1985 and ends in December 2003. Panel B analyzes conditional volatility’s asymmetric response to good earnings announcements and bad earnings announcements. Robust Newey-West (1987) t-statistics are reported in square brackets. EVol: EGARCH estimation of the conditional volatility of stock return. SSUE: The square of standardized unexpected earnings (SUE). Gao (2005) following Doyle, Lundholm and Soliman (2003) measures SUE as the standardized difference of the actual reported earnings and the median of the analysts’ forecast from the I/B/E/S data set. \( SUE_{i,t} = \frac{(Actual_{i,t} - Forecast_{i,t})}{Price_{i,t}} \). SSUEP: The square of positive SUE; it equals zero for all non-positive SUE stocks. \( SSUEP = \left( \frac{SUE+|SUE|}{2} \right)^2 \). SSUEN: The square of negative SUE; it equals zero for all non-negative SUE stocks. \( SSUEN = \left( \frac{|SUE| - SUE}{2} \right)^2 \).

\[ \text{lmvlag2: the lagged two month natural log of the price times the shares outstanding.} \]
\[ \text{nyamdvol: the lagged two month natural log of the price times the trading volume of the NYSE and AMEX stocks; it equals zero for all NASDAQ stocks. nasdvol: the lagged two month natural log of the price times the trading volume of the NYSE and AMEX stocks; it equals zero for all NASDAQ stocks. Gibbs: Hasbrouck (2005) estimates the Gibbs Sampler estimates of effective trading costs using the return under Roll’s (1984) model.} \]

<table>
<thead>
<tr>
<th>SSUE</th>
<th>lmvlag2</th>
<th>nyamdvol</th>
<th>nasdvol</th>
<th>Gibbs</th>
<th>Adjusted R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Standardized Unexpected Earnings and Conditional Return Volatility</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0019***</td>
<td>0.0002</td>
<td>0.0111***</td>
<td>0.0144***</td>
<td>0.0147***</td>
<td>0.7268***</td>
</tr>
<tr>
<td>[11.97]</td>
<td>[7.04]</td>
<td>[−13.97]</td>
<td>[11.73]</td>
<td>[12.14]</td>
<td>[18.64]</td>
</tr>
<tr>
<td>SSUEP</td>
<td>SSUEN</td>
<td>lmvlag2</td>
<td>nyamdvol</td>
<td>nasdvol</td>
<td>Gibbs</td>
</tr>
<tr>
<td>Panel B: Asymmetric Response to Good News and Bad News</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0006***</td>
<td>0.0012***</td>
<td>0.0194***</td>
<td>0.0144***</td>
<td>0.0147***</td>
<td>0.7260***</td>
</tr>
<tr>
<td>[4.51]</td>
<td>[6.89]</td>
<td>[−14.07]</td>
<td>[11.81]</td>
<td>[12.21]</td>
<td>[19.04]</td>
</tr>
<tr>
<td>F-test of SSUEP = SSUEN: ( F(1,554766) = 14.28; P\text{-value}:0.0002. )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-test of SSUEP = SSUEN: ( F(1,549128) = 6.73; P\text{-value}:0.0095. )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5: Controlling for Governance

