Homework 20: Due Tuesday, November 19

1. Find the tangent “plane” at (3, 2, 6) of the function

\[ f(x, y, z) = \sqrt{x^2 + y^2 + z^2} \]

and use it to approximate \( f(3.02, 1.97, 5.99) \).

Hint: Instead of \( z = f(x, y) \), extrapolate the techniques of tangent planes and linear approximations in the case where \( w = f(x, y, z) \).

2. (This is easier than it seems) Suppose that the price of a carburetor (a car part) is a function of how many cars are owned internationally and the price of steel. You’ve collected data on carburetor prices in 2011 and found that

- When there were $1 billion cars on the planet and the price of steel was $90 a ton, the price of a carburetor was $200. That is, \( f(1 \text{ billion}, 90) = 200 \).
- When the number of cars increased by 1 million (=0.001 billion), the price of steel increased by $2. To keep your units in billions of cars, you estimate that \( f_x(1 \text{ billion}, 90) = 2000 \).
- When the price of steel increased by $10, the price of a carburetor increased by $5. Therefore, you estimate that \( f_y(1 \text{ billion}, 90) = 5 \).

Estimate the cost of a carburetor if there are 2 billion cars on the road and the price of steel jumps to $140 per ton. Is this a good estimate?

3. Suppose \( f(3, 1) = 2 \), \( f_x(3, 1) = -1 \), and \( f_y(3, 1) = 10 \). How much does \( z \) change if \( x \) increases by 0.5 and \( y \) increases by 0.7?

4. Use the Chain Rule to find \( dz/dt \) for the following.

(a) \( f(x, y) = x^3 + 3x^2y + 3xy^2 + y^3 \), where \( x = 3t \) and \( y = t^2 \)
(b) \( f(x, y) = \cos(xy) \), where \( x = 1/t \) and \( y = t^3 + t \)
(c) \( f(x, y) = x^2 + y^2 \), where \( x = \sin(2t) \) and \( y = \cos(2t) \)

5. Use the Chain Rule to find \( \partial z/\partial t \) and \( \partial z/\partial s \) for the following.

(a) \( f(x, y) = \arctan(x - y) \), where \( x = t^2 + s^2 \) and \( y = 2st \)
(b) \( f(x, y) = \cos(x) \sin(y) \), where \( x = s^2t^2 \) and \( y = st \)