

Signalling, Reputation and Spinoffs*

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Abstract

Employees often leave their firm to form a new firm (spinoff) of their own. The literature attributes most of these spinoffs to the worker getting a private new idea. In this paper, I propose a different channel for new firm formation based on signalling and reputation concerns. If high ability workers are mistakenly perceived to be low type then they would like to signal their ability to earn more. Consider a two period principal-worker problem where the worker's type is his private knowledge. If the prior belief about the worker's type is low (high type with low probability) then, despite the principal's ability to offer contracts to persuade the worker to stay, there may exist a separating equilibrium where the high type worker signals his ability by forming his own firm. This result provides theoretical support to the findings of Skogstrøm (2012), who observed high rates of entrepreneurship amongst Norwegian workers with low education and high ability. When moral hazard is introduced into the environment, I show that the separating equilibrium may generate the highest incentives to work. This may have policy implications for non-compete clauses.

JEL Codes - L14, L26, D81, D02

1 Introduction

Employees often leave their firm to form a rival firm of their own. Firms formed in this manner are called spinoffs and the firms from which they spawn are called parent firms¹. Such firms have been observed in industries ranging from semiconductors (National, AMD, Intel are all spinoffs) to auto-mobiles (Klepper (2007)) to law firms (Phillips (2002)). The literature on spinoffs has offered a range of explanations for the formation of spinoffs. Not surprisingly, most of these explanations have to do with an employee getting a

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¹There seems to be some disagreement in the literature about the definition of spinoffs. In this paper, I do not consider "sponsored spin-offs" (Cooper (1971)), in which a parent firm voluntarily establishes and holds stocks in a newly formed company intended to perform some of the business of the parent company

private new idea and then forming the spinoff because of asymmetric information about the idea's profitability (the employee knows more than the employer) - Chatterjee and Rossi-Hansberg (2012), Anton and Yao (1995). Such models have been studied in environments where the parent firm has capacity constraints (Cassiman and Ueda (2006)), the new idea is far from the core business of the parent firm (Pakes and Nitzan (1982), Hellman 2007) etc. Other explanations for spinoff formation include disagreements about which technology to pursue (Klepper and Thompson (2010)) or about the wage contracts, an employee having the 'entrepreneurial spirit' and imminent failure of the parent firm.

In this paper, I suggest a different channel for spinoff formation based on signalling, contracting and reputation concerns. Frustration among employees because their talents are not recognized has long been known to be a spur to entrepreneurship (Cooper (1971)). This leads to the primary question of this paper - If the worker type is known only to the worker, under what conditions can high ability workers signal their type to the market by forming their own firms? It is important to answer this question for the following reasons. One, asymmetric information about worker type and contracting limitations² may lead to the high ability worker getting much less than his marginal product as wages. Not only does this reduce the welfare of the good type worker, it could also hamper the regional economy by encouraging brain drain. Two, if good workers get low wages because they are believed to be low ability types then they may not find it incentive compatible to put in high efforts. As I show in this paper, in an environment with moral hazard, the equilibrium in which the good type worker separates and forms a new firm may generate the highest incentives to work. This may have implications for covenants not to compete which restrict signalling behaviour by disallowing the worker from forming competing firms. These inefficiencies can be avoided if the good type worker can signal his ability to the market. Finally, a signalling model of firm formation may be able to explain some empirical observations better than other models³. For example, Skogstrøm (2012) uses Norwegian data to show that entrepreneurship rates are particularly high among workers with low education and high ability. While there may be other reasons for these statistics, I believe that signalling and reputation are an important component of the explanation for this data. High ability workers with low education may be perceived as low ability workers. In this case, the high ability-low education workers can improve their payoffs by becoming entrepreneurs. Not only can this signal their ability to the market, they will be able to earn more in the future by maintaining their reputation with good performances. An intuitive idea of my model follows.

I consider a simple two period principal-worker model. The worker may be good or bad. The type is known only to the worker. The worker's type affects the probability of success of the job which the worker performs for the customer. The good type worker always succeeds at the job while the bad type worker succeeds with a lower probability⁴. The outcome of the job is publicly observed after each period. At the beginning of each period, the worker may accept a one period contract from the principal or form his own firm. Setting up a firm is costly and requires a one time investment⁵. If the worker forms his own firm he

²Contract cannot ask the worker to pay to work/charge the worker for failing. Principal cannot commit to long term outcome contingent contracts

³A caveat here. Identification of the cause of entrepreneurship is extremely hard owing to the many possible explanations. In particular, unless explicitly asked in the survey, it may be very difficult to distinguish between entrepreneurship because the worker wanted to be his own boss and any other reason.

⁴This particular formulation is not essential for the qualitative results.