Each year each individual stock is sorted into one of the four groups based on the “Governance Index” (Gindex) formed by Gompers, Ishii and Metrick (2003) of the same year. Group 1, 2, 3 and 4 contain firms whose Gindex fall into interval [2, 5], [6, 9], [10, 13] and [14, 17] respectively. The smaller the Gindex is, the stronger the shareholders’ power is. The pooled OLS regression of the conditional volatility (Evol) on stock characteristics is performed for each of the four governance-index-based groups. The sample starts in January 1990 and ends in December 2003. Robust Newey-West (1987) t-statistics are reported in square brackets. Gibbs: Using publications of the Investor Responsibility Research Center, Gompers, Ishii and Metrick (2003) construct a “Governance Index” as a proxy for the balance of power between shareholders and managers. For each firm, they add one point for every provision that reduces shareholder rights. EVol: EGARCH estimates of the conditional volatility of stock return. SSUE: The square of standardized unexpected earnings (SUE). Gao (2005) following Doyle, Lundholm and Soliman (2003) measures SUE as the standardized difference of the actual reported earnings and the median of the analysts’ forecast from the I/B/E/S data set. SSUEP: The square of positive SUE; it equals zero for all non-positive SUE stocks. SSUEN: The square of negative SUE; it equals zero for all non-negative SUE stocks. lmvlag2: the lagged two month natural log of the price times the shares outstanding. nyamdvol2: the lagged two month natural log of the price times the trading volume of the NYSE and AMEX stocks; it equals zero for all NASDAQ stocks. nasdvol2: the lagged two month natural log of the price times the trading volume of the NYSE and AMEX stocks; it equals zero for all NASDAQ stocks. Gibbs: Hashbrouck (2005) estimates the Gibbs Sampler estimates of effective trading costs using the return under Roll’s (1984) model.

<table>
<thead>
<tr>
<th>Group</th>
<th>SSUE</th>
<th>lmvlag2</th>
<th>nyamdvol2</th>
<th>nasdvol2</th>
<th>Gibbs</th>
<th>Adjusted $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Impact of Earnings News on Conditional Volatility</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0000</td>
<td>-0.0538***</td>
<td>0.0426***</td>
<td>0.0418***</td>
<td>0.2127</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>[0.02]</td>
<td>[-2.87]</td>
<td>[4.25]</td>
<td>[3.58]</td>
<td>[1.02]</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.0016***</td>
<td>-0.0131***</td>
<td>0.0100***</td>
<td>0.0103***</td>
<td>1.8630***</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td>[2.70]</td>
<td>[-13.44]</td>
<td>[16.62]</td>
<td>[15.44]</td>
<td>[3.22]</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0027***</td>
<td>-0.0103***</td>
<td>0.0078***</td>
<td>0.0076***</td>
<td>1.3481***</td>
<td>0.134</td>
</tr>
<tr>
<td></td>
<td>[3.65]</td>
<td>[-38.55]</td>
<td>[40.30]</td>
<td>[40.66]</td>
<td>[12.80]</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.0251***</td>
<td>-0.0076***</td>
<td>0.0052***</td>
<td>0.0052***</td>
<td>0.6483***</td>
<td>0.653</td>
</tr>
<tr>
<td></td>
<td>[22.54]</td>
<td>[-18.44]</td>
<td>[17.21]</td>
<td>[17.16]</td>
<td>[8.91]</td>
<td></td>
</tr>
</tbody>
</table>

F-test of $SSUE_1 = SSUE_4$: $F(1, 189096) = 7.33$; P-value:0.0078.
F-test of $SSUE_1 = SSUE_2 = SSUE_3 = SSUE_4$: $F(3, 189094) = 5.24$; P-value:0.0013.

<table>
<thead>
<tr>
<th>Group</th>
<th>SSUEP</th>
<th>SSUEN</th>
<th>lmvlag2</th>
<th>nyamdvol2</th>
<th>nasdvol2</th>
<th>Gibbs</th>
<th>Gibbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel B: Asymmetric Response to Good News and Bad News</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.1415</td>
<td>0.0000</td>
<td>-0.0054***</td>
<td>0.0426***</td>
<td>0.0418***</td>
<td>0.2172</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[-0.44]</td>
<td>[0.02]</td>
<td>[-4.25]</td>
<td>[4.31]</td>
<td>[3.25]</td>
<td>[1.06]</td>
<td></td>
</tr>
</tbody>
</table>

F-test of $SSUEP = SSUEN$: $F(1, 17478) = 0.19$; P-value:0.6605.

| 2 | 0.0081 | 0.0016*** | -0.0131*** | 0.0101*** | 0.0103*** | 1.8582*** | |
|  | [0.67] | [2.68] | [-13.46] | [16.64] | [15.45] | [3.21] | |

