MATH 497 INTRODUCTION TO APPLIED ALGEBRAIC GEOMETRY
HOMEWORK 5

Assigned 10/9, due 10/16 in class.

This week we introduce some applications, particularly phylogenetics.

Problem 1. Define the neural ring corresponding to a code $C$.

Problem 2. Let $r \leq \max(n,m)$ be positive integers. Show that an $n \times m$ matrix $M$ has rank at most $r$ if and only if all the $(r + 1) \times (r + 1)$ minors of $M$ vanish.

Problem 3. Recall that a split $A|B$ of a leaf set $[n] = \{1, 2, \ldots, n\}$ is a partition of $[n]$ into two sets $A, B \subset [n]$ with $A \cap B = \emptyset, A \cup B = [n]$. Prove that if $A|B$ and $C|D$ are splits coming from two edges of a tree with leaf set $[n]$, then they are compatible.

Problem 4. In Sage (or Singular, or Macaulay2), find the ideal of the GMM on two letters (all $\kappa = 2$) with tree given in Cayley notation by $(((i)(j))(k)(\ell))$ using elimination. Show that this answer is the same as the one from class (Allman-Rhodes-Draisma-Kuttler-Raicu Theorem).

Problem 5. In Sage (or Singular, or Macaulay2), find the ideal of the GMM on two letters (all $\kappa = 2$) with claw tree with five leaves, given in Cayley notation by $(i)(j)(k)(\ell)(m)$. Show that this answer is the same as the one from class (Allman-Rhodes-Draisma-Kuttler-Raicu Theorem).