⁵In general, a model of spinoffs should have a principal and a worker. The worker works for the principal's firm. However, since the worker may have some fixed costs of firm formation and the principal may also have some fixed costs of operations (like getting licenses/permissions), I have simplified this environment and assumed that the principal is trying to recruit a worker and both have a

has to incur a one time fixed cost of R_w (interpret this as the cost of acquiring land/office space) whereas if the worker accepts the contract offered by the principal, the principal incurs a fixed one time cost of R_p .⁶ The cost of setting up a firm is higher for the worker as compared to the principal ($R_w > R_p$). This can be interpreted in many ways - the principal may already have the land/capital required for setting up shop while the worker may not. Or, we could interpret this difference in cost to networking differences, that is the principal may know the right people which would guarantee that the principal can start his firm more easily. However, the principal cannot form the firm without the worker. So at any time there is at most one firm (principal-worker firm or worker owned firm). If the prior belief about the worker's type is low (worker is more likely to be bad), then a good worker would like to signal his type and earn more as his reputation grows. He may be able to do this by making the costly investment and forming his own firm. However, the bad type worker may be tempted to copy the good worker's strategy in the hope of fooling the market into paying him more. Thus, to obtain conditions for a separating equilibrium, we must argue why a bad type worker will not find it optimal to copy the good type worker's strategy. Moreover, to get a separating equilibrium where the good type worker forms his own firm, the principal must be unwilling to offer a contract which would get accepted by the good worker.

Two ideas are key to my results showing when separating equilibria may exist. One, the customer will pay the firm based on his beliefs about the worker type. The further the beliefs of the customer are from the truth, the more incentive a good type worker will have to signal his ability by making the costly investment. The principal will not find it incentive compatible to offer him a better contract since he would be unwilling to pay wages substantially above the price he expects to get from the customer. Two, since it is a two period model, a bad type worker will think twice before imitating the costly strategy of a good worker. This is because the bad worker realizes that the customer may learn his type after observing the outcome of the job at the end of period 1. This would give him a much lower expected payoff tomorrow than the good type worker. As an example, think of the people who leave their high paying jobs, invest their own money and start firms. Their move may be interpreted as a signal indicating that they are really high ability people (because low ability people will not find it profitable to make an investment, get found out later and then earn low returns). Another example of this is the belief held by movie goers when a film is released in 3-D ("*the producers must have real faith in the film*") or when a firm spends a lot of money on advertising a good whose efficacy is revealed soon after use. These two ideas will also work in more general models. In particular, if I relax the assumption of a good type worker never failing then we can obtain similar results in an infinite horizon model too. Note that the important elements in the above examples are - signalling should be costly, the outcome in any period should be informative of the unknown type and that there should be more than one period so that the good type can have a higher expected future payoff than the bad type. My model reflects these ideas in a simple set up.

I consider a baseline case first and then two possible extensions. In the baseline model, we have a principal-worker problem where the principal is trying to recruit a worker whose type is not known. In all

fixed (but different) cost of firm formation.

⁶It may be asked - If the worker was working for the principal, why did he not form his firm before? The opportunity to form a firm comes very rarely. For example, Blanchflower and Oswald (1990) points out that people who get inheritances are more likely to be entrepreneurs. Obviously, these are somewhat random events. In terms of the model, the cost of firm formation may be prohibitive for the worker in most periods until that one period when it is feasible for the worker to incur this cost. I start my analysis at this period.

three environments, I consider a benchmark case first where the worker type is common knowledge. In this case, irrespective of the environment, the worker never forms his own firm. Since the principal has a lower cost of setting up a firm, he is always able to offer a contract which will be accepted by the worker. This result does not hold when the worker's type is his private information. In the principal-worker model, there exists a separating equilibrium where the good type worker is able to signal his ability by making the costly investment of forming his own firm. I find sufficient refinements under which this is the unique equilibrium outcome.

In the first extension, the market for labour is competitive (two principals offering contracts to one worker). There is a separating equilibrium where the good type worker leaves to form his own firm. However, separating is much harder in this environment. In fact, under some conditions, there exists a separating equilibrium in the principal-worker model whenever one exists in the competitive labour market environment but not vice-versa. One reason for this is that the worker gets paid a lot more because of competition between principals. This reduces the incentive of the good type worker to signal his type by making a costly investment.

In the second extension, I introduce moral hazard (the worker can put in unobservable effort which affects the probability of success of the job) into the model and analyze the principal-worker model again. A planner or a policy maker may be interested in knowing if the separating equilibrium actually generates higher incentives to put in effort (and therefore leads to higher probability of success for the job) as compared to other equilibria which may exist under the same conditions. In particular, this may be of interest from the standpoint of covenants not to compete. Traditionally, non compete clauses have been used to protect trade secrets and customer lists. However, they may induce effort inefficiency if the worker puts in higher efforts when he forms his own firm as compared to when he works for a principal. I show that this result holds under some parametric conditions. This seems a little counter intuitive, specially in light of the observation that the worker may find it difficult to convince the customers that he will put in effort after he forms his own firm. This is because, unlike the principal, the worker is unable to credibly offer himself outcome dependent contracts when he forms his own firm. However, I show that in any separating equilibrium, the good type worker will put in full effort in the first period after separating. This is to avoid failure and being thought of as the bad type worker subsequently, as this will reduce payoffs in period 2. If effort is valuable but not extremely important for success, then there exist conditions under which the principal does not find it worthwhile to offer high wage contracts to encourage effort. Under these conditions, the separating equilibrium is the best equilibrium as it generates the most effort which leads to the highest probability of success for the job. This may add to explanation for the rise of Silicon Valley over Massachusetts Route 128 (unlike Massachusetts, California did not enforce non-compete covenants⁷).

