

To Learn or Not Learn?

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ABSTRACT

This paper focuses on the firm decision maker's knowledge management behavior, a learning process which is used to acquire additional knowledge and improve the firm's profit capacity. A conceptual model is developed to address the ways of executing learning process and presents a model incorporating knowledge management into firm's profit maximization problem. We decompose the marginal benefit and cost associated with knowledge management behavior focusing on immediate and future gains/losses that guides the decision maker in allocating learning effort. The numerical solutions provide the optimal learning effort at each point in time illustrating the decision maker's learning strategies. In addition, the shadow value of knowledge follows different trajectories as different learning strategies are adopted.

Key Words: Knowledge Management, Knowledge Acquisition, Dynamic Programming

JEL Classification No.: C61, D21, D83.

INTRODUCTION

Today's global market-driven economy imposes greater competitive pressures on firm decision makers. Sustaining competitiveness over the long run involves attention to growth prospects associated with the innovations needed to keep pushing the competitive envelope, and the efficiency gains needed to ensure that implemented technologies can succeed. The effective management of knowledge contributes to both sources of profitability growth.

Starting from Arrow's learning-by-doing model (Arrow, 1962), knowledge entered the decision making model as a firm-specific capital good generated from the firm's previous production experience, which is usually represented by cumulative gross investment (Arrow, 1962; Sheshinski, 1967), cumulative output (Sheshinski, 1967; Rosen, 1972), or cumulative market inputs (Rosen, 1972). Under this setting, knowledge may appear in the production function along with other production inputs or as a productivity index which is multiplied by the production function in the economic model. Aside from the influence on the production function, the cumulative knowledge from previous production experience may also influence production decisions from the cost side as the marginal cost decreases with the firm's production experience represented by the cumulative output (Spence, 1981).

Economists soon realized that the production of knowledge does not emanate from the firm alone. The experience of other firms in the same industry may offer valuable information and influence other's production behavior as well. The learning-from-others phenomenon is more easily observed when new technology is introduced to the industry because the firm's previous experience may not offer as much insight as the new technology comes on line. Empirical efforts indicate that decision makers may learn from early adopters (Besley & Case,

1993), the peers with similar socio-economic background (Ueda, 2002), or other decision makers in the same social communication networks (Conley & Udry, 2001).

The learning-by-doing and learning-from-others models reflect the concept that knowledge needs to be acquired (Arrow, 1962). Despite the information from own or others' experiences, the decision maker can also invest in information from other sources, such as the outreach services offered by government sponsored research institutes (Wozniak, 1987). The situation that the decision maker may acquire knowledge intentionally raises the issue of passive versus active learning. Malerba (1992) defines that active learning represents activities that the firm focus at learning consciously and expressly. R&D is one of the examples. Feder and Slade (1984) emphasize the difference between passive and active information acquisition activities, noting that while both activities contribute to knowledge accumulation, they imply different costs, lead to different results of knowledge accumulation and have different impact on production decisions.

Nowadays, firms are usually viewed as a dynamic system of knowledge management with a variety of different strategies for the knowledge acquisition. The possibility of information exchange across firms is raised by the appearance of strategic alliances formed by R&D contracts, joint development agreements, or consumer-supplier partnerships (Mowery, Oxley, & Silverman, 1996). Strategic alliances broaden the scope of information flows. Not only can the firms producing the same product be bound by the strategic alliances, the firms that have consumer-supplier relationships can be connected through it as well. Along with these various knowledge strategies, different knowledge strategies bring diversity in knowledge type, learning speed, and breadth of knowledge base (Bierly & Chakrabarti, 1996).

Knowledge plays an important role in firm decision making as it involves the selection of technological systems and the execution of technologies that are implemented. As such, knowledge management is a learning behavior that is undertaken to acquire additional knowledge and improve the firm's profit potential. The decision maker can undertake different knowledge management activities to learn. However, the question of whether the decision maker should allocate efforts to learn arises when the costs of learning are considered. Additionally, learning at a certain period of time does not mean that learning in the following period is rational. The decision maker may stop allocating effort to learn with continuous and discontinuous learning strategies for decision makers being admissible.

The objective of this study is to formalize the decision maker's knowledge management behavior and investigate the decision maker's learning decision over time. Although some studies (Grant, 1996; Spender, 1996; Spender & Grant, 1996) apply a "knowledge-based approach" to investigate the firm's knowledge management behavior, defining the characteristics of knowledge and analyzing the role of hierarchy in knowledge accumulation, sharing, and transferring within the firm. In this paper, the focus is on how knowledge is acquired and applied to improve the firm's profitability. The knowledge sharing and transferring within the firm is not the main concern. A pilot survey of the use of knowledge management practices conducted by Statistics Canada shows that the size of a firm plays an important role in knowledge management. For over 70% of micro and small firms, the manager takes the responsibility for knowledge management activities (Earl & Gault, 2003). To focus on the learning decision in terms of whether the effort is allocated to knowledge management activities, we assume that the decision maker (or the manager) is the central agent undertaking the knowledge management activities while other employees follow the orders from the manager. In

this situation, we can focus on the roles of learning costs and benefits and how the learning decisions are made. The conceptual model addresses different ways of executing the learning process. This is followed by a decision making model illustrating the firm's profit maximization problem based on its production and knowledge management constraints. Numerical solutions to a dynamic programming model are presented, providing more concrete aspects for knowledge management behavior and quantifying the shadow value of knowledge that is distributed overtime.

CONCEPTUAL FRAMEWORK

The conceptual framework addresses how the firm decision maker obtains additional knowledge by engaging in knowledge management behavior. The core elements of knowledge management involve i) the learner, to whom the learning process is attributed, ii) the learning process, which consists of acquiring information and processing information to additional knowledge, and iii) the learning outcome, which is what we obtain from learning.

The focus is on the firm decision maker as the learner while the learning outcome is the updated knowledge base playing an important role in the firm's decision making and can lead, potentially, to profitability growth. Although knowledge is treated often as information in the economic literature, knowledge is in fact distinguished from information since information only represents a single and isolated element while knowledge is a web, which is a result of information connection and integration (Mayer 2005, Loasby 2005). More specifically, information is an input, which is used for knowledge generation (Kirzner, 2005). Based on this concept, the learning process, where the learner obtains additional knowledge, involves two phases. The first is the information acquisition phase involving different kinds of information acquisition activities. Some information acquisition activities are generated internally from past

experience and can involve lower learning costs, such as learning-by-doing. Other information acquisition activities involve the firm engaging in socially-oriented learning activities as external information is acquired. This form of information acquisition activity brings valuable information from outside the firm, but it can involve higher learning costs as the learner puts forth greater cognitive effort as he communicates with others.

After collecting the internal and external information from information acquisition activities, the learner still needs to filter this information through a learning mechanism as a means of translating the information set into additional knowledge that can support production decision making. The second phase of the learning process is the knowledge updating phase which is associated with the use of a learning mechanism that translates the information collected into additional useful knowledge feeding into the existing knowledge base. The learning process is complete when one recalls and successfully adapts already existing experiences, new experiences and information to new situations, re-indexing them and making them part of the new production operations procedures. Consequently, the knowledge base is updated so that the learner (the firm decision maker) can use it in future production decision making.

The decision maker may engage in social learning during both information acquisition and knowledge updating phases. In the information acquisition phase, internal information can be collected from historical data while external information can be acquired by the decision maker engaging in socially-oriented activities. In the knowledge updating phase, the decision maker can either employ a learning mechanism to process collected information by himself or employ a joint learning mechanism with other decision makers so that they can update the knowledge base together. The possibility that the decision maker can involve in different levels of social learning extends the options of knowledge management scheme. Table 1 describes

different types of knowledge management behavior that can be undertaken by the decision maker during the learning process. Behaviors (i) and (iii) describe internal learning where the information is collected from past experiences or R&D experiments. Although behavior (ii) does collect external information from outside the firm, it is characterized as non-social learning because the decision maker does not allocate effort to communicate with others and the information is collected solely by observation. The rest of the knowledge management behaviors described in table 1 involve social learning at different levels. Learning via conversation illustrates the situation that the decision maker acquires external information by having a conversation with other decision makers. After the conversation, the decision makers construct their own information set and update the knowledge base by themselves. Learning from cooperation is a more formal social learning activity, resulting in creating a common information set (e.g., a memorandum detailing a common conclusions from the formal meetings) shared by the cooperating decision makers. Learning via collaboration is the knowledge management behavior most dependent on social learning. The collaborative firms are integrated into a more extensive system where the decision makers share the same information set, update the knowledge base together, and even make the common production decisions. In fact, the actual knowledge management behavior may be more complex than what is described in table 1 as it may involve a mixture of different types of learning activities.

Insert Table 1 Here

THEORETICAL MODEL

Knowledge management is a learning process involving information acquisition and the use of a learning mechanism, which transfers the collected information into the firm's knowledge

base. The conceptual framework in the previous section, however, does not reflect the costs and benefits of knowledge management and therefore stops short of offering optimal rules for the decision maker regarding knowledge management decisions. This section presents a dynamic decision model involving costs of information acquisition and costs of employing the firm's learning mechanism. Marginal benefits and costs associated with knowledge management are decomposed and guide the decision maker to allocate the optimal effort for knowledge management.

