1. Let \( \mathbf{v} = \langle 1, 1, 2 \rangle \) and \( \mathbf{w} = \langle -2, 3, 1 \rangle \).

   a. Find the unit vector in the same direction as \( \mathbf{v} \).
   
   b. Find the dot product \( \mathbf{v} \cdot \mathbf{w} \).
   
   c. Find the cross product \( \mathbf{v} \times \mathbf{w} \).
   
   d. Find the vector projection of \( \mathbf{v} \) onto \( \mathbf{w} \).

2. What is the area of the parallelogram determined by \( \mathbf{a} \) and \( \mathbf{b} \) in terms of \( |\mathbf{a}|, |\mathbf{b}| \), and \( \theta \) (the angle between the vectors)?

3. a. Given two vectors, how can you tell if they are parallel?
   
   b. Given two lines, how can you tell if they are parallel?
   
   c. Given two planes, how can you tell if they are parallel?
   
   d. Given a vector and a line, how can you tell if they are parallel?
   
   e. Given a vector and a plane, how can you tell if they are parallel?
   
   f. Given a line and a plane, how can you tell if they are parallel?

4. a. Given two vectors, how can you tell if they are orthogonal?
   
   b. Given two lines, how can you tell if they are perpendicular?
   
   c. Given two planes, how can you tell if they are orthogonal?
   
   d. Given a vector and a line, how can you tell if they are perpendicular?
   
   e. Given a vector and a plane, how can you tell if they are normal?
   
   f. Given a line and a plane, how can you tell if they are normal?

5. Consider the lines with parametric equations

\[
\ell_1 : \quad x = t, \quad y = 2t - 2, \quad z = t + 10 \\
\ell_2 : \quad x = 1 - s, \quad y = -2s, \quad z = 7 - 3s
\]

   a. Find the point of intersection of \( \ell_1 \) and \( \ell_2 \).
   
   b. Find a vector normal to the plane containing \( \ell_1 \) and \( \ell_2 \).
   
   c. Write an equation for the plane containing \( \ell_1 \) and \( \ell_2 \) in “standard form” (that is, \( ax + by + cz + d = 0 \)).
6. a. Sketch and identify the surface \( x^2 + y^2 - (z + 1)^2 = 0 \).
   
b. Sketch and identify the surface \( z = 2 \).
   
c. Sketch the region in space bounded by \( x^2 + y^2 - (z + 1)^2 = 0 \) and \( z = 2 \).

7. a. Sketch and identify the surface \( x^2 + y^2 + z^2 = 4 \).
   
b. Sketch and identify the surface \( z = \sqrt{4 - x^2 - y^2} \).

8. a. Sketch and identify the surface \( y^2 + z^2 = 9 \).
   
b. Sketch and identify the surface \( x + y = 1 \).
   
c. Sketch both \( y^2 + z^2 = 9 \) and \( x + y = 1 \) and mark their curve of intersection.
   
d. Give a vector function that traces out the intersection of \( y^2 + z^2 = 9 \) and \( x + y = 1 \).

9. Consider the ellipse \( x^2 + \frac{y^2}{4} = 1 \) in the \( xy \)-plane.
   
a. Give a vector function in 2D that traces out the ellipse counter-clockwise.
   
b. Give a vector function in 2D that traces out the ellipse clockwise.

10. Consider the helix \( \mathbf{r}(t) = (12 \cos t, 5t, 12 \sin t) \).
    
a. Calculate the unit tangent vector \( \mathbf{T} \) at the the point \((-12, 5\pi, 0)\).
    
b. Calculate the unit normal vector \( \mathbf{N} \) at the the point \((-12, 5\pi, 0)\).
    
c. Calculate the unit binormal vector \( \mathbf{B} \) at the the point \((-12, 5\pi, 0)\).
    
d. Calculate the curvature at the point \((-12, 5\pi, 0)\).
    
e. Calculate the distance travelled along the curve from \((12, 0, 0)\) to \((-12, 5\pi, 0)\).
    
f. Reparametrize \( \mathbf{r}(t) \) by arclength starting from \((12, 0, 0)\).

11. A particle moves along the path \( \mathbf{r}(t) = (\sqrt{t+1}, t - 5, t^2) \). At what point is the velocity of this particle parallel to vector \( (1, 4, 24) \)?

12. Find the position vector \( \mathbf{r}(t) \) for a particle that starts at the origin and has velocity vector \( \mathbf{v}(t) = (t, 4, 1 - 2t) \).
13. A particle starts at the origin and has velocity \( \mathbf{v}(t) = \langle t, 3t^2, 2t - 1 \rangle \). Which of the following is a point through which the point travels?

(a) (2, 8, 2)  
(b) (1, 3, 1)  
(c) (1, 0, 2)  
(d) (2, 6, 1)  
(e) (2, 3, 2)

14. Which of the following statements is NOT true?

(a) At any point on a curve, the curvature \( \kappa \) is a scalar that is greater than or equal to 0.  
(b) Two vectors are orthogonal if and only if their dot product is zero.  
(c) The equation \( x^2 - y + z^2 = 1 \) describes an elliptic paraboloid.  
(d) The equation \( ax + by + cz = 0 \) describes a plane that passes through the origin.  
(e) If two lines are not parallel, they intersect at exactly one point.

15. Suppose \( |\mathbf{a}| = 6, |\mathbf{b}| = \sqrt{3} \), and the angle between \( \mathbf{a} \) and \( \mathbf{b} \) is \( \pi/6 \). Which of the following is the value of \( \mathbf{a} \cdot \mathbf{b} \)?

(a) -1  
(b) 3\sqrt{3}  
(c) 0  
(d) 9  
(e) Not enough information.  
(f) None of the above.
16. Which of the following planes is normal to the line \( x = t + 1, \ y = 3t - 1, \ z = 2t - 3? \)

(a) \( x - 3y - 2z = 0 \)
(b) \( x - y - 3z + 2 = 0 \)
(c) \( 2x + 6y + 4z - 1 = 0 \)
(d) \( 2x + 6y + z = 0 \)
(e) \( 2x + y - 3z + 3 = 0 \)

17. What is the vector projection of \( \mathbf{v} = \langle 5, -1/2, 0 \rangle \) onto \( \mathbf{w} = \langle 8, -1, 4 \rangle? \)

(a) \( \langle 4, -1/2, 2 \rangle \)
(b) \( \langle 16, -2, 4 \rangle \)
(c) \( \langle 10, -1, 0 \rangle \)
(d) \( \langle -8, 1, -4 \rangle \)
(e) \( \frac{1}{101} \langle 810, 81, 0 \rangle \)
(f) None of the above.

18. Which of the following pairs of vectors are orthogonal?

(a) \( \langle 23, 4, 1 \rangle \) and \( \langle 0, 1, -3 \rangle \)
(b) \( \mathbf{i} + 2 \mathbf{j} - 3 \mathbf{k} \) and \( 19 \mathbf{i} + \mathbf{j} + 7 \mathbf{k} \)
(c) \( \langle \frac{1}{2}, 3, -1 \rangle \) and \( \langle -18, 4, 1 \rangle \)
(d) \( -6 \mathbf{i} + 3 \mathbf{j} + \mathbf{k} \) and \( \mathbf{i} + 4 \mathbf{j} + 2 \mathbf{k} \)
(e) \( \langle 3, -5, 2 \rangle \) and \( \langle -1, 3, 2 \rangle \)
(f) \( 2 \mathbf{i} - \mathbf{j} + 12 \mathbf{k} \) and \( -4 \mathbf{i} + 3 \mathbf{j} \)