Homework 24: Due Thursday, December 5

Please note that you will not be able to turn in revisions for this homework assignment.

1. (Extra Credit – Worth 5 points; all or nothing) Find the maximum and minimum values of the function \( f(x, y, z) = x^4 + y^4 + z^4 \) subject to \( x^2 + y^2 + z^2 = 1 \).
   
   Hint: Be sure to consider the case when variables are zero. You should get a total of 26 points. Some of them will be maximums, some minimums, some in between.

2. Find the extreme value(s) of \( f(x, y, z) = x^2 + y^2 + z^2 \) subject to \( x + y + z = 12 \).

3. Find the extreme value(s) of \( f(x, y, z) = yz + xy \) subject to the constraints \( xy = 1 \) and \( y^2 + z^2 = 1 \).

4. Find the absolute minimum(s) and absolute maximum(s) of \( f(x, y) = 2x^2 + 3y^2 - 4x - 5 \) over the domain
   
   \[ D = \{(x, y) \mid x^2 + y^2 \leq 1\} \]

   Hint: Find the critical points of the function on \( D \). Then find the extreme values of \( f \) restricted to \( x^2 + y^2 = 1 \), which is the boundary of \( D \). Compare the points you find to determine which is the absolute maximum and absolute minimum. Be sure to consider the cases when the variables are zero.