Profit Maximization Model

Assume the decision maker maximizes the net present value of firm i 's profit over time under its production and knowledge management constraint (Table 2). In every period, the stock of knowledge (or knowledge base), KB_t^i , presents how well the decision maker understands the executed production technology, and along with the physical input (X_t^i) and the effort of general management for production process ($e_t^{i.M}$), the quantity of physical output (Y_t^i) can be determined. The firm can also devote some effort to its knowledge management behavior improving the decision maker's understanding about production technology in the future.

Knowledge management is a learning process involving two phases: information acquisition and knowledge updating. The information collected in the information acquisition phase comes from two sources: internal information acquisition and external social learning. The internal information is denoted as firm-specific information, ω_t^i , generated from past production experience represented by past input use. The external information arises from the decision maker engaging in social learning activities, such as having a conversation or cooperating with other decision makers. The accumulated effort associated with social learning

activities, $E_t^{i,\Psi}$, equals current effort plus the depreciated effort of past social learning activities, or $E_t^{i,\Psi} = e_t^{i,\Psi} + (1 - \delta) \cdot E_{t-1}^{i,\Psi}$. The accumulated effort is used in the information generation function since past efforts associated with social learning still impact current information generation. Combining the firm-specific information and accumulated effort associated with social learning leads to the information set, which is what the decision maker manages in the knowledge updating phase, or $\Omega_t^i = \phi_t^i(\omega_t^i, E_t^{i,\Psi})$. The information acquisition phase has thresholds assigned to both firm-specific information and accumulated effort associated with social learning. If the firm-specific information does not achieve a minimal hurdle or threshold, then the firm does not possess a sufficient foundation of experience and, thus, is not capable of benefiting from social learning. If the accumulated effort for social learning does not exceed the threshold, the decision maker does not allocate sufficient effort to social learning activities. In both cases, external information cannot be obtained and the information set only reflects the firm's internal information from historical data.

The information collected from internal information acquisition and social learning activities are stored as an information set, and the issue is whether the decision maker allocates effort to use the learning mechanism and transfer information into useful knowledge. Once the information set is transferred by the learning mechanism, additional knowledge is generated and incorporated into the existing knowledge base. The updated knowledge base then enters the production function next period and influences future decision making. Let Λ_t^i denote the change of knowledge base; i.e., $KB_{t+1}^i = KB_t^i + \Lambda_t^i$. Knowledge generation is also viewed as a production process where the information set (Ω_t^i) and effort in applying the learning mechanism ($e_t^{i,LM}$) are inputs, or $\Lambda_t^i = LM_t^i(\Omega_t^i, e_t^{i,LM})$. In addition, if effort to the learning

mechanism or the information set does not achieve the threshold level, the learning mechanism cannot be implemented and no additional knowledge is generated, or $\Lambda_t^i = 0$ if $e_t^{i,LM} \leq e_{\min}^{i,LM}$ or $\Omega_t^i \leq \Omega_{\min}^i$.

Let $C_t^I(\omega_t^i, E_t^{i,\Psi})$ denote the cost of information acquisition, which is increasing in ω_t^i and $E_t^{i,\Psi}$, and $C_t^{LM}(\Omega_t^i, e_t^{i,LM})$ denotes the cost of employing the learning mechanism, which is increasing in Ω_t^i and $e_t^{i,LM}$. Considering a T -period time horizon,

$$\begin{aligned} \text{Max} \quad \pi = & \sum_{t=1}^{T-1} \left(\frac{1}{1+r} \right)^{t-1} \left[P_t \cdot F_t(X_t^i, KB_t^i, e_t^{i,M}) \right. \\ & \left. - W_t X_t^i - C_t^I(\omega_t^i, E_t^{i,\Psi}) - C_t^{LM}(\Omega_t^i, e_t^{i,LM}) \right] \\ & + \left(\frac{1}{1+r} \right)^{T-1} \cdot [P_T \cdot F_T(X_T^i, KB_T^i, e_T^{i,M}) - W_T X_T^i] \end{aligned} \quad (1)$$

subject to the knowledge management constraints in table 2.

Insert Table 2 Here

Properties of Information Acquisition and Knowledge Generation

Properties of information acquisition

Some properties of the information acquisition and knowledge updating can be gleaned by graphical analysis. The information set (Ω_t^i) is the result from an information generation process where the firm-specific information (ω_t^i) and the accumulated effort associated with social learning activities ($E_t^{i,\Psi}$) are inputs. Figure 1 illustrates this production relationship.

The firm-specific information is accumulated continuously from past production experience and reflected by the firm's historical input data up to the present. Further, if the firm-specific information or the accumulated effort associated with social learning does not achieve the given thresholds, ω_0 and $E_{\min}^{i,\Psi}$, respectively, the external information cannot be accessed; thus, the information set will only reflect the firm's historical data.

We assume that the information set has a linear relationship with firm-specific information if either the firm-specific information or the accumulated effort associated with social learning does not achieve their respective thresholds. Notice that for a given information set (Ω^{B1}), it can be generated either by (i) $\omega = \omega'$ and $E^{i,\Psi} > E_{\min}^{i,\Psi}$ or (ii) $\omega = \omega''$ and $E^{i,\Psi} \leq E_{\min}^{i,\Psi}$. The case $\omega = \omega'$ and $E^{i,\Psi} > E_{\min}^{i,\Psi}$ denotes the situation when the decision maker obtains the external information via social learning activities where he applies sufficient effort to communicate with the decision makers of other firms (e.g., cooperating with others). The case $\omega = \omega''$ and $E^{i,\Psi} \leq E_{\min}^{i,\Psi}$ denotes the situation when the decision maker does not allocate sufficient effort to social learning activities and the external information cannot be obtained. As such, the decision maker generates the information set by using the firm-specific information only. We further assume that $\Omega_t^i \big|_{\text{with external information}} > \Omega_t^i \big|_{\text{without external information}}$ for $\omega^i > \omega_0$ implying that as the firm-specific information exceeds the threshold, ω_0 , the decision maker can add to the information set from having both internal and external information than from internal information only.

Insert Figure 1 Here

Figure 2 establishes the relationship between firm-specific information, ω_t^i , and the accumulated effort associated with social learning, $E_t^{i,\Psi}$, which reflects the kink in the relationship between ω_t^i and Ω_t^i in figure 1.

The thresholds of firm-specific information and accumulated efforts associated with social learning activities are represented by the ω_{\min} line and $E_{\min}^{i,\Psi}$ line, respectively. Presented in terms of the isoquant of information acquisition, the isoquants below ω_{\min} are horizontal lines indicating that the information sets only depend on firm-specific information when the firm-specific information is not greater than the threshold ω_{\min} . $E_{\min}^{i,\Psi}$ line is downward sloping instead of a vertical line allowing for the prospect where the more firm-specific the information the decision maker has, the less effort the decision maker must engage in social learning activity. In addition, the isoquants are horizontal lines when the accumulated effort for social learning does not achieve the threshold, $E_{\min}^{i,\Psi}$. If both firm-specific information and accumulated efforts for social learning exceed their thresholds, the isoquant is assumed to be downward sloping and convex toward the origin as the information acquisition function reflected in table 2.

Insert Figure 2 Here

Properties of knowledge generation

Similarly, the change of the knowledge base (Λ_t^i) can be influenced by a learning mechanism where the information set (Ω_t^i) and current period effort allocated to the learning mechanism ($e_t^{i,LM}$) are inputs. It is also assumed that the learning mechanism cannot be active if either the

information set or the current period effort to the learning mechanism does not achieve either of their respective thresholds. In this case, the additional knowledge cannot be generated from the learning mechanism, and there is no change in the knowledge base. Figure 3 illustrates this knowledge generation relationship.

Insert Figure 3 Here

The isoquant of knowledge generation associated with figure 3 is illustrated in figure 4. The thresholds of the information set and effort for the learning mechanism are denoted by the Ω_{\min} line and $e_{\min}^{i,LM}$ line, respectively. As long as the information set and effort for learning mechanism exceed their respective thresholds, the isoquant is assumed to be downward-sloping and convex toward the origin; otherwise, the learning mechanism is not activated and no additional knowledge can be generated ($\Lambda_t^i = 0$).

Insert Figure 4 Here

Optimization analysis

The optimal conditions of the profit maximization model usually show that the decision maker allocates inputs so that the associated marginal benefit and cost are balanced. In this study, the decision maker has to decide on the optimal level for physical input (X_t^i), efforts associated with social learning activities ($e_t^{i,\Psi}$), and efforts to use the learning mechanism ($e_t^{i,LM}$). Since all three variables contribute to future knowledge accumulation, the potential

benefit from knowledge gains and the opportunity cost arising from devoting effort to knowledge management must be addressed when investigating the optimal decisions.

