Homework 7 solutions

2. Determine if the surface is a cylinder.

(a) \( z = y \)

[Yes] This is a plane.

(b) \( x + y + 2z = 12 \)

[Yes] This is also a plane.

(c) \( x^2 + y^2 = 16 - z \)

[No] This is a parabola.

3. Write the equation for the following surfaces.

(a) A parabola centered at the point \((1,0,0)\) along the y-axis

\[(x-1)^2 + y^2 = z \quad \text{or} \quad -(x-1)^2 - y^2 = z \]

\[(x-1)^2 + y^2 = z \quad \text{or} \quad (x-1)^2 - y^2 = z \]

(or any constants in front of the terms)
(b) A hyperboloid of two sheets opening along the z-axis, centered at \((0, 1, 0)\).

\[
-x^2 - (y-1)^2 + z^2 = 1
\]

(terms can be divided by constants)

(c) An ellipse centered at \((1, 1, 1)\) where the figure is longer along the z-axis.

\[
\frac{(x-1)^2}{2} + \frac{(y-1)^2}{2} + \frac{(z-1)^2}{2} = 1
\]

Any # bigger than one

4. Classify and sketch the curves

(a) \(x^2 - y^2 + z^2 - 2x + 4y - 6z + 5 = 0\)

\[
(x^2 - 2x + 1) - (y^2 - 4y + 4) + (z^2 - 6z + 9) + 5 = 1 - 4 + 9
\]

\[
(x-1)^2 + (y - 2)^2 + (z - 3)^2 = 9
\]

\[
- \frac{5}{5}
\]

\[
(x-1)^2 - (y - 2)^2 + (z - 3)^2 = 4
\]

\[
\Rightarrow \frac{(x-1)^2}{4} - \frac{(y - 2)^2}{4} + \frac{(z - 3)^2}{4} = 1
\]

Hyperboloid of One Sheet
(4a continued)

Center: \((1, 2, 3)\)

Opens along \(-y\)-axis

\(z^2 + \frac{y^2}{9} + \frac{x^2}{5} = 1\)

\(4b)\)

\(-x^2 - y^2 + z^2 + 4x + 2y - 6z + 4 = 0\)

\[-(x^2 - 4x + 4) - (y^2 - 2y + 1) + (z^2 - 6z + 9) + 4 = -4 - 1 + 9\]

\[-(x-2)^2 - (y-1)^2 + (z-3)^2 + 4 = 4\]

\[-(x-2)^2 - (y-1)^2 + (z-3)^2 = 0\]

**Cone**

Centered at \((2, 1, 3)\)

Opening along \(z\)-axis
\[ -x^2 + y^2 + 4x - 2y - 2 - 3 = 0 \]

Centre: \((0, 2, -1)\)

\[ -(x^2 - 4x + 4) + (y^2 - 2y + 1) - 2 - 3 = -4 + 1 \]
\[ -(x - 2)^2 + (y - 1)^2 - 2 - 3 = -3 \]

\[ z = -(x - 2)^2 + (y - 1)^2 \]

Saddle

\[ p \| p = p \cdot (p + x\cdot x - y) + (p + y\cdot y - z) \cdot (p + x\cdot x - z) - \]
\[ y = p + z(2 - z) + (1 - p) - z(2 - x) - \]

\[ 0 = z(2 - x) + (1 - p) - z(2 - x) - \]

\((2, 1, 0)\) in Limit