Problem 4-33

Known: Temperature distribution in a planar slab

\[ T(x) = T_0 \left( 1 - \frac{x^2}{L^2} \right) + \frac{T_2 - T_1}{L} \left( \frac{x}{L} \right) + \frac{T_1 + T_2}{2} \]

Find:
A) Draw a graph of \( T(x) \) for \( T_1 = 350 \text{ K} \), \( T_2 = 450 \text{ K} \), \( L = 22 \text{ mm} \)
B) Value of \( \frac{dT}{dx} \) at \( x = -L \)
C) Calculate \( \dot{q}(-L) \) with \( k = 25 \text{ W/m.K} \) (line direction)
D) Location of \( \dot{q}(x) = 0 \), and \( T \) at this location
E) At \( x = +L \), is \( \dot{q} \) from slab to surroundings or from surroundings to the slab?

Sketch:

Assumptions:
- 1-D Conduction
- Constant thermal conductivity
- Steady-state

Analysis:
A)
B) To find \( \frac{dT}{dx} \bigg|_{x=-L} \), first find \( \frac{dT}{dx} \):

\[
T(x) = 50 \left(1 - \frac{x^2}{L^2}\right) + \frac{T_2 - T_1}{2} \left(\frac{x}{L}\right) + \frac{T_2 + T_1}{2}
\]

\[
\frac{dT}{dx} = -2 (50) \frac{x}{L^2} + \frac{T_2 - T_1}{2L}
\]

\[
\left. \frac{dT}{dx} \right|_{x=-L} = -2 (50) (-L) \frac{L}{L^2} + \frac{T_2 - T_1}{2L}
\]

\[
\left. \frac{dT}{dx} \right|_{x=-L} = \frac{-2 (50) (-L)}{0.022} + \frac{450 - 350}{2 (0.022)}
\]

\[
\left. \frac{dT}{dx} \right|_{x=-L} = 6810 \frac{K}{m}
\]

C) \( \dot{Q}''(-L) = -k \left. \frac{dT}{dx} \right|_{x=-L} \) as given by Fourier's Law

\[
\dot{Q}''(-L) = -25 \frac{W}{m \cdot K} \left(6810 \frac{K}{m}\right)
\]

\[
\dot{Q}''(-L) = -170,450 \frac{W}{m^2} \quad \text{or} \quad 170,450 \frac{W}{m^2} \quad \text{in the -x}
\]

D) Set \( \frac{dT}{dx} = 0 \) and solve for \( x \):

\[
\frac{dT}{dx} = -\frac{100x}{L^2} + \frac{T_2 - T_1}{2L} = 0
\]

\[
-\frac{100x}{(0.022)^2} = -\frac{100}{2 (0.022)}
\]

\[
x = 0.011 \text{ m}
\]

\[
T(0.011) = 50 \left(1 - \frac{0.011^2}{0.022^2}\right) + \frac{100 (0.011)}{2 (0.022)} + \frac{450 + 350}{2}
\]

\[
T(0.011) = 462.5 \text{ K}
\]

E) At \( x = L \), heat transfer is from the slab to the surroundings. Note \( \frac{dT}{dx} < 0 \) at \( x = L \).

Comments: Pay attention to the sign convention for Fourier's Law.
PROBLEM 4.40

**Known:**
\[ \dot{Q}_{\text{conv}} = 20 \, \frac{\text{W}}{\text{m}^2} \]
\[ T_s = 35^\circ \text{C} \]
\[ T_\infty = 32^\circ \text{C} \]

**Find:** Heat-transfer coefficient

**Assumptions:** Steady-state

**Analysis:**
\[ \dot{Q} = h_{\text{conv}} (T_s - T_\infty) \]
\[ 20 \, \frac{\text{W}}{\text{m}^2} = h_{\text{conv}} (35^\circ \text{C} - 32^\circ \text{C}) \]

\[ h_{\text{conv}} = 6.67 \, \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \]

**Comments:** Remember not to confuse heat flow with heat flux.

\[ \theta \approx 1.5 \]
Problem 4.47

Known: \( T_S = 100^\circ C \)
\( T_{\text{sur}} = 80^\circ C \)
\( \varepsilon_S = (a) 1.0 \quad (b) 0.8 \)

Find: Net radiant flux from surface

Assumptions: steady state

Analysis:
\[
\dot{Q}_{\text{rad}} = \varepsilon_S A_S \sigma \left( T_S^4 - T_{\text{sur}}^4 \right)
\]
\[
\dot{Q}_{\text{rad}}'' = \frac{\dot{Q}_{\text{rad}}}{A_S}
\]
\[
\dot{Q}_{\text{rad}}'' = \varepsilon_S \sigma \left( T_S^4 - T_{\text{sur}}^4 \right)
\]

(a) \( \dot{Q}_{\text{rad}}'' = (1.0) \left( 5.67051 \times 10^{-8} \frac{W}{m^2 \cdot K^4} \right) \left( (100 + 273K)^4 - (80 + 273K)^4 \right) \)

(b) \( \dot{Q}_{\text{rad}}'' = (0.8) \left( 5.67051 \times 10^{-8} \frac{W}{m^2 \cdot K^4} \right) \left( (100 + 273K)^4 - (80 + 273K)^4 \right) \)

\[
\begin{align*}
(a) \ & \dot{Q}_{\text{rad}}'' = 217.1 \ \frac{W}{m^2} \\
(b) \ & \dot{Q}_{\text{rad}}'' = 173.7 \ \frac{W}{m^2}
\end{align*}
\]

Comments: Since the only difference between (a) and (b) is the emissivity, simply find the blackbody radiant flux and then multiply by the necessary emissivity.