We need the following optimization conditions associated with the planning problem in (1) to show how the decision maker allocates physical inputs, the effort associated with social learning activities, and the effort for employing the learning mechanism when considering the knowledge management constraints:

$$\begin{aligned} \frac{\partial \pi}{\partial X_t^i} = & \left(\frac{1}{1+r} \right)^{t-1} \cdot \left(P_t \frac{\partial F_t}{\partial X_t^i} - W_t \right) - \sum_{\tau=t}^{T-1} \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left[\frac{\partial C_\tau^I}{\partial \omega_\tau^i} \frac{\partial \omega_\tau^i}{\partial X_t^i} + \frac{\partial C_\tau^{LM}}{\partial \varphi_\tau^i} \frac{\partial \varphi_\tau^i}{\partial \omega_\tau^i} \frac{\partial \omega_\tau^i}{\partial X_t^i} \right] \\ & + \sum_{\tau=t+1}^T \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left[P_\tau \cdot \frac{\partial F_\tau}{\partial KB_\tau^i} \cdot \left(\sum_{d=t}^{\tau-1} \frac{\partial LM_d^i}{\partial \varphi_d^i} \frac{\partial \varphi_d^i}{\partial \omega_d^i} \frac{\partial \omega_d^i}{\partial X_t^i} \right) \right] = 0 \quad \forall t=1, \dots, T-1 \end{aligned} \quad (2)$$

$$\frac{\partial \pi}{\partial X_t^i} = \left(\frac{1}{1+r} \right)^{t-1} \cdot \left[P_t \frac{\partial F_t}{\partial X_t^i} - W_t \right] = 0 \quad \forall t=T \quad (3)$$

$$\begin{aligned} \frac{\partial \pi}{\partial e_t^{i,\Psi}} = & - \left(\frac{1}{1+r} \right)^{t-1} \cdot \left(P_t \frac{\partial F_t}{\partial e_t^{i,M}} \right) - \sum_{\tau=t}^{T-1} \left[\frac{(1-\delta)^{t-\tau}}{(1+r)^{\tau-1}} \right] \cdot \left[\frac{\partial C_\tau^I}{\partial E_\tau^\Psi} + \frac{\partial C_\tau^{LM}}{\partial \varphi_\tau^i} \frac{\partial \varphi_\tau^i}{\partial E_\tau^\Psi} \right] \\ & + \sum_{\tau=t+1}^T \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left\{ P_\tau \frac{\partial F_\tau}{\partial KB_\tau^i} \left[\sum_{d=t}^{\tau-1} (1-\delta)^{d-t} \frac{\partial LM_d^i}{\partial \varphi_d^i} \frac{\partial \varphi_d^i}{\partial E_d^{i,\Psi}} \right] \right\} \leq 0, \\ e_t^{i,\Psi} \geq 0, \quad e_t^{i,\Psi} \left(\frac{\partial \pi}{\partial e_t^{i,\Psi}} \right) = 0 \quad \forall t=1, \dots, T-1 \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{\partial \pi}{\partial e_t^{i,LM}} = & - \left(\frac{1}{1+r} \right)^{t-1} \cdot \left(P_t \frac{\partial F_t}{\partial e_t^{i,M}} \right) - \left(\frac{1}{1+r} \right)^{t-1} \cdot \frac{\partial C_t^{LM}}{\partial e_t^{i,LM}} \\ & + \sum_{\tau=t+1}^T \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left[P_\tau \frac{\partial F_\tau}{\partial KB_\tau^i} \frac{\partial LM_\tau^i}{\partial e_t^{i,LM}} \right] \leq 0, \\ e_t^{i,LM} \geq 0, \quad e_t^{i,LM} \left(\frac{\partial \pi}{\partial e_t^{i,LM}} \right) = 0 \quad \forall t=1, \dots, T-1 \end{aligned} \quad (5)$$

Since the physical input is assumed to be greater than zero, conditions (2) and (3) show that the decision maker will use the physical input until its marginal profit is zero. Condition (4) reflects how the decision maker allocates the effort associated with social learning over time, while condition (5) represents how the decision maker chooses the optimal level of effort to allocate to the learning mechanism. As the effort associated with social learning and effort to employ the learning mechanism contribute to knowledge management by acquiring external information and updating information collected into additional knowledge, the use of physical input also contributes to knowledge management by cumulating internal information from past production experience. For each variable, the elements of marginal benefits and costs associated with knowledge management are decomposed from the mathematical conditions and listed in table 3.

Take condition (4) as an example. The component,

$$\sum_{\tau=t+1}^T \left(\frac{1}{1+r} \right)^{\tau-t} \cdot \left\{ P_{\tau} \frac{\partial F_{\tau}}{\partial KB_{\tau}^i} \left[\sum_{d=t}^{\tau-1} (1-\delta)^{d-t} \frac{\partial LM_d^i}{\partial \varphi_d^i} \frac{\partial \varphi_d^i}{\partial E_d^{i,\Psi}} \right] \right\}, \text{ denotes the potential future benefits from one}$$

more unit of the effort associated with social learning. The component,

$$\sum_{\tau=t}^{T-1} \left[\frac{(1-\delta)^{\tau-t}}{(1+r)^{\tau-1}} \right] \cdot \left[\frac{\partial C_{\tau}^I}{\partial E_{\tau}^{\Psi}} + \frac{\partial C_{\tau}^{LM}}{\partial \varphi_{\tau}^i} \frac{\partial \varphi_{\tau}^i}{\partial E_{\tau}^{i,\Psi}} \right], \text{ denotes the direct marginal cost which follows from the}$$

increase of current and future cost because of one more unit of effort associated with social

learning, while the first component, $\left(\frac{1}{1+r} \right)^{t-1} \cdot \left(P_t \frac{\partial F_t}{\partial e_t^{i,M}} \right)$, is the opportunity cost which is

measured as the decrease in current revenue arising from diverting resources to this social learning activity and away from current productive efforts.

The role of learning benefits and costs in learning decisions are revealed in these conditions as well. It is shown that the decision maker chooses positive social learning or

learning mechanism effort only when the marginal benefit and marginal cost are balanced. In each period, if marginal benefits associated with social learning effort are less than its marginal cost, the decision maker does not allocate effort to engage in social learning activities. Similarly, if the marginal benefit associated with effort for employing the learning mechanism does not exceed its corresponding marginal cost, the decision maker does not allocate effort to employ the learning mechanism.

Insert Table 3 Here

Learning strategies over time

The optimization mathematical conditions reveal the roles of marginal benefits and costs associated with effort for social learning activities ($e_t^{i,\Psi}$) and effort to use the learning mechanism ($e_t^{i,LM}$), which guide decision making. Although the internal information is accumulated continuously from past production experiences represented by the accumulated input use, the decision maker may not engage in social learning activities or employ the learning mechanism to update the knowledge base at every period; i.e., knowledge management may not be a continuous process.

To consider the learning strategies over time, we define *learning* as the situation when the learning mechanism is employed and the knowledge base is updated, while *not learning* is the situation when the decision maker does not allocate effort to employ the learning mechanism. The reason is that, even though both effort for social learning activities and effort to use a learning mechanism are associated with knowledge generation, the effort to use a learning mechanism directly results in the increase of knowledge base. Since the decision maker can

choose to *learn* or *not learn* in each period, there are 2^T combinations of learning decisions from $t = 1$ to $t = T$, illustrating the decision maker's various learning strategies.

Focusing on the decision maker's optimal effort to employ the learning mechanism from $t = 1$ to $t = 3$, there are eight possible learning strategies presented in table 4 that can be summarized into 5 types: (i) *always learn*, (ii) *never learn*, (iii) *wait to learn*, (iv) *quit learning*, and (v) *learn-in-bursts*. The first and the second learning strategies in table 4 illustrate the situations that the decision maker chooses to either update the knowledge base continuously or never update the knowledge base. Strategies (3) and (4) denote the *wait to learn* behavior when the decision maker does not use the learning mechanism in the early periods, but chooses to employ it later. Strategies (5) and (6) are the opposite situation where the decision maker employs the learning mechanism in the early periods, then quits using the learning mechanism later. The last two strategies are *learn-in-bursts* cases, reflecting the situation that the decision maker changes the learning decision over time.

Insert Table 4 Here

The learning strategies presented in table 4 show that learning at a given period of time does not mean that the learning in the following period is rational, meaning that learning behavior can be continuous or discontinuous.

NUMERICAL MODEL: DYNAMIC PROGRAMMING APPROACH

The conceptual framework describes knowledge management schemes addressing different ways of executing the learning process. Although knowledge is important in production decisions, it may not always be profitable for the firm to build upon the knowledge

base at each potential learning opportunity. The theoretical model indicates that the potential marginal benefits and the associated marginal costs of learning clearly impact learning decisions. Additionally, knowledge updating may not be a continuous process since the decision maker must accumulate sufficient information before he employs a learning mechanism to transform the collected information into additional knowledge. Whether the decision maker should allocate additional effort to knowledge management depends on the potential benefits and costs associated with the effort.

In this section, a tractable numerical dynamic programming model is constructed from the theoretical model in section 3 and investigates how the decision maker manages to allocate effort to knowledge management over time. The decision maker's learning strategies in response to the changes in the output price are presented in this section as well.

Model Setting

The numerical model is similar to the theoretical model where the effort for information acquisition and the effort for using the learning mechanism are combined and renamed as knowledge management effort, e_t^L . In each period, the physical input, X_t , and the effort of general management for production process, $(\bar{e}_t - e_t^L)$, is used to produce the physical output, Y_t . The knowledge management effort, e_t^L , is used to collect the production information and employ the learning mechanism so that additional knowledge can be generated in the future. Thus knowledge management effort is defined as learning effort since it represents effort employed in the learning process to generate additional knowledge.