The rest of the paper is organized as follows. Section 2 describes the relevant literature. Section 3 contains the baseline model and analysis for the principal-worker problem. In section 4, an extension of the model is analysed where the market for labour is competitive. Section 5 introduces the moral hazard dimension to the principal agent problem and section 6 concludes the paper.

⁷Also see Saxenian (1994), Gilson (1999)

2 Literature

This paper is related to several branches of the economics and business literature. I have already mentioned some of the papers which talk about spinoff formation in the introduction. In this section, I will mention a few other papers this paper is related to. The idea in my paper is similar to that of job market signalling as studied in the celebrated paper by Spence (Spence (1973)). However, the question here becomes all the more interesting because the worker's employer is aware of the possibility of the worker forming his own firm and can offer contracts to make the worker stay. Thus, the opportunity cost of leaving to form a spinoff is endogenously determined in my model. Moreover, the model here is dynamic in nature where the unknown worker type may get revealed by the worker's performance at the end of each period. This brings reputation concerns into play. Also, unlike Spence's (and many others) paper which is concerned with the idea of using education as a signal to potential employers, in this paper I look at the signalling opportunities after the education stage. As Skogstrøm (2012) points out, education may not signal ability perfectly. Skogstrøm (2012) has a simple model of signalling to explain entrepreneurship. The author points out that if the cost to education and ability are not perfectly correlated then it is possible that high ability workers who have a high cost to education may take up entrepreneurship in equilibrium. However, the returns to entrepreneurship for high ability workers is assumed to be an exogenous function of ability. In fact, Skogstrøm (2012) specifically rules out the possibility of entrepreneurship itself acting as a valuable signal of ability. This paper attempts to fill this gap.

The idea of investing personal wealth to indicate quality of new firm has been explored in other papers like Brealey et al. (1977), Prasad et al. (2000) and Han et al. (2008). In Brealey et al. (1977), the amount of equity kept by the owner helps solve the adverse selection problem. Players who have higher expected returns keep more of the equity. Similarly, in Han et al. (2008), good type players signal their type by taking up high collateral-low interest loans from the bank. Prasad et al. (2000) claims that it is not the portion of the equity kept by the new entrepreneur but the proportion of the entrepreneur's net worth invested in a new venture which could act as a signal of quality for the new venture. However, none of these papers allow for a principal who has the power to stop a worker from forming a new firm by offering him a better contract. Moreover, the reasoning in my paper depends crucially on the reputation aspect as the worker's type may get revealed through performance. This reputation angle is missing in these papers. For example, in Han et al. (2008), the bank is able to screen the workers perfectly so that they self select into different contracts. Other papers have also looked at quality signals *after* a new venture is formed. For example, in Hsu and Ziedonis (2008) and Audretsch et al. (2009), patents can be used as a signal for quality.

I should also differentiate this paper from those in the literature which attribute spinoff formation to a worker getting a private new idea (Chatterjee and Rossi-Hansberg (2012), Anton and Yao (1995)). Asymmetric information about a worker's type is different from asymmetric information about a new idea which the worker may get. This is because the (average) profitability of a new idea is generally exogenous and known to the worker. For example, in Anton and Yao (1995), the payoff to the worker from the new idea is exogenously given as the payoff from a Duopoly (the worker's old firm and his new firm are in the market). In my model, the payoff to a type is strongly linked to the signalling aspect of the problem. Even a good type worker will earn much less than his ability if the market believes him to be a bad type worker. Note that it is

possible to model the signalling channel of firm formation with a new idea story as well. However, this has not been addressed in the literature.

Finally, my work is related to the literature on non-compete covenants. While Rubin and Shedd (1981) shows that covenants not to compete may encourage worker training investments by firms, Garmaise (2009) points out that managers may have less incentives to invest in their own human capital after they have signed non-compete covenants. If the latter is more valuable then the efficiency of non compete clauses is reduced. Marx et al. (2010) uses the inadvertent reversal of Michigans non-compete enforcement policy to show that there is internal (within country) brain drain from states which enforce non-compete covenants to states which do not. Another paper which questions the benefits of non-compete clauses is Gilson (1999). Gilson builds on the ideas in Saxenian (1994) which compares the rise of Silicon Valley with Massachusett's Route 128 and attributes a lot of the credit for the growth of the former to the legal environment in California which does not enforce non-compete covenants. Saxenian (1994) had pointed out the importance of a culture of high job mobility which leads to valuable knowledge transfers and affects the economic growth of a region. In this paper, I describe conditions under which the worker puts in the most effort when he forms his own firm. Since non-compete covenants rule out the possibility of rival entrepreneurship for the worker, they may generate an effort inefficiency leading to a lower probability of success for jobs executed by workers under some conditions.

