

**MATH 497 INTRODUCTION TO APPLIED ALGEBRAIC
GEOMETRY
HOMEWORK 1**

Assigned 8/24, due 8/28 in class.

The goal of this homework is to review some background material on fields and polynomial rings, and begin to get comfortable with some of the core concepts of the class including the idea of an algebraic variety. This course will have a large computational element, and this week you will get started with the software as well.

Reading this week is page 1-23 in your book, Cox, Little, and O'Shea's *Ideals, Varieties, and Algorithms*. A pdf of the book is available for free from Penn State Libraries by going to <http://link.springer.com/book/10.1007/978-0-387-35651-8>. You should also go over the sage worksheet I shared with you.

Problem 1. *State the fundamental theorem of algebra.*

Problem 2. *Consider the variety X cut out of three-dimensional affine space by the vanishing of the polynomials $2x^2+y^3$ and $4x-y^5$. Use `sagemath-cloud` or some other program to show the dimension is zero, and compute the points that make up X . Does your answer depend on the field?*

Problem 3. *Is every linear subspace of a complex vector space an affine algebraic variety? Why or why not?*

Problem 4. *How do you think we should define the dimension of an affine algebraic variety? This problem has many right answers, but don't worry about looking up and understanding the correct definitions that are out there (we'll get there). Just think about it and come up with a proposal, no wrong answers here.*

Now in Cox, Little, and O'Shea, in Section 1, do exercises 2a, 5, and 6. In Section 2, do exercises 1, 3, 6, 8, 9, and 15. Most of these should be reasonably quick, please ask for help if you get stuck.