MATH 497 INTRODUCTION TO APPLIED ALGEBRAIC GEOMETRY
HOMEWORK 8

Assigned 10/23, due 10/30 in class.

Problem 1. Explain what Bell’s Theorem is and why it shows how entanglement cannot be explained by a local hidden variable model. How does this create a distinction between secant varieties interpreted as (Zariski closures of) probability models vs. interpreted as sets of quantum states (ignoring marginalization)?

Problem 2. Classify entanglement for two qubits, up to an independent change of basis ($\text{GL}_2(\mathbb{C})$ action) for each qubit.

Problem 3. Classify entanglement for two qudits, one with $m$ classical states and one with $n$ classical states, up to an independent change of basis for each qubit ($\text{GL}_n(\mathbb{C}) \times \text{GL}_m(\mathbb{C})$ action).

Problem 4. Can you classify entanglement for three qubits, up to an independent change of basis for each qubit ($\text{GL}_2(\mathbb{C}) \times \text{GL}_2(\mathbb{C}) \times \text{GL}_2(\mathbb{C})$ action)? This is hard, so feel free to give a partial answer. Hint: just as in the previous two questions, you should get finitely many “types” of entanglement.

Problem 5. Find the ideal of the 3 and 4 qubit translation-invariant matrix product state. That is, let $A_0$ and $A_1$ be two $2 \times 2$ matrices, which are the parameters. Given $i, j, k, \ell$ each 0 or 1, $A_i A_j A_k A_\ell$ is a $2 \times 2$ matrix given by matrix multiplying the four matrices indicated. We define the sixteen complex coefficients of the 4-qubit MPS state corresponding to these parameters by the polynomial parameterization given by, ignoring normalization,

$$\Psi_{ijk\ell} = \text{tr}(A_i A_j A_k A_\ell)$$

and the eight complex amplitudes of the three-qubit case are given by

$$\Psi_{ijk} = \text{tr}(A_i A_j A_k).$$