3 Principal Agent Problem

3.1 Model

There are four risk neutral players - one principal, one worker and two customers. It is a two period game. At time 0, nature moves and assigns a type from the set $\{G, B\}$ (good and bad) to the worker. Only the worker knows his type. The other players know that nature made this choice according to a probability distribution where the probability of being type G is p_g and probability of being type B is given by $1 - p_g$. There is a job/project which both customers want executed each period. If the job is successful then the customer receives utility V and if it is unsuccessful then the customer gets zero utility. Worker type G is better than worker type B at performing the job. Let the probability of success at job given worker type t be given by $P(S/t)$. Then $P(S/G) = 1$ and $P(S/B) = \lambda_b$ where λ_b is a real number between zero and one. Thus, both workers can succeed at the project but only the B worker can fail. These probabilities are common knowledge.

In period one, the principal moves first and offers either no contract or a one period contract(s)⁸ to the worker. Each contract is a tuple (s, f) . If the worker accepts the contract (s, f) then the worker's wage for that period is s if the job is successful and f if it is not. All contract offers are observable by all players. Having observed the menu of contracts that the principal has offered, the worker chooses between three

⁸One period here may be long time. In particular, it could be the time required to complete the project. This assumption restricts the principal's ability to write long run contracts. Also, note that the principal can offer more than one contract to screen the two types of workers.

actions - accepting one of the contracts, forming his own firm or doing nothing. I will assume that the worker will not choose doing nothing unless he strictly prefers this action. The formation of a firm requires a one time fixed investment. The principal is better suited to forming the firm. This is reflected in my assumption that the fixed investment needed to start the firm is higher for the worker as compared to the principal. Let R_w and R_p denote the fixed cost of firm formation for the worker and principal respectively. By assumption $R_w > R_p$. Additionally, I assume that $R_w > V$ and $V(1 + \delta) > R_w > \lambda_b V(1 + \delta)$. These assumptions imply that one period payoff is not enough to make any worker form his own firm and if types are known, then only a G type worker may find it individually rational to form his own firm. This is a natural assumption. The cost of forming a firm will generally be high enough to dissuade workers from incurring this cost if the reward is just profits from a single period. Moreover, a worker who is known to be inefficient may never find it worthwhile to incur costs and form his own firm. From the principal's point of view, he may⁹ be willing to form the firm even with a bad type worker in period 1 but not if he gets only one period payoffs i.e. $\lambda_b V < R_p < \lambda_b V(1 + \delta)$. Let the principal's costs be low enough so that he is willing to invest in a one period game if the worker is definitely G type i.e. $V > R_p$. If the worker accepts a contract then the principal incurs the cost R_p and forms the firm with the worker. If the worker forms his own firm then the worker has to suffer an initial one time cost of R_w . Note that the principal cannot form the firm without the worker. So, in any period there is at most one firm in the market (principal-worker firm or worker owned firm). The customers move next and make bids indicating how much they are willing to pay to the firm to execute the job for them. The worker can work on only one job in one time period so only one customer's job can be performed. The customer with the highest bid wins and his job is given to the worker.¹⁰ The success of the job is chosen by nature according to the probabilities $P(S/G = 1)$, $P(S/B) = \lambda_b$. The success or failure of the job is observed by all players. Wages are paid to the worker¹¹. In period two, the same process is repeated i.e. the principal moves first, then the worker, then the customers and then the outcome is realized. Note that if the worker had already formed his firm in period one then his action set in period two consists of choosing one of the contracts, staying with his own firm or doing nothing. All players have a discount factor of δ . Players maximize discounted sum of payoffs. In any period, the reputation of the worker will indicate the commonly held belief about the worker being type G .

In any period, the worker can make a mistake¹². Formally, this means that, in every period, after the worker makes his action decision, nature moves and assigns a probability to every action in the worker's choice set. If the worker is type B and his strategy is to choose the action b , then nature chooses action b with probability $1 - \epsilon$ and all other actions with equal probability (probability weights adding up to ϵ). If the worker is type G and his strategy is to choose action g , then nature chooses action g with probability $1 - \alpha\epsilon$ and the rest of the weight is distributed equally among all other actions. Here ϵ is a small real positive number and $\alpha \in [0, 1]$. Note that if α takes the value one then both types are just as likely to make mistakes.

⁹At zero wages.

¹⁰In case both customers make the same bid, a customer is selected at random to be the winner.

¹¹If it's a principal-worker firm. In the case in which the worker forms his own firm, he gets the price which the customer has paid when the payment is made.

¹²I have to allow for the possibility of mistakes for the following reason. It will get rid of the problem of having to update on zero probability events. For example, consider the situation when everyone expects the separating equilibrium to be played in period 1. Suppose the worker is actually B type and suppose he deviates and imitates the G worker's action and subsequently fails at the project. This would mean that when the action was taken, the worker's reputation would go to 1 (since the separating equilibrium is being played) and when the worker fails at the project, the customers and principal have to somehow update on this zero probability event (since G type worker never fails).