Knowledge generation is a discrete production process since the additional knowledge is not necessarily produced in every period. If the decision maker does not accumulate sufficient

production information, the learning mechanism is ineffective and no additional knowledge is attained. The accumulated effort for knowledge management, E_t^L , represents the decision maker's accumulated effort in learning. If the accumulated learning effort reaches the assigned threshold(s), then the decision maker devotes enough effort to learning so that the collected information is sufficient to be transformed into additional knowledge successfully. If the accumulated learning effort does not meet the assigned threshold(s), then the decision maker does not devote enough effort to learning and no additional knowledge is generated. Once the additional knowledge is generated, it will contribute to the firm's future knowledge base and influence the firm's production performance and decision making activities.

Assume that the cost associated with the learning effort is a linear function, and the dynamic programming model is constructed as following:

$$V = \underset{X_1, \dots, X_T, e_1^L, \dots, e_T^L}{Max} \sum_{t=1}^T \frac{1}{(1+r)^{t-1}} \cdot [P_t \cdot Y_t(X_t, KB_t, e_t^M) - W_t X_t - \alpha_4 \cdot e_t^L] \quad (6)$$

subject to production and knowledge management constraints listed in table 5.

Insert Table 5 Here

Focusing on a 25-period time horizon, the available effort in each period is denoted as \bar{e}_t and the decision maker can choose to devote either one or zero unit of effort to learning, indicating whether the decision maker decides to *learn* or to *not learn*. The thresholds for the knowledge generation process are denoted by em_1 and em_2 while the parameters, W_t and α_4 , indicate the unit cost of the physical input and the learning effort, respectively.

The decision maker's understanding about the executed production technology is represented by the knowledge base, KB_t , and the function, $h(KB_t)$, is defined as a knowledge function illustrating the way the knowledge base influences production performance. The idea of knowledge function is similar to that of Feder and Slade (1984). Although the knowledge base has a positive impact on production performance, the impact increases at a decreasing rate. Thus the firm's knowledge function converges to a limiting saturation level as the knowledge base becomes very large.

The firm's initial knowledge base is assumed to be at the minimum level implying the decision maker only has basic understanding about the production technology at the beginning of the time horizon. In each period, the decision maker decides whether he wants to allocate learning effort ($e_t^L = 0$ or 1). If no effort is allocated to learning, all the available effort will be used for the management of production process, i.e., $e_t^L = 0$ and $e_t^M = \bar{e}_t = 2$. If the decision maker decides to allocate effort for knowledge management, i.e., $e_t^L = 1$, this learning effort will contribute to the improvement of the firm's knowledge base through the function $g(E_t^L)$ at the time the accumulated learning effort hits the threshold. Two thresholds, em_1 and em_2 , are assigned in this numerical model to illustrate the discrete property of knowledge generation. The thresholds also reflect that there are two opportunities for knowledge improvement. The knowledge base increases marginally once the accumulated learning effort hits the first threshold (em_1), and the knowledge base increases considerably when the second threshold (em_2) is reached.

Numerical Results

In the numerical model, the effort for information acquisition and the effort for using the learning mechanism are combined and renamed as knowledge management effort, which is regarded as learning effort since it is used to acquire more knowledge. In addition, the assumption that learning effort is either one or zero indicates whether the decision maker decides to *learn* or to *not learn*.

The types of the learning strategies need to be reconsidered as well. Two thresholds are assigned in the numerical model indicating that the decision maker faces two opportunities for knowledge base improvements. As the accumulated learning effort hits the first threshold, the decision maker gains a little more knowledge. The second threshold is assumed to be much greater than the first one, so it takes more time to reach the second one. But once the decision maker accumulates enough learning effort and reaches the second threshold, considerably more knowledge improvement can be obtained from this learning behavior. After reaching the second threshold, the decision maker will not employ any learning effort since no additional knowledge can be generated afterward.

Thus, the learning strategy category is based on the decision maker's learning behavior from the beginning until the time the second threshold is reached. *Always learn* denotes the situation where the decision maker applies the learning effort from the beginning until the accumulated learning effort hits the second threshold. *Wait to learn* denotes the situation where the decision maker does not apply the learning effort in the first few periods, but once he applies the learning effort, he keeps applying it until the second threshold is reached. *Quit learning* indicates the situation where the decision maker only applies learning effort to complete the first knowledge base improvement, and then the learning effort is never employed afterward. Finally,

the *learn-in-bursts* strategy states the situation in which the decision maker temporarily stops the learning process, but after a few periods, restarts the learning effort and then completes the second knowledge base improvement.

Basic case

In the basic case, the output price is assumed to be constant over time, i.e., $P_t = 5$ for $t = 1, \dots, T$. The paths of optimal learning effort and knowledge base are shown in figure 5. It is shown that the decision maker employs the learning effort from the beginning until the second threshold is reached (*Always learning* strategy). The path of optimal knowledge base, KB_t^* , indicates that the firm reaches the first threshold and acquires more knowledge at the end of period 2. After six more periods of learning, the firm gains much more knowledge making the knowledge base jump from 2 to 8.5 at the end of period 8. Once the second threshold is reached, the firm cannot acquire more knowledge by employing the learning effort. Thus, no additional learning effort is employed afterward.

Insert Figure 5 Here

The path of optimal learning effort in figure 5 indicates that the decision maker chooses the *always learn* strategy if the output price is constant over time. The output price is one of the factors impacting potential learning benefits and costs. For example, a decrease in future output price implies that the opportunity cost of learning is higher in the earlier periods than it is in the later periods. In early periods (while the output price is still high), the decision maker gives up relatively more profit if he decides to divert resources away from physical output production to

learning. The change in output price alters the net learning benefits, and therefore influences learning behavior. In the next section, the decision maker's learning decisions are investigated under the condition that the output price is allowed to change in different periods.

Learning strategies as output price changes

In this section, the decision maker is assumed to make production and learning decisions under the situation where he knows that the output price is decreasing from \$5 to \$3 at time t^D . There is no uncertainty in this case since the timing of the decrease of the output price, t^D , is known. Various learning strategies can be observed as t^D changes. To see this, the results of optimal learning effort for case $t^D=2$ to case $t^D=11$ are listed in figures 6 to 15.

Take $t^D=2$ as an example. The decision maker knows that the output price decreases from \$5 to \$3 at period 2, and he makes production and learning decisions based on this understanding. Figure 6 illustrates the decision maker adopting a *wait to learn* strategy. Since the decision maker knows that the output price is decreasing at period 2, he decides to allocate total effort to produce the physical output while the price is still high in period 1. The use of learning effort is delayed until period 2, but once the decision maker starts to use learning effort, he keeps using it until the accumulated learning effort hits the second threshold. Thus, the first improvement of the knowledge base is completed at the end of period 3 and the second improvement is completed at the end of period 9.

Insert Figures 6 through 15 Here

Summarizing the results in figures 6 to 15, four learning strategies as the output price decreases at different period emerge:

- ***Wait to Learn***: If the output price decreases from \$5 to \$3 before period 4, the decision maker delays the entire learning process (the *wait to learn* strategy). The decision maker devotes the total effort to producing the physical output while the output price is still high, and starts to allocate the learning effort from the time the output price decreases. Once the decision maker starts to devote effort to learning, he keeps using the learning effort until the second threshold is reached.
- ***Learn-in-bursts***: If the output price decreases between period 4 and period 9, the decision maker decides to *learn-in-bursts*. The learning effort is employed in the first two periods so that the decision maker can complete the first knowledge base improvement and obtain the profit gain from production enhancement. Then the learning process stops temporarily so that the decision maker can allocate all effort to produce the physical output while the output price is still high. The decision maker starts allocating the learning effort again when the output price decreases. It is shown that the later the output price decreases, the longer the decision maker shut down the learning process.
- ***Quit Learning***: If the output price decreases at period 10, the decision maker adopts a *quit learning* strategy, which means the learning effort is employed only for the first two periods. The first knowledge base improvement is completed at the end of period 2, and since no more learning effort is employed afterward, the knowledge base remains at the same level for the rest of the periods.

- ***Always Learn***: If the output price decreases at an even later period (in our case, if the output price decreases at or after period 11), the decision maker adopts an *always learn* strategy, which means he allocates the learning effort from the beginning until the accumulated effort reaches the second threshold.

Among the learning strategies shown in figures 6 to 15, the *quit learning* strategy in figure 14 may not appear consistent with other learning strategies. As the output price drops at period 10, the decision maker does not choose a *learn-in-bursts* strategy (such as figures 8 to 13), where he can temporarily shut down the learning process after finishing the first knowledge improvement and restart to use the learning effort to complete the second knowledge improvement. Nor does he choose an *always learn* strategy (such as figure 15) to finish the entire learning process as soon as possible. With a fixed terminal time and discounting, the decision maker does not choose a *learn-in-bursts* strategy since less time remains for the firm to enjoy the profit gain from the second knowledge improvement. The decision maker does not choose an *always learn* strategy, because it sacrifices too much profit in the early periods when the output price is still high. As such, the *quit learning* strategy in figure 14 is a critical case where neither the *learn-in-bursts* strategy nor the *always learn* strategy is more attractive than the *quit learning* strategy.

THE SHADOW VALUE OF KNOWLEDGE

The dynamic programming model yields the shadow value of knowledge, which indicates the change of the value function if the decision maker has one more unit of knowledge. As the optimal production and learning decisions are made, the shadow value of knowledge is simultaneously determined by the profit maximization model.