F-test of $SSUEP = SSUEN$: $F(1, 81495)) = 0.30$; P-value:0.5869.

| 3 | 0.0023 | 0.0026*** | -0.0103*** | 0.0078*** | 0.0078*** | 1.3483*** | |
|  | [0.46] | [3.63] | [38.55] | [40.30] | [40.66] | [12.74] | |

F-test of $SSUEP = SSUEN$: $F(1, 78409)) = 0.00$; P-value:0.9476.

| 4 | 0.0129*** | 0.0250*** | -0.0077*** | 0.0052*** | 0.0052*** | 0.6561*** | |
|  | [2.85] | [22.40] | [-18.42] | [17.19] | [17.14] | [8.94] | |

F-test of $SSUEP = SSUEN$: $F(1, 11693)) = 9.03$; P-value:0.0027.
Table 6: **Earnings Forecast Error Distribution**

This table reports the distribution of earnings forecast errors measured in two different ways. Panel A reports the distribution of earnings forecast errors measured by following Abarhanell and Lehavy (2003). For each firm quarter, the standardized unexpected earnings is measured as the actual earnings per share (as reported in I/B/E/S) minus the average of the median of analysts’ forecasts for the given quarter, scaled by the stock price at the end of this quarter and multiplied by 100. Forecast error in Panel A is defined as: \( \frac{\text{Actual EPS} - \text{Forecast EPS}}{\text{Price}} \times 100. \) Panel B reports the distribution of earnings forecast errors measured by following Doyle, Lundholm and Soliman (2003). For each firm quarter, the standardized unexpected earnings is calculated as the actual earnings per share (as reported in I/B/E/S) minus the average of the median of analysts’ forecasts for the given quarter, scaled by the stock price at the end of this quarter. Forecast error in Panel B is defined as: \( \frac{\text{Actual EPS} - \text{Forecast EPS}}{\text{Price}}. \)

<table>
<thead>
<tr>
<th>Range of Earnings Forecast Errors</th>
<th>% of total number of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Forecast Error (Abarhanell and Lehavy (2003)) Distribution</strong></td>
<td></td>
</tr>
<tr>
<td>Forecast errors = 0</td>
<td>2.27</td>
</tr>
<tr>
<td>[-0.1, 0)</td>
<td>13.69</td>
</tr>
<tr>
<td>(0, 0.1]</td>
<td>18.07</td>
</tr>
<tr>
<td>[-0.2, -0.1)</td>
<td>6.56</td>
</tr>
<tr>
<td>(0.1, 0.2]</td>
<td>7.34</td>
</tr>
<tr>
<td>[-0.3, -0.2)</td>
<td>4.38</td>
</tr>
<tr>
<td>(0.2, 0.3]</td>
<td>3.97</td>
</tr>
<tr>
<td>[-0.4, -0.3)</td>
<td>3.23</td>
</tr>
<tr>
<td>(0.3, 0.4]</td>
<td>2.46</td>
</tr>
<tr>
<td>[-0.5, -0.4)</td>
<td>2.61</td>
</tr>
<tr>
<td>(0.4, 0.5]</td>
<td>1.61</td>
</tr>
<tr>
<td>[-1, -0.5)</td>
<td>7.64</td>
</tr>
<tr>
<td>(0.5, 1)</td>
<td>3.77</td>
</tr>
<tr>
<td>[min, -1)</td>
<td>18.04</td>
</tr>
<tr>
<td>(1, max]</td>
<td>4.36</td>
</tr>
</tbody>
</table>

| **Panel B: Forecast Error (Doyle, Lundholm and Soliman (2003)) Distribution** |                                   |
| Forecast errors = 0               | 2.27                             |
| [-0.01, 0)                         | 38.11                            |
| (0, 0.01]                          | 37.22                            |
| [-0.02, -0.01)                     | 6.42                             |
| (0.01, 0.02]                       | 2.05                             |
| [-0.03, -0.02)                     | 2.95                             |
| (0.02, 0.03]                       | 0.69                             |
| [-0.04, -0.03)                     | 1.74                             |
| (0.03, 0.04]                       | 0.39                             |
| [-0.05, -0.04)                     | 1.14                             |
| (0.04, 0.05]                       | 0.20                             |
| [-1, -0.05)                        | 5.29                             |
| (0.05, 1)                          | 0.93                             |
| [min, -1)                          | 0.50                             |
| (1, max]                           | 0.10                             |
Table 7: Controlling for “Meeting the Analysts’ Forecast” and “Big Bath” Effect