3.1.1 Game Tree

3.1.2 Strategies and Payoffs

¹³ $s, f \geq 0$ reflects limited liability for the worker. Due to labour laws or some other limitation, the principal may not charge the worker for failing.

tion to do nothing (i.e. do not accept a contract and do not form own firm), acc_i is the action to accept contract (s_i, f_i) and L refers to the action of leaving and forming own firm. At $t = 2$, the action set for the worker is given by either $\{N, acc_1, \dots, acc_n, L\}$ (if the worker had accepted a contract or chosen N and therefore not formed his firm in a period one) or $\{N, acc_1, \dots, acc_n, S\}$ (if the worker had formed his own firm in the previous period. Here S refers to staying with own firm). The strategy for the worker is a function from the history of play and his own type to his action set. Let $a_w(t)$ denote the action eventually¹⁴ taken by the worker in period t . Denote by $A_w(t)$ the action that worker wants to take. The first period outcome of the job can be denoted by o_1 . $o_1 \in \{Success, Failure, No Outcome\}$ where *No Outcome* refers to the event where the worker's realized action was N in period 1. Since it is a simple two period game, I can describe the history observed by all players when they have to make a decision. For $i \in \{Principal, Worker, Customers\}$ and $t \in \{1, 2\}$, let $H(i, t)$ denote the history observed by player i when they have to make a decision in time period t . Then $H(Principal, 1) = \phi$, $H(Worker, 1) = \{a_p(1)\}$, $H(Customers, 1) = \{a_p(1), a_w(1)\}$, $H(Principal, 2) = \{a_p(1), a_w(1), a_c(1, 1), a_c(1, 2), o_1\}$, $H(Worker, 2) = \{a_p(1), a_w(1), a_c(1, 1), a_c(1, 2), o_1, a_p(2)\}$, $H(Customers, 2) = \{a_p(1), a_w(1), a_c(1, 1), a_c(1, 2), o_1, a_p(2), a_w(2)\}$

Next we describe the payoff functions for each player. In any period, if the worker's realized action is N , then the payoffs to all players in that period is zero. If the worker does not choose the action N , there are four cases: 1 - worker accepts contract in period 1 and 2, 2 - Worker accepts contract in period 1 and forms own firm in period 2, 3 - worker forms own firm in period 1 and accepts contract in period 2, 4 - worker forms own firm in period 1 and stays with own firm in period 2.

Without loss of generality, let the relevant contracts (the contracts which the worker accepts whenever his action is to accept a contract) in period 1 and 2 be (s_1, f_1) and (s_2, f_2) respectively. WLOG, let customer 1 win both bids. Let $I(t)$ be an indicator function which takes the value 1 if the job was successful in period t and zero if the job was unsuccessful. Then the aggregate (over both periods) ex-post payoff for the four players is represented by a quadruple $(P, W, C1, C2)$ where the first coordinate is the payoff to the principal, the second coordinate is the payoff to the worker, the third coordinate represents the payoff received by customer 1 and the fourth is for payoff received by customer 2. Then,

Payoff in case 1:

$$(a_c(1, 1) - s_1 I(1) - f_1(1 - I(1)) - R_p + \delta(a_c(2, 1) - s_2 I(2) - f_2(1 - I(2))), \\ s_1 I(1) + f_1(1 - I(1)) + \delta(s_2 I(2) + f_2(1 - I(2))), VI(1) - a_c(1, 1) + \delta(VI(2) - a_c(2, 1)), 0)$$

Payoff in case 2:

$$(a_c(1, 1) - s_1 I(1) - f_1(1 - I(1)) - R_p, s_1 I(1) + f_1(1 - I(1)) + \delta(a_c(2, 1) - R_w), VI(1) - a_c(1, 1) + \delta(VI(2) - a_c(2, 1)), 0)$$

Payoff in case 3

$$(\delta(a_c(2, 1) - s_2 I(2) - f_2(1 - I(2))) - R_p, a_c(1, 1) - R_w + \delta(s_2 I(2) + f_2(1 - I(2))), VI(1) - a_c(1, 1) + \delta(VI(2) - a_c(2, 1)), 0)$$

¹⁴This is the action that actually gets taken after nature takes the mistake move.

Payoff in case 4

$$(0, a_c(1, 1) - R_w + \delta(a_c(2, 1)), VI(1) - a_c(1, 1) + \delta(VI(2) - a_c(2, 1)), 0)$$

Note that the model has two customers but the firm can accept only one job. This will imply that on the equilibrium path, both customers will bid their expected value. Therefore, if, at time t , the reputation of the worker is π_t , then both customers will bid the price $\pi_t V + (1 - \pi_t)\lambda_b V$ in equilibrium. The strategies of the principal and worker depends heavily on the price they expect to get and this assumption (of more customers than the firm can handle) makes the model much more tractable. Similar assumptions of customers always paying their expected utility have been made in many papers in the past, example Holmstrom (1999).