There are two state variables in the dynamic programming model: the knowledge base, KB_t , and the accumulated learning effort, $\sum_t e_t^L$. The initial conditions of the state variables indicate that the decision maker only has basic knowledge about the production technology ($KB_1 = 1$) and he does not have any accumulated learning effort before making production and learning decisions. The decision variables, physical input (X_t) and learning effort (e_t^L), are chosen under these initial conditions to maximize the sum of the net present value of the firm's profit over time. Once the optimal physical input and the learning effort are chosen, the optimal path for the state variables, KB_t^* and $\left(\sum_t e_t^L\right)^*$, can be drawn for each time t . The value function (V) represents the sum of net present value of the firm's profit over 25 periods as the optimal decision variables are employed.

Assume that the decision maker has one more unit of knowledge at the beginning of the period. Different optimal decisions will be made to reach the maximization goal and a different value function will be obtained. Thus the shadow value of the knowledge at $t = 1$ is defined as the difference between the value functions since it represents how much the sum of the net present value will increase if the decision maker has one more unit of knowledge. Appendix A provides details for calculating the shadow value of knowledge.

The Basic Case

The shadow value of the knowledge at each point in time is illustrated in figure 16. As the output price is constant ($P_t = 5$), the decision maker adopts an *always learn* strategy, which makes the knowledge base increase from 1 to 2 at the end of second period and then jumps from 2 to 8.5 when the second knowledge base improvement is completed at the end of period 8. The

value of additional knowledge is the increase of the sum of the net present value at time τ when having one more unit of knowledge base than the optimal level. It is shown that the shadow value of knowledge decreases smoothly overtime and the curve is slightly concave after the second knowledge base improvement is revealed at period 9.

Insert Figure 16 Here

Shadow Value of Knowledge as Output Price Changes

The numerical results already indicate that the change of the output price influences the firm's learning strategies. In fact, the change of the output price has an impact on the shadow value of the knowledge as well. The values of the knowledge for price-decreasing cases are presented in figures 17 to 20. Since the decision maker adopts different learning strategies as the output price decreases in different periods, the value of the knowledge corresponding to the same learning strategy are presented in the same figure. Figures 17 and 18 show the patterns of the shadow value of knowledge corresponding to the *wait-to-learn* and the *learn-in-bursts* strategies, respectively. Again, t^D indicates the time the output price decreases from \$5 to \$3. In figure 17, cases $t^D=2$ and $t^D=3$ present the same shadow value of the knowledge after period 11, since both cases face the same knowledge base, accumulated learning effort, and output price after period 11. As for the periods earlier than period 11, we can see that the value of the knowledge in case $t^D=3$ is greater than that in case $t^D=2$ in every period.

Insert Figure 17 Here

A similar situation is observed in figure 18 where the decision maker adopts a *learn-in-bursts* strategy. The decision maker learns in the first two periods and then stops learning temporarily while the output price remains at \$5. The decision maker starts learning again at the time the output price decreases. In addition, the later the output price decreases, the later the decision maker restarts the learning process. All the cases in figure 18 have the same shadow value of the knowledge after period 14, which is when all the cases finish the second knowledge improvement. In addition, for the period earlier than period 14, the later the output price decreases, the greater the shadow value of the knowledge at each period.

Insert Figure 18 Here

The patterns of the shadow value of knowledge for case $t^D=10$ and case $t^D=11$ are shown in figures 19 and 20, respectively. In case $t^D=10$, the decision maker takes the *quit learning* strategy; the value of the knowledge increase very slightly before period 3 and then decreases over time. In case $t^D=11$, the output price drops at period 11, and the decision maker takes an *always learn* strategy. The shadow value of the knowledge in figure 20, however, shows a very different pattern than other cases. It drops rapidly at period 4 and then decreases slowly after period 11.

Insert Figure 19 Here

Insert Figure 20 Here

Figures 17 to 20 show the patterns of the shadow value of the knowledge as the output price decrease at different periods. The shadow value of knowledge is influenced by the change of output price and the learning strategy adopted by the decision maker. Generally, the shadow value of the knowledge decreases at an increasing rate after the second knowledge base improvement is finished. However the pattern of the shadow value of knowledge may not be smooth before the second knowledge improvement is completed. The possible reason is that the decision maker might change his current and future learning decision if he has one more unit of knowledge. Take $t^D=11$ as an example. The decision maker will choose a *quit learning* strategy instead of an *always learn* strategy if he has one more unit of knowledge at the beginning of time (i.e., $KB_1=2$). Similarly, the assumption that the decision maker has one more unit of knowledge at time t could change the optimal learning decision from that point on. At each point of time, if the assumption of having one more unit of knowledge does not alter the optimal learning decision, the pattern of the shadow value of knowledge decreases smoothly like the patterns in figures 16 and 19. However, if the increase of knowledge base at a given period alters the optimal learning decisions from that point on, the pattern of the shadow value of knowledge might be bumpy as what is shown in figure 18 or figure 20. Take figure 20 as an example, the assumption that the decision maker has one more unit of knowledge at period 4 (or any period after period 4) does not influence the optimal learning decision from that point on, so the pattern of the shadow value decreases smoothly after period 4, presenting the profit gain of having one more unit of knowledge and the influence from the discounting factor. However, the assumption that the decision maker has one more unit of knowledge at period 3 (or any period earlier than

period 3) alters the learning option decisions. The change of learning decision reflects on the value function and leads to the significant difference in the shadow value of knowledge before and after period 4.

CONCLUDING COMMENTS

In this paper our intent is to organize our thinking on how learning takes place; specifically, describing the allocation of effort to learn over time. The conceptual framework addresses different ways of executing learning process and points out the possibilities of acquiring external information from social learning activities. The importance of marginal benefits and costs associated with the learning effort are shown in the theoretical model and guides the decision maker to make the optimal learning decisions. The possibility that the decision maker can choose to *learn* or *not learn* results in different types of learning strategies, such as *always learn*, *wait to learn*, *learn in burst*, and *quit learning*. These learning strategies are reflected by the numerical study as the decision maker faces different market circumstances. Finally, the numerical study quantifies the shadow value of knowledge, which is implicated in the knowledge managing behavior.

We find that different learning strategies can emerge and that these strategies depend on various factors involved in the learning process. The knowledge management constraints presented in the theoretical model illustrate how the decision maker generates knowledge from the knowledge managing behavior. Knowledge management involves the information acquisition phase where the decision maker generates the information set and the knowledge updating phase where the learning mechanism is employed to transfer information collected into addition knowledge. The thresholds in both phases imply that knowledge accumulation is not a continuous process. The decision maker has to accumulate sufficient foundations from past

production experience and contribute sufficient effort to social learning activities so that the firm can benefit from external information. Additionally, the decision maker has to build a sufficient information set and allocate sufficient effort to employ the learning mechanism to generate additional knowledge.

The constraints in the decision making model also imply that the learning strategies are influenced by firm-specific characteristics since the outcome of knowledge management is affected by the decision maker's learning ability. Different decision makers may have different information acquisition and knowledge generating functions, as well as different assigned thresholds. One can expect that the decision maker with greater learning ability usually faces smaller thresholds or generates more knowledge when he applies the same information set and the same effort to employ the learning mechanism.

How the decision maker implements knowledge into production behavior also has an impact on the decision maker's learning strategies. The knowledge function in the numerical study can be viewed as the effective knowledge which is associated with the decision maker's ability to implement his knowledge into actual production behavior. Two decision makers with the same knowledge about the production technology may choose different ways to apply the knowledge into action. As a result, one decision maker may have more effective knowledge than the other.

The learning strategies may also be influenced by factors that are not controlled by the decision maker, such as the changes in the output price. The numerical results show that the decision maker adopts different learning strategies (e.g. *always learn*, *wait to learn*, *learn-in-bursts*, and *quit learning*) as he knows about the changes in the future output price. Furthermore,

the shadow value of knowledge follows different patterns as the decision maker adopts different learning strategies in response to different market circumstances.

One should note that these factors influence learning decisions because they alter the net benefit of learning directly or indirectly. With knowledge management behavior being a means to improve the firm's profit capacity, the decision maker will adopt a learning strategy that is believed to bring the largest benefit to the firm. The decision maker takes different learning decisions in response to different knowledge management constraints or different market conditions. As a result, continuous learning and discontinuous learning can be optimal learning decisions.

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APPENDIX A: CALCULATING THE SHADOW VALUE OF KNOWLEDGE

Assume that $\left(V_\tau \left| \begin{matrix} KB_\tau, \sum_{t=1}^{\tau-1} e_t^L \end{matrix} \right. \right)$ indicates the value function at time τ , obtained as the decision maker employs the optimal physical input and learning effort to maximize the sum of the net present value of the firm's profit from time τ to time T under the conditions that KB_τ and $\sum_{t=1}^{\tau-1} e_t^L$ are

$$\text{given, i.e., } V_\tau \left| \begin{matrix} KB_\tau, \sum_{t=1}^{\tau-1} e_t^L \end{matrix} \right. = \text{Max}_{X_\tau, \dots, X_T, e_\tau^L, \dots, e_T^L} \sum_{t=\tau}^T \frac{1}{(1+r)^{t-\tau}} \cdot [P_t \cdot Y_t(X_t, KB_t, e_t^M) - W_t X_t - \alpha_4 \cdot e_t^L].$$

The shadow value of the knowledge at time τ is represented by V_{KB_τ} which is defined as the difference of the value functions for having one more unit of knowledge:

$$V_{KB_\tau} = V_\tau \left| \begin{matrix} KB_\tau = KB_\tau^* + 1, \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L \right)^* \end{matrix} \right. - V_\tau \left| \begin{matrix} KB_\tau = KB_\tau^*, \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L \right)^* \end{matrix} \right.$$

where KB_τ^* and $\left(\sum_{t=1}^{\tau-1} e_t^L \right)^*$ are the optimal level of the knowledge base and the optimal

accumulated learning effort at the beginning of time τ provided by the deterministic numerical results.