Each quarter all stocks are sorted into two groups according to how much their deviations from the analysts’ forecasts and their dispersions of the analysts’ forecasts are. A stock is sorted into group 1 (“Not Smoothed”) if it appears not being actively engaged in earnings smoothing. Group 2 contains firms that appear to practice earnings smoothing actively. A stock is sorted into group 1 if (1) Its $SU\ E$ is less than 0.01 and more than −0.01 while the dispersion of its analysts’ forecasts is less than 0.007; (2) its $SU\ E$ is more than −0.43 but less than −0.01 or its $SU\ E$ is more than 0.01; and (3) its $SU\ E$ falls in interval (−0.01, 0.01) while the dispersion of its analysts’ forecasts is less than 0.007. The rest of the stocks are sorted into the “Smoothed” group. These are stocks that either appear to take “big bath” (i.e., $SU\ E < -0.433$), or just deviate from analysts’ forecasts by 0.01 and their dispersion of analysts’ forecasts is more than 0.007 (i.e., $SU\ E \in [-0.01,0.01]$ and $Dispersion \geq 0.007$). The cutoff point for “big bath” is $SU\ E$’s 1 percentile (−0.033). The cutoff point for the dispersion of analysts’ forecasts is $Dispersion$’s 10 percentile (0.007). Panel A reports the pooled OLS regression of the conditional volatility (Evol) on stock characteristics performed for each of the two groups. The sample starts in April 1985 and ends in December 2003. Robust Newey-West (1987) t-statistics are reported in square brackets. Panel B compares the difference of the “Governance Index” between the “smoothed” group and the “not smoothed” group. P-values for the Mann-Whitney test and Kolmogorov-Smirnov test are for the hypothesis that the “Governance Index” of the two groups come from identical populations. The sample starts in January 1990 and ends in December 2003. Gindex: Using publications of the Investor Responsibility Research Center, Gompers, Ishii and Metrick (2003) construct a “Governance Index” as a proxy for the balance of power between shareholders and managers. For each firm, they add one point for every provision that reduces shareholder rights.


<table>
<thead>
<tr>
<th>Group</th>
<th>SSUE</th>
<th>lnvlag2</th>
<th>nymdvol2</th>
<th>nasdvol2</th>
<th>Gibbs</th>
<th>Adjusted R²</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Impact of Earnings News on Conditional Volatility</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not Smoothed</td>
<td>0.0005</td>
<td>−0.0239***</td>
<td>0.0176***</td>
<td>0.0178***</td>
<td>0.6160***</td>
<td>0.005</td>
</tr>
<tr>
<td>1</td>
<td>[1.81]</td>
<td>[−9.72]</td>
<td>[7.84]</td>
<td>[7.98]</td>
<td>[12.06]</td>
<td></td>
</tr>
<tr>
<td>Smoothed</td>
<td>0.0026***</td>
<td>−0.0137***</td>
<td>0.0107***</td>
<td>0.0112***</td>
<td>1.3492***</td>
<td>0.049</td>
</tr>
<tr>
<td>2</td>
<td>[5.85]</td>
<td>[−56.80]</td>
<td>[51.06]</td>
<td>[53.54]</td>
<td>[6.63]</td>
<td></td>
</tr>
<tr>
<td>F-test of $SSUE$ in group 1 vs $SSUE$ in group 2 : $F(1,549128) = 17.79$, P-Value:0.0000.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
<th>P-Value Mann-Whitney Test</th>
<th>Kolmogorov-Smirnov Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not Smoothed</td>
<td>8.95</td>
<td>9.00</td>
<td>2.77</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Smoothed</td>
<td>9.52</td>
<td>10.00</td>
<td>2.64</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Number of Observations(firm-year): Not Smoothed Group (1): 2464; Smoothed Group (2): 12389
Table 8: Controlling for the Dispersion of Analysts’ Forecast