3.2 Assumptions

1. Principal offers at most two contracts in every period. This is just for simplicity. The only reason principal may wish to offer more than two contracts is because he may want to rope in the worker in a bad contract if the worker makes a mistake and selects a contract at random. Since mistakes probabilities will be close to zero, this will not play a big role on the equilibrium path. This can also be motivated by assuming a small cost of offering every contract.
2. When G worker chooses to accept a contract and is indifferent between contracts $(s_1, f_1), \dots, (s_n, f_n)$, he accepts the contract $i \Leftrightarrow i = \min\{j \in \{1, \dots, n\}\}$.
3. When B worker chooses to accept a contract and is indifferent between contracts $(s_1, f_1), \dots, (s_n, f_n)$, he accepts the contract $i \Leftrightarrow i = \max\{j \in \{1, \dots, n\}\}$.

3.3 Types are Common Knowledge

In this subsection we deal with the case where there is no information asymmetry about worker type. The type (G, B) is common knowledge to all players in the game. The main finding of this section is that when the types are common knowledge, the worker never forms his own firm in equilibrium. Since the principal has a lower cost of firm formation, he can always offer a contract which the worker will accept. I present the main result below. The details are in the appendix.

Proposition 1. *In the unique sub-game perfect equilibrium of the game, when the worker type is B , the principal offers the worker a contract with zero wages $(\{0, 0\})$ at period 1 and the worker accepts. If the worker type is G then the principal offers the contract $\{V - R_w + \delta V, 0\}$ in period 1 and it is accepted by the worker. In period 2, the principal offers a zero wage contract which the worker accepts.*

Proof. It is not individually rational for any worker type to form their own firm in period 2. This follows from the assumption that $V < R_w$. Therefore, since the worker's outside option is negative, if the worker accepts a contract in period 1 then the worker will be offered a zero wage contract in period 2 which will be accepted.

By assumption $\lambda_b V(1 + \delta) < R_w$, so it is not IR for B type worker to invest in forming his own firm at period 1. Therefore, the principal will offer a zero wage contract which will be accepted. Note that it is IR for the principal to offer this contract by the assumption that $\lambda_b V(1 + \delta) - R_p > 0$. When the worker type is G , the worker gets the following payoff by investing R_w and starting his own firm:

$$V - R_w + \delta V (> 0)$$

The principal must offer him this amount in period 1 to dissuade him from leaving. This is because it is known that the principal cannot commit to offering more than zero wages in period 2 if the worker accepts a contract in period 1. Offering more than $V - R_w + \delta V$ will clearly not be optimal. Therefore, the principal offers him a contract which pays him $V - R_w + \delta V$ upon success (since G type always succeeds at V , there is no point in offering him any payoff for failure).

Offering this contract is also IR for the principal as the principal's payoff is $V - (V - R_w + \delta V) - R_p + \delta V = R_w - R_p (> 0)$. \square

Corollary 1. *Worker never forms his own firm if the worker's type is known.*

Thus equilibrium payoffs to principal and workers in the game where types are known are given by the following matrix. The first coordinate refers to the principal's payoff.

Table 1: Payoffs if Types are Known

Principal, G worker	Principal, B worker
$(R_w - R_p, V - R_w + \delta V)$	$(\lambda_b V(1 + \delta) - R_p, 0)$

3.4 Types are Private Knowledge

The central message from the previous section is that if the worker's type is known then the worker never forms his own firm. This is because the principal has a cost advantage in setting up the firm which grants him the ability to offer a contract which the worker will accept. This result does not hold when the worker type is private knowledge of the worker. I will describe sufficient conditions under which there is a separating equilibrium where the G type worker's strategy is to leave in period one to form his own firm and the B type worker's strategy is to accept a contract in period one.

Let the contracts offered in period t be $\{(s_1^t, f_1^t), (s_2^t, f_2^t)\}$. Let $s_{p,t}, s_{w,t}, s_{c,t}$ denote the strategy function for the principal, worker and customers at time t respectively. I will look for conditions under which the worker forms his own firm in period one. This can happen via a separating equilibrium where the G worker leaves to form own firm in period 1 and the B worker accepts a contract in period 1. Alternatively, we could have a pooling equilibrium where workers of both types choose to form a firm in period 1. Note that we can never have a separating perfect Bayesian equilibrium where the B type worker leaves to form his own firm in period one and G type worker accepts a contract. This is because of the assumption $\lambda_b V(1 + \delta) < R_w$. Also

note that the principal has no incentives to offer two contracts to separate the G and B type workers since he can get the same expected payoff by offering one contract¹⁵.

3.5 Equilibria

As is common in signalling games there are many equilibria in this game under different parameter conditions. These equilibria include outcomes where both type workers pool on accepting a contract in period one and others where there is separation. Note that the positive probability of mistakes guarantees that there is never a full separation of types. Separating equilibrium here will refer to equilibria in which the actions chosen by the two types (before nature makes its mistake move) are different. A more detailed discussion of the possible equilibria is in the appendix.

Two results must be pointed out now:

Result 1

If the worker accepts a contract in period 1 then, in any equilibrium, the principal will offer a zero wage contract in period 2 and the worker will accept.