Table 1: Knowledge management behavior

Possible Knowledge Management Behavior	Description	Level of Social Learning
(i) Learning-by-doing	The decision maker acquires internal information from: (1) Experiences from manipulative behavior in past production process. (2) Learning from reflecting the past production decisions and past production realization (input use and output level). Both (1) and (2) represent “learning by own experience”, and relate to learning from past mistakes.	Non-Social Learning
(ii) Learning via Observation	The decision maker acquires external information by observing others’ production decision and realization directly.	Non-Social Learning
(iii) Experiments (R&D)	The decision maker sets up his own experiments. When doing so, he sacrifices short-term income for long-term understanding of the process.	Non Social Learning
(iv) Externally Contracted	The decision maker acquires external information by going to Seminar or Workshop.	Social Learning in the information acquisition phase
(v) Directed (planned)	The decision maker knows what he wants to study / develop, so he acts on a plan to learn. Consulting experts and buying specific book/software to address specific issues are examples.	Social Learning in the information acquisition phase
(vi) Learning via Conversation	Decision makers share information by having a conversation. This is the chance to compare each other’s experience. From this conversation, decision makers construct their own information sets and update knowledge base by employing their own learning mechanisms.	Social Learning in the information acquisition phase
(vii) Learning via Cooperation	Decision makers actively discuss common issues and come to common conclusions (i.e., walk away with the same information set), but they transfer the information set into knowledge by employing “own” learning mechanism and make “own” final decisions.	Social Learning in information acquisition phase
(viii) Learning via Collaboration	Decision makers actively plan to generate data and translate it into learning information “together”. These collaborators generate common information set, employ the same learning mechanism, and make joint production decisions.	Social Learning in both information acquisition and knowledge updating phases.

Table 2: Knowledge management constraints

Knowledge Management Constraints	Description
$\omega_t^i = \omega_t^i (\sum_{\tau=1}^t X_\tau^i) = \sum_{\tau=1}^t X_\tau^i$	The firm-specific information is generated from the past input using.
$E_t^{i,\Psi} = e_t^{i,\Psi} + (1 - \delta)E_{t-1}^{i,\Psi}$	The accumulated effort associated with social learning is the current effort plus the depreciated effort of past social learning activities. ^a
$\Omega_t^i = \varphi_t^i(\omega_t^i, E_t^{i,\Psi})$	Information acquisition, which is determined by the firm-specific information and the accumulated effort associated with social learning.
$\Omega_t^i = \varphi_t^i(\omega_t^i) \text{ if } E_t^{i,\Psi} \leq E_{\min}^{i,\Psi} \text{ or if } \omega_t^i \leq \omega_{\min}$	If the firm-specific information or the accumulated effort associated with social learning does not reach the threshold, the information set only reflects the firm's historical data.
$KB_{t+1}^i = KB_t^i + \Lambda_t^i$	The knowledge base updating
$\Lambda_t^i = LM_t^i(\Omega_t^i, e_t^{i,LM})$	The knowledge generation depends on the information set and the effort for learning mechanism
$\Lambda_t^i = 0 \text{ if } e_t^{i,LM} \leq e_{\min}^{i,LM} \text{ or } \Omega_t^i \leq \Omega_{\min}^i$	No additional knowledge can be generated if either the information set or the effort for learning mechanism does not exceed the threshold.
$\bar{e}_t^i = e_t^{i,M} + e_t^{i,\Psi} + e_t^{i,LM}$	Total effort constraint
$X_t^i > 0 \quad e_t^{i,\Psi} \geq 0 \quad e_t^{i,LM} \geq 0$	The non-negative constraint for choice variables.
$t = 1, \dots, T$	Index for each period of time

a. Ψ denotes the external information acquisition associated with social learning activities (such as cooperation, collaboration, or going to a seminar).

Table 3: Marginal benefits and costs associated with knowledge management

Inputs	Marginal Benefits/ Marginal Costs	Mathematical Components
X_t^i	Current and Future Marginal Cost	$\sum_{\tau=t}^{T-1} \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left[\frac{\partial C_\tau^I}{\partial \omega_\tau^i} \frac{\partial \omega_\tau^i}{\partial X_t^i} + \frac{\partial C_\tau^{LM}}{\partial \phi_\tau^i} \frac{\partial \phi_\tau^i}{\partial \omega_\tau^i} \frac{\partial \omega_\tau^i}{\partial X_t^i} \right]$
	Potential Future Benefit	$\sum_{\tau=t+1}^T \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left[P_\tau \cdot \frac{\partial F_\tau}{\partial KB_\tau^i} \cdot \left(\sum_{d=t}^{\tau-1} \frac{\partial LM_d^i}{\partial \phi_d^i} \frac{\partial \phi_d^i}{\partial \omega_d^i} \frac{\partial \omega_d^i}{\partial X_t^i} \right) \right]$
$e_t^{i,\Psi}$	Current Opportunity Cost	$\left(\frac{1}{1+r} \right)^{t-1} \cdot \left(P_t \frac{\partial F_t}{\partial e_t^{i,M}} \right)$
	Current and Future Direct Cost	$\sum_{\tau=t}^{T-1} \left[\frac{(1-\delta)^{\tau-t}}{(1+r)^{\tau-1}} \right] \cdot \left[\frac{\partial C_\tau^I}{\partial E_\tau^\Psi} + \frac{\partial C_\tau^{LM}}{\partial \phi_\tau^i} \frac{\partial \phi_\tau^i}{\partial E_\tau^{i,\Psi}} \right]$
	Potential Future Benefit	$\sum_{\tau=t+1}^T \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left\{ P_\tau \frac{\partial F_\tau}{\partial KB_\tau^i} \left[\sum_{d=t}^{\tau-1} (1-\delta)^{d-t} \frac{\partial LM_d^i}{\partial \phi_d^i} \frac{\partial \phi_d^i}{\partial E_d^{i,\Psi}} \right] \right\}$
$e_t^{i,LM}$	Current Opportunity Cost	$\left(\frac{1}{1+r} \right)^{t-1} \cdot \left(P_t \frac{\partial F_t}{\partial e_t^{i,M}} \right)$
	Current Direct Cost	$\left(\frac{1}{1+r} \right)^{t-1} \cdot \frac{\partial C_t^{LM}}{\partial e_t^{i,LM}}$
	Potential Future Benefit	$\sum_{\tau=t+1}^T \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left[P_\tau \frac{\partial F_\tau}{\partial KB_\tau^i} \frac{\partial LM_\tau^i}{\partial e_\tau^{i,LM}} \right]$

Table 4: Learning strategies

Learning Strategies	$t = 1$	$t = 2$	$t = 3$	Learning Type
(1)	Learn	Learn	Learn	<i>Always Learn</i>
(2)	Don't Learn	Don't Learn	Don't Learn	<i>Never learn</i>
(3)	Don't Learn	Learn	Learn	<i>Wait to Learn</i>
(4)	Don't Learn	Don't Learn	Learn	<i>Wait to Learn</i>
(5)	Learn	Don't Learn	Don't Learn	<i>Quit Learning</i>
(6)	Learn	Learn	Don't Learn	<i>Quit Learning</i>
(7)	Don't Learn	Learn	Don't Learn	<i>Learn-in-Bursts</i>
(8)	Learn	Don't Learn	Learn	<i>Learn-in-Bursts</i>

Table 5: Production and knowledge management constraints

Production and Knowledge Management Constraints	Descriptions
$Y_t(X_t, KB_t, e_t^M) = h(KB_t) \cdot (X_t^{0.3} \cdot (e_t^M)^{0.5})$ $= h(KB_t) \cdot (X_t^{0.3} \cdot (\bar{e}_t - e_t^L)^{0.5})$	Production function
$h(KB_t) = 1 + 1.7 \cdot \{1 - \exp[-0.15(KB_t - 1)]\}$	Knowledge function
$KB_{t=1} = KB_{\min} = 1$	Initial state of knowledge base.
$KB_{t+1} = KB_t + \Lambda_t$	Knowledge base accumulation (knowledge base updating).
$E_t^L = \sum_{\tau=1}^{\tau=t} e_{\tau}^L$	Accumulated learning effort at the end of time t .
$\Lambda_t = g_1(E_t^L) = (E_t^L)^{0.1} \quad \text{if} \quad \sum_{\tau=1}^{\tau=t-1} e_{\tau}^L < em1 \quad \text{and} \quad \sum_{\tau=1}^{\tau=t} e_{\tau}^L = em1$ $\Lambda_t = g_2(E_t^L) = (E_t^L)^{0.9} \quad \text{if} \quad \sum_{\tau=1}^{\tau=t-1} e_{\tau}^L < em2 \quad \text{and} \quad \sum_{\tau=1}^{\tau=t} e_{\tau}^L = em2$ $\text{otherwise } \Lambda_s^i = 0$	Knowledge generation as the accumulated learning effort hit the threshold(s).
$W_t = 2, \quad \alpha_4 = 2, \quad r = 0.1, \quad X_t \geq 0, \quad \bar{e}_t = 2,$ $e_t^L = 0 \quad \text{or} \quad 1, \quad em_1 = 2, \quad em_2 = 8, \quad T = 25$	Given parameters.

