Homework 16: Due Tuesday, November 5

1. For the following function, show that \( f_{xy} = f_{yx} \). That is, show that the order in which you take the partial derivatives doesn’t matter.

\[
f(x, y) = \cos(xy + y)
\]

2. Find \( f_{xx}, f_{yy} \) and \( f_{xy} \) for the following equations.

(a) \( f(x, y) = x^3 + x^2y + xy^2 + y^3 \)
(b) \( f(x, y) = e^{xy+1} \)
(c) \( f(x, y) = x \cos(y) + y \sin(x) + (xy)^2 \)

3. Consider the function below and answer the following questions

\[
f(x, y) = x^2 + y^2
\]

(a) What is this surface?
(b) Find the first partial derivatives of \( f \).
(c) Graph the level curves. At the point (1, 1), draw the vector \( \langle f_x(1, 1), f_y(1, 1) \rangle \).
(d) In what direction does the vector \( \langle f_x(1, 1), f_y(1, 1) \rangle \) point (is \( z \) increasing or decreasing)?
(e) Let \( F(x, y, z) = x^2 + y^2 - z \). Find the first partial derivatives of \( F \) (i.e. \( F_x, F_y, \) and \( F_z \)). How do these partials relate to the partials of \( f \)?
(f) Graph the surface. At the point (1, 1, 2), draw the vector \( \langle F_x(1, 1, 2), F_y(1, 1, 2), F_z(1, 1, 2) \rangle \).
(g) Is the vector \( \langle F_x(1, 1, 2), F_y(1, 1, 2), F_z(1, 1, 2) \rangle \) perpendicular to the surface?

4. If the limit exists, find it. If it does not, prove it.

\[
\lim_{(x,y)\to(1,0)} \frac{4y^4 \cos(x - 1)}{(x - 1)^4 + y^4}
\]

5. If the limit exists, find it. If it does not, prove it.

\[
\lim_{(x,y)\to(1,0)} \frac{5xy}{\sqrt{x^2 + y^2}}
\]
6. Let $F(x, yz) = x^2 + y^2 - z^2 - 1$. Answer the following questions.

(a) When $F(x, y, z) = 0$, what surface do you have? Is the point $(-1, 1, 1)$ on this surface?

(b) Find $\langle F_x(-1, 1, 1), F_y(-1, 1, 1), F_z(-1, 1, 1) \rangle$.

(c) Find the equation for the plane who is normal to the vector found in part (6b) and contains the point $(-1, 1, 1)$.

(d) Justify the claim that the plane in part (6c) is tangent to the surface in (6a) at the point $(-1, 1, 1)$. (Graphing is the best way to do this. You may graph the surface in two dimensions by setting one of the variables fixed, like $y = 1$.)