Proof. In period two, the worker never wants to form his own firm. This follows from the assumption $V < R_w$. The worker is indifferent between the zero wage contract and the action N . By assumption, he will accept the zero wage contract. Given that the worker will accept any contract, it is optimal for the principal to offer the zero wage contract. Note that it is IR for the principal to offer this contract in period 2 because he has already made his firm forming investment in period 1. Hence it is costless for him to offer the zero wage contract in period 2. \square

Result 2

If the worker chooses the action N in period 1, then, in any equilibrium, period two play will have the following features: One, the worker will always accept any contract if a contract is offered. Else, the worker will play N in period 2. Two, if the principal offers a contract, then it will always be the zero wage contract. Three, the principal offers a contract in period 2 only if $p_g^2 \geq \frac{R_p - \lambda_b V}{V(1 - \lambda_b)}$.

Proof. The first two points follow from the same argument as in result 1. For the third point, since the principal has not formed his firm in period 1, he will only be willing to make the costly investment of forming a firm in period 2 if the probability that the worker is G type is high enough. This can be easily seen from

¹⁵This can be easily seen through this example - Suppose the optimal menu of contracts for the principal in period 1 is $\{x_g, y_g\}$ and $\{x_b, y_b\}$ where G type worker accepts the first contract and B type the second. Incentive compatibility for separation of types demands that $\lambda_b x_b + (1 - \lambda_b)y_b \geq \lambda_b x_g + (1 - \lambda_b)y_g$ and $x_g \geq x_b$. In fact, since the constraint for the B type will bind in equilibrium, $\lambda_b x_b + (1 - \lambda_b)y_b = \lambda_b x_g + (1 - \lambda_b)y_g$. Thus, the B type worker is indifferent between the two contracts. Also, note that since the outside option for both type workers is negative in period 2 if they accept a contract in period 1, the principal will never offer the worker a positive wage contract in period 2. Then we can easily see that since the principal is risk neutral, the payoff (for the principal) when he offers a single contract $\{x_g, y_g\}$ is the same as the expected payoff when he offers the two contracts - $\{x_g, y_g\}$ and $\{x_b, y_b\}$.

the principal's IR condition below. Note that the principal will get $p_g^2 V + (1 - p_g^2) \lambda_b V$ as price from the customers because a worker of any type will accept the zero wage contract.

$$\begin{aligned} p_g^2 V + (1 - p_g^2) \lambda_b V - R_p &\geq 0 \\ \Leftrightarrow p_g^2 &\geq \frac{R_p - \lambda_b V}{V(1 - \lambda_b)} \end{aligned}$$

□

3.6 Separating Equilibrium

In this subsection, we will be interested in determining the conditions under which the good type worker can signal his type by making the costly decision of forming his own firm in period 1. We will also be interested to know if there are conditions under which such an outcome can be the unique equilibrium outcome of the game. The following proposition highlights sufficient conditions needed to get a separating equilibrium.

Proposition 2. *Let $V < \frac{R_w}{1 + \delta \lambda_b (2 - \lambda_b)}$. Then, if $p_g < \frac{R_p - \lambda_b R_w + (\lambda_b V(1 + \delta) - R_p)}{R_w - \lambda_b R_w + (\lambda_b V(1 + \delta) - R_p)}$, there exists a separating equilibrium where the G worker forms his own firm in period 1 and the B worker accepts a zero wage contract offered by the principal.*

The details are in the appendix¹⁶. Intuitively, if the value of the project is not too high ($V \leq \frac{R_w}{1 + \delta \lambda_b (2 - \lambda)}$), then there may be a separating equilibrium where the G type worker forms his own firm in period one and the B type worker accepts a contract with the principal. The idea is that with these low payoffs to forming own firm ($V \leq \frac{R_w}{1 + \delta \lambda_b (2 - \lambda)}$), the B type worker's gains from imitating G type and forming own firm becomes negative. This is because the B type worker realizes that there is a chance that he would fail at the project and therefore be found out to be B type worker. In this case, his payoff tomorrow will be low and the payoff today is negative ($V - R_w$). The G type on the other hand is willing to invest R_w because he knows that even though current payoff is negative ($V - R_w$) the total payoff from forming his own firm is positive ($V(1 + \delta) - R_w$) for him since he will succeed at the project. The principal cannot offer a contract to attract the G type worker because he will have to pay high wages and get low returns. This is because it is known that B type always accepts any contract. Therefore, if the principal offers a contract to attract the G type worker then the customers will realize that both type workers would accept it. Thus, the customers will be willing to pay a price according to the prior probability of worker being good type i.e. risk neutral customers will be willing to pay not more than $p_g V + (1 - p_g) \lambda_b V$. If p_g is low then the customers will be willing to pay a low price only and therefore it will not be worthwhile for the principal to try and attract the G type with high wages.

Along with low p_g , the condition $V < \frac{R_w}{1 + \delta \lambda_b (2 - \lambda)}$ is a sufficient condition to get a separating equilibrium. It is definitely not a necessary one. As an example, consider an environment where $\frac{R_w}{1 + \delta \lambda_b (2 - \lambda)} < V < \frac{R_w}{1 + \delta \lambda_b (2 - \lambda)} + \nu$ where ν is small. In this case, there will be a separating equilibrium as well. However, since the outside option for the B worker may be positive now, the principal may offer a small amount to the B

¹⁶See proposition 14 in the appendix.

worker to compensate him for staying. I use the condition $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda)}$ for simplicity and to bring out the intuition cleanly.