Figure 1: Information Acquisition

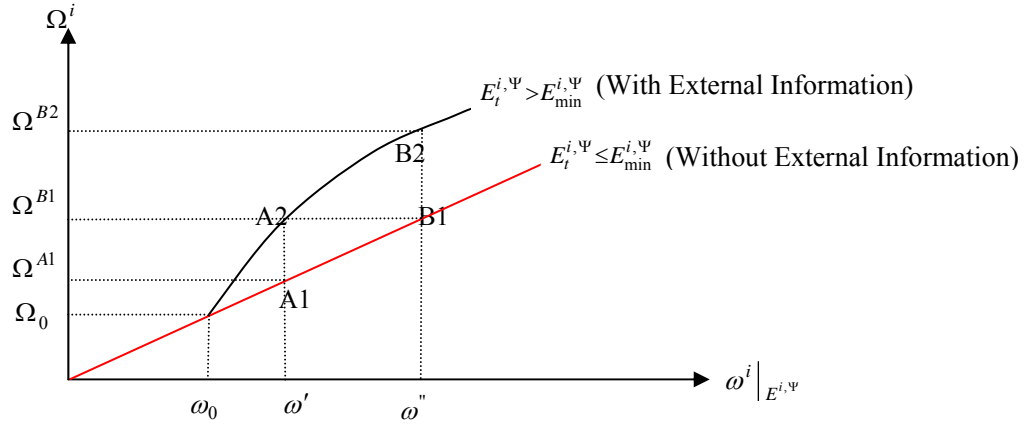


Figure 2: The isoquant of information acquisition

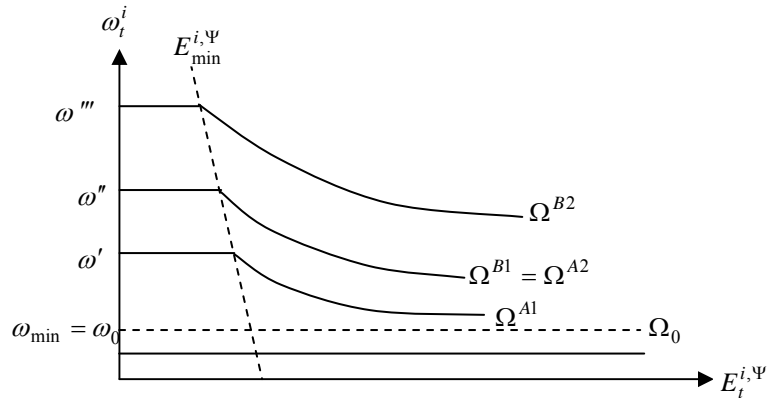


Figure 3: Knowledge generation

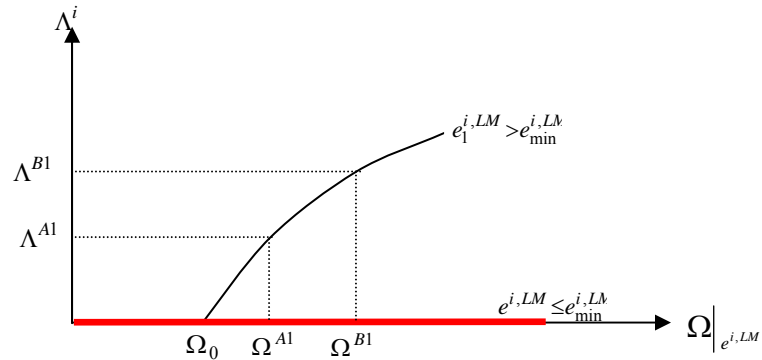


Figure 4: The isoquant of knowledge generation

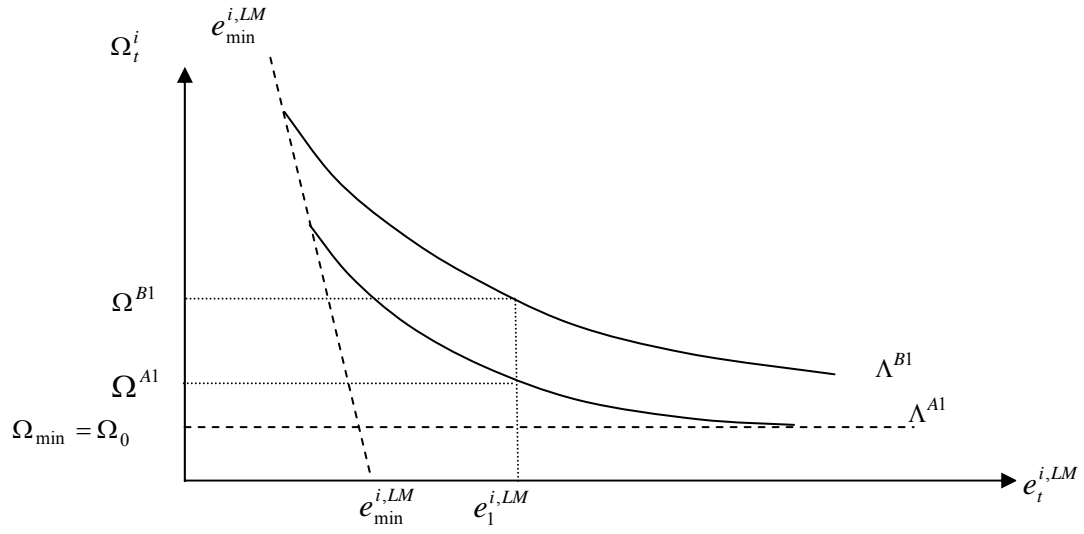


Figure 5: The paths of optimal learning effort and knowledge base – basic case

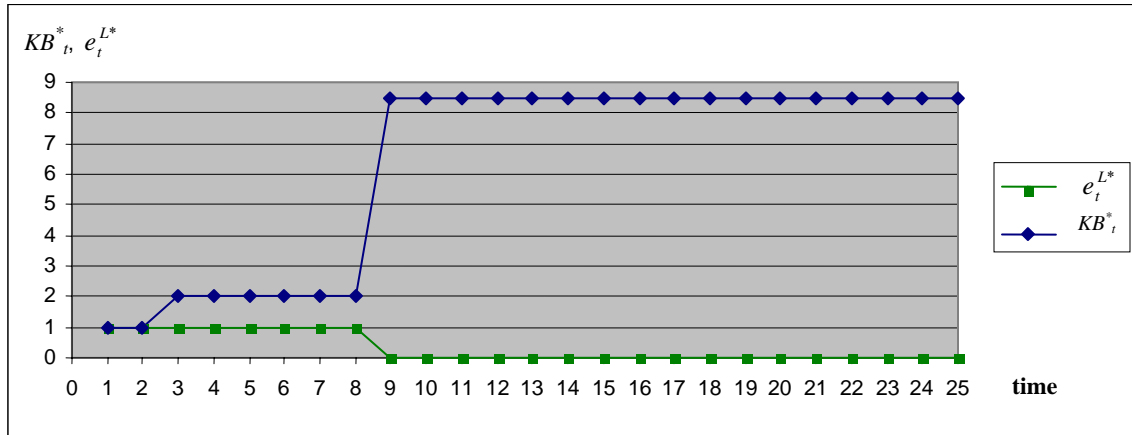


Figure 6: A wait to learn strategy ($t^D=2$)

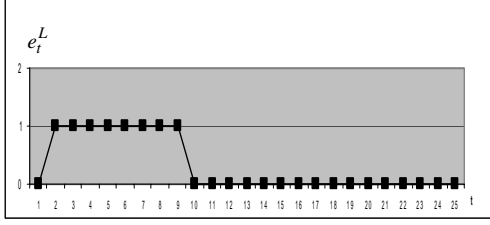


Figure 7: A wait to learn strategy ($t^D=3$)

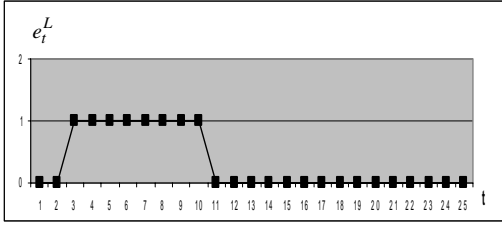


Figure 8: A learn-in-bursts strategy ($t^D=4$)

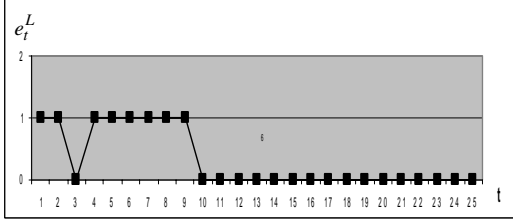


Figure 9: A learn-in-bursts strategy ($t^D=5$)

