HW #10 Solutions:

1) 

1st plot 

flat in $z$-direction

2nd plot

increasing in $z$-direction
(a line)

3rd plot

a parabola in the $z$-direction.

a.) The degree of $z$ affects the shape of the graph.

b.) If $z(t) = t^3$, we'd get a cubic shape:

like
2) 1st plot

straight line up (only in z direction)

2nd plot

straight line sloped in z and y direction

3rd plot

a straight line in all directions equally

a) as each dimension goes from 0 to t, the line has slope in that direction. Basically, it goes from moving in one dimension to moving in 3 dimensions.
3.)

1st plot

oscillating line

2nd plot

spiral moving up.

3rd plot

4th plot

< just a circle

5th plot

a.) As the plots go from t to trig functions, we see that the graph becomes bounded in each direction.

b.) A bounded, oscillating graph.

c.) When the frequencies match (5t), there are fewer oscillations. The closer the frequencies, the fewer the oscillations.

4.) Bounded by 1 and -1 in all directions

E.g. 10t is just 5t x 2, so we see 2 oscillations.

5t and t differ by 5. So we end up with 5 oscillations.
4. The two graphs are similar in proportion but they're not the same surface:

\[ x^2 + y^2 + \frac{z^2}{a} = 1 \quad \text{Both equal 1.} \]

\[ 3x^2 + 2y^2 + z^2 = 1 \]

They're not located in the same place.

5. a) The first two graphs are hyperboloids of two sheets. Adding in the factor of 3 in the z direction elongates the graph:

This makes sense because it is a scaling factor.

The third graph changes the sign of the first by moving 1 to the other side.

That makes sense since changing the sign makes it go from a hyperboloid of 2 sheets to 1 sheet.
6. As the scaling factor gets smaller and smaller
\[-\frac{x^2}{10000} - \frac{y^2}{10000} + \frac{z^2}{10000} = 1\]

versus
\[-\frac{x^2}{1000000} - \frac{y^2}{1000000} + \frac{z^2}{1000000} = 1\]

we get closer and closer to a cone
\[x^2 + y^2 - z^2 = 0\]

7. \[x^2 - x + y^2 - (\omega y + z^2 + 2z + \frac{25}{4}) = 0\]
\[(x^2 - x + \frac{1}{4}) + (y^2 - (\omega y + 9) + (z^2 + 2z + 1)) + \frac{25}{4} = \frac{1}{4} + 9 + 1\]
\[(x - \frac{1}{2})^2 + (y - 3)^2 + (z + 1)^2 = -\frac{25}{4} + \frac{1}{4} + 10\]
\[(x - \frac{1}{2})^2 + (y - 3)^2 + (z + 1)^2 = -6 + 10\]
\[(x - \frac{1}{2})^2 + (y - 3)^2 + (z + 1)^2 = 4\]

Center: \((\frac{1}{2}, 3, -1)\) radius: 2
8. \( \vec{v} = \langle 1, 0, 1 \rangle \)
   
   a.) \( \langle \sqrt{2}, 0, \sqrt{2} \rangle \cdot \langle 1, 0, 1 \rangle = 2 \sqrt{2} \)  \( \text{Not } \perp \)  \( \vec{x} \)
   
   b.) \( \langle 0, \frac{1}{2}, 0 \rangle \cdot \langle 1, 0, 1 \rangle = 0 \)
   
   But \( |\langle 0, \frac{1}{2}, 0 \rangle| = \frac{1}{\sqrt{3}} \neq \sqrt{2} \)  \( \checkmark \)
   
   c.) \( \langle \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}, -\frac{\sqrt{2}}{\sqrt{3}} \rangle \cdot \langle 1, 0, 1 \rangle = 0 \)
   
   \[ |\langle \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}, -\frac{\sqrt{2}}{\sqrt{3}} \rangle| = \sqrt{\frac{2}{3} + \frac{2}{3} + \frac{2}{3}} = \sqrt{2} \]  \( \checkmark \)
   
   d.) \( \langle -1, 0, 1 \rangle \cdot \langle 1, 0, 1 \rangle = 0 \)
   
   \[ |\langle -1, 0, 1 \rangle| = \sqrt{1 + 1} = \sqrt{2} \]  \( \checkmark \)

   \( c \) and \( d \)

9.) plane \( \parallel \) to \( \langle 1, 1, 1 \rangle \) and contains the point \( (0, 0, 0) \)

   a.) No. \( \langle 1, 1, 1 \rangle \) is not \( \perp \) to itself.
   
   b.) \( \langle -1, 1, 0 \rangle \perp \langle 1, 1, 1 \rangle \)  \( \checkmark \)
   
   c.) No. Same as \( a \).

10.) Yes. For line 1, pick \( t = 0 \). For line 2, pick \( t = -1 \).
    All the same points!

Remember, there is no unique formula for a line.

Any multiple of the same slope and any point will give you the same line.