Each quarter each individual stock is sorted into one of the five quintiles based on the dispersion of analysts' forecasts over this quarter. The pooled OLS regression of the conditional volatility (Evol) on stock characteristics is performed for each of the five dispersion-based quintiles. Panel A reports the impacts of earnings news on conditional volatility. Panel B analyzes conditional volatility’s asymmetric response to good earnings announcement and bad earnings release. The sample starts in April 1985 and ends in December 2003. Robust Newey-West (1987) t-statistics are reported in square brackets. Dispersion: I/B/E/S statistical measure of dispersion of the estimates of analysts’ forecasts for the fiscal period indicated. Evol: EGARCH estimates of the conditional volatility of stock return. SSUE: The square of standardized unexpected earnings (SUE). Gao (2005) following Doyle, Lundholm and Soliman (2003) measures SUE as the standardized difference of the actual reported earnings and the median of the analysts’ forecast from the I/B/E/S data set. SSUEP: The square of positive SUE; it equals zero for all non-positive SUE stocks. SSUEN: The square of negative SUE; it equals zero for all non-negative SUE stocks. lnvlag2: the lagged two month natural log of the price times the shares outstanding. nyamdvol2: the lagged two month natural log of the price times the trading volume of the NYSE and AMEX stocks; it equals zero for all NASDAQ stocks. nasdvol2: the lagged two month natural log of the price times the trading volume of the NYSE and AMEX stocks; it equals zero for all NASDAQ stocks. Gibbs: Hasbrouck (2005) estimates the Gibbs Sampler estimates of effective trading costs using the return under Roll’s (1984) model.

<table>
<thead>
<tr>
<th>Quintile</th>
<th>SSUE</th>
<th>lnvlag2</th>
<th>nyamdvol2</th>
<th>nasdvol2</th>
<th>Gibbs</th>
<th>Adjusted R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Impact of Earnings News on Conditional Volatility</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0007**</td>
<td>−0.0167***</td>
<td>0.0138***</td>
<td>0.0139***</td>
<td>0.9768***</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>[2.10]</td>
<td>[−7.74]</td>
<td>[7.10]</td>
<td>[7.31]</td>
<td>[11.99]</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.0002</td>
<td>−0.0253**</td>
<td>0.0208**</td>
<td>0.0209**</td>
<td>0.7024***</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>[0.32]</td>
<td>[−2.31]</td>
<td>[2.14]</td>
<td>[2.20]</td>
<td>[13.17]</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0071**</td>
<td>−0.0157***</td>
<td>0.0115***</td>
<td>0.0119***</td>
<td>0.6307***</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>[2.54]</td>
<td>[−34.70]</td>
<td>[35.31]</td>
<td>[36.35]</td>
<td>[15.75]</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.0131*</td>
<td>−0.0151***</td>
<td>0.0109***</td>
<td>0.0114***</td>
<td>0.7032***</td>
<td>0.068</td>
</tr>
<tr>
<td></td>
<td>[1.78]</td>
<td>[−39.17]</td>
<td>[24.85]</td>
<td>[38.57]</td>
<td>[13.64]</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.0016***</td>
<td>−0.0226***</td>
<td>0.0149***</td>
<td>0.0157***</td>
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<td>Panel B: Asymmetric Response to Good News and Bad News</td>
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<td>0.045***</td>
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</table>

F-test of SSUE1 = SSUE5: F (1, 425806) = 8.90; P-value:0.0029.
F-test of SSUE1 = SSUE2 = SSUE3 = SSUE4 = SSUE5: F (4, 425803) = 4.98; P-value:0.0005.