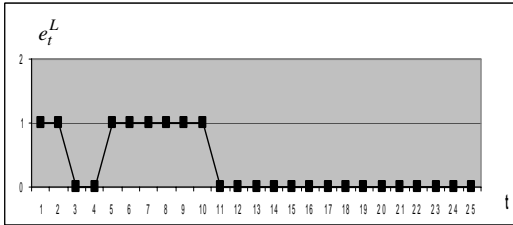


Figure 10: A learn-in-bursts strategy ($t^D=6$)

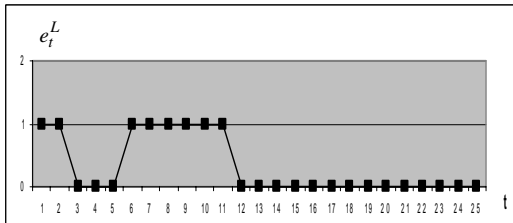


Figure 11: A learn-in-bursts strategy ($t^D=7$)

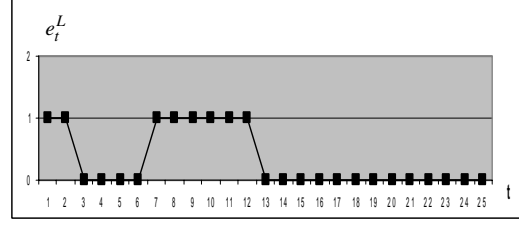


Figure 12: A learn-in-bursts strategy ($t^D=8$)

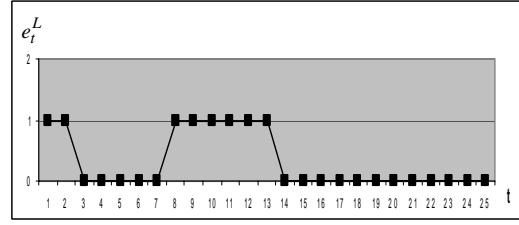


Figure 13: A learn-in-bursts strategy ($t^D=9$)

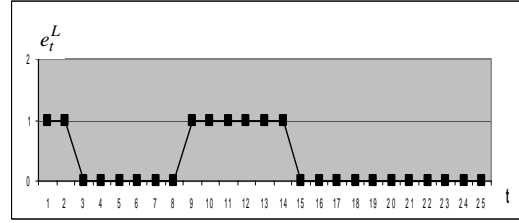


Figure 14: A quit learning strategy ($t^D=10$)

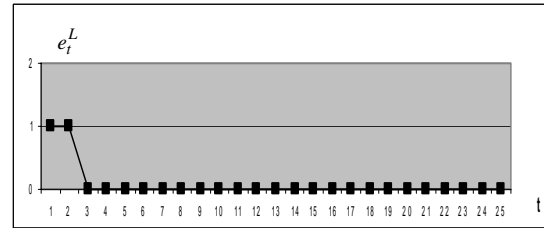


Figure 15: An always learn strategy ($t^D=11$)

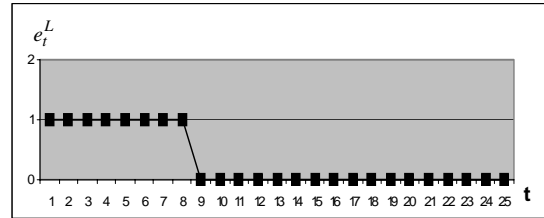


Figure 16: The shadow value of the knowledge – basic case

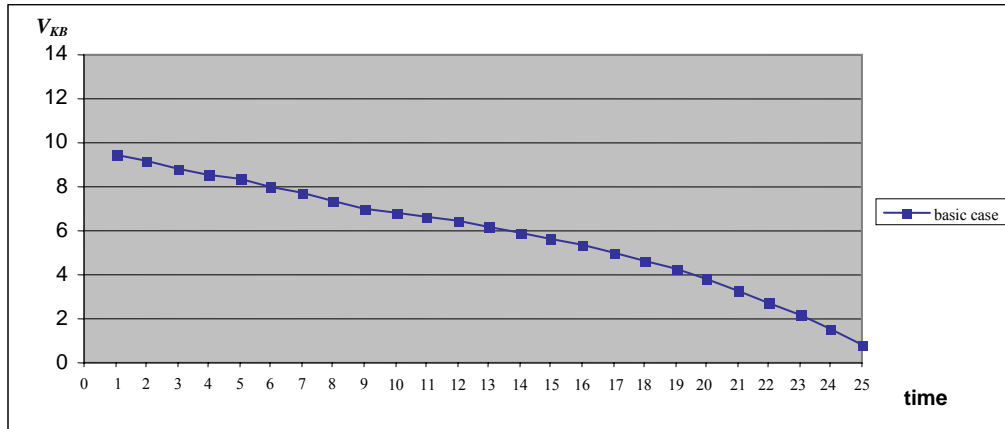


Figure 17: The shadow value of the knowledge – price decreasing cases, *wait to learn*

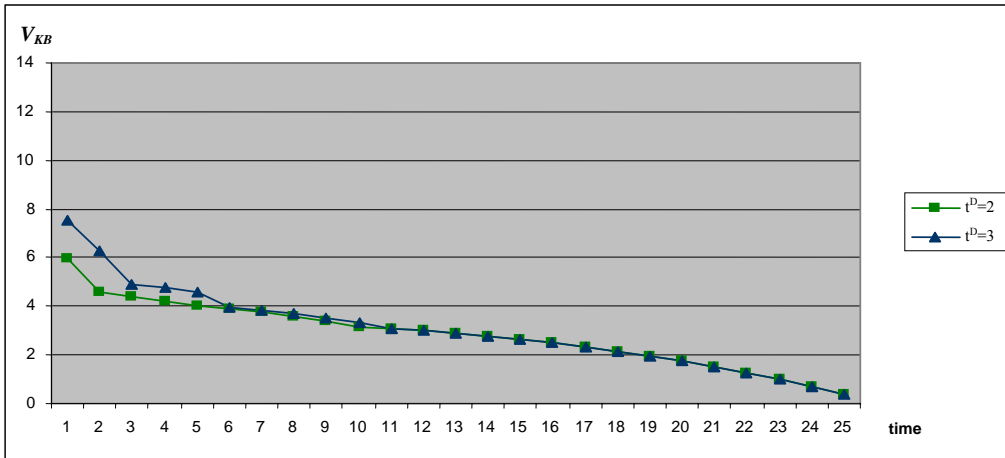


Figure 18: The shadow value of the knowledge – price decreasing cases, *learn-in-bursts*

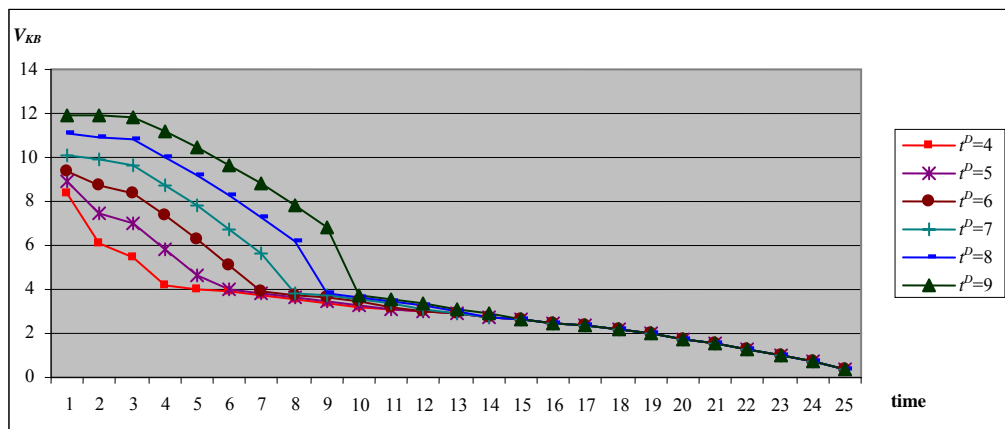


Figure 19: The shadow value of the knowledge – price decreasing cases, *quit learning*

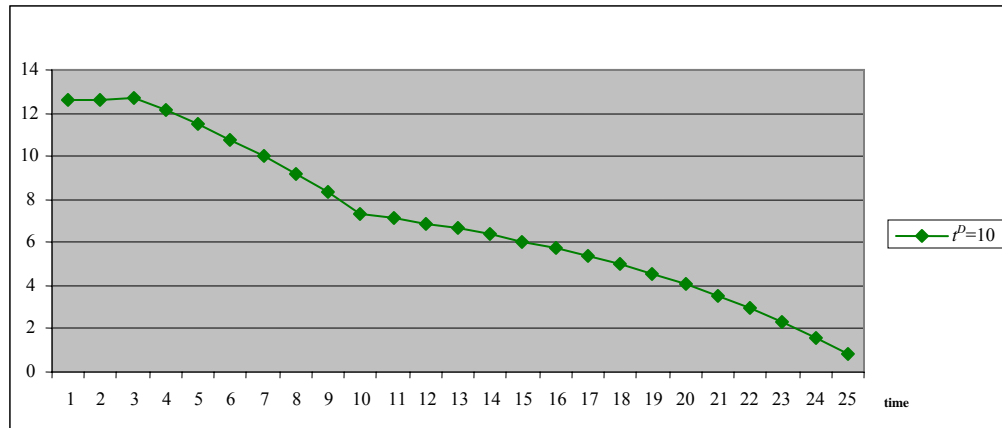


Figure 20: The shadow value of the knowledge – price decreasing cases, *always learn*