Note that the principal has the ability to offer contracts which will screen the workers (different types choose different contracts). However, he may not want to do this. The reason is that if the worker strategies are such that the good type worker is going to separate unless offered a better contract - the principal will have to offer the good type at least as much as he would get by forming his own firm. Moreover, he will have to offer the bad type worker at least as much as the bad type worker can get by accepting the contract meant for the good type worker. If the prior about the worker is that the worker is bad type with very high probability, then the principal will prefer to offer a single low paying contract which only the bad type worker will accept while the good worker may leave.

The payoffs in this equilibrium are given by the following table. The payoff for the principal is given by the first coordinate.

Table 2: Payoffs if private types and separating equilibrium

Principal, G worker	Principal, B worker
$(0, V - R_w + \delta V)$	$(\lambda_b V(1 + \delta) - R_p, 0)$

The expected payoff for the principal is given by $(1 - p_g)(\lambda_b V(1 + \delta) - R_p)$.

3.6.1 Uniqueness of Equilibrium

Is it possible to get the equilibrium outcome in proposition 2 as the unique equilibrium outcome¹⁷ of the game? The above equilibrium is of particular interest for the following reasons. One, unlike the complete information model where the worker never forms his own firm, here the G worker always leaves and forms his own firm. Two, the G type worker can credibly signal his type in the above equilibrium. Next, I describe sufficient restrictions/refinements under which the separating equilibrium will generate the unique equilibrium outcome of the game. I show that if the following two assumptions hold and the conditions required in proposition 2 hold then the separating equilibrium outcome is unique. Assumption B is going to be the more important assumption here so let me give some intuition for it first. Since the bad worker's type could get revealed if the project fails in period 1, the expected future payoff from forming the firm is much higher for the good type worker as compared to the bad type worker. Assumption B states that therefore, we should believe that the worker who forms his own firm is much more likely to be good type. I choose the simplest formulation to achieve this result. A discussion of the assumptions and their impact follows after the statement of the assumptions.

Assume the following:

- Assumption A - If a worker leaves to form his own firm in period one then he may not sign a contract with the principal in period 2.

¹⁷Whenever, I say unique equilibrium outcome, I will refer to unique equilibrium outcome if no mistake actually happens.

- Assumption B - Given an equilibrium, if a mistake (non-equilibrium action) gives the worker strictly negative payoffs for all possible beliefs that the customers may have about the worker who makes that mistake, then the probability of such a mistake is much lower than the probability of a mistake where the worker may get positive payoffs for some beliefs. In particular, I assume the following mathematical specification¹⁸ - if a mistake gives the worker strictly negative payoffs for all possible beliefs that the customers may have about the worker who makes that mistake, then the probability of such a mistake is ϵ^2 (independent of type). Else, the probability of the mistake is ϵ (independent of type)

Essentially, if Assumption B and the conditions required in proposition 2 hold, then the only equilibrium outcomes possible are separating equilibria where the G worker forms his own firm in period 1 and the B worker's strategy is to accept a contract in period 1. Adding assumption A to the fray makes sure that the play in period 2 following separation corresponds to the equilibrium in which both worker types choose to stay with the new firm instead of accepting a contract. An intuitive idea of this claim follows.

Assumption A above guarantees that the second period play following the worker forming his own firm in period one and then succeeding is that of pooling on S ¹⁹. The separating equilibrium described before has the feature that the good type worker forms his own firm in period 1 whereas the bad type worker would accept a contract in period 1. In period 2, if the history is one where the worker has formed a firm and then succeeded at the project, the worker's choice along the equilibrium path is to stay at his own firm. Assumption A eliminates equilibria of the following form²⁰ : the good type worker forms his own firm in period one but signs a contract with the principal in period 2. The bad type worker accepts a contract in period one and does the same in period 2. In case the bad type worker makes a mistake in period one and forms his own firm, then in period 2, this worker stays with his own firm.

Assumption A is not an implausible one. It would be odd for the customers to hold beliefs that a worker who has invested a lot of money into building his own firm in period 1, decides to forfeit that investment and accept a contract from the principal in period 2. Such beliefs would be even more implausible if the worker was successful in period one and has a very high reputation (since in this case the principal could not be offering more than what the worker could have earned by having a strategy of staying with his own firm). Moreover, we can obtain similar results if we simply assume that $V < R_p$. In this case, it would not be individually rational for the principal to form the firm in period 2. This assumption would be intuitive in environments where the fixed cost of starting a firm is recovered only after several periods of operation (as would be the case in many industries). Other authors have also used such assumptions to simplify analysis, example (Golan (2009)).

Now lets assume the conditions required in proposition 2. In particular, $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$ combined with assumption B helps us remove equilibria of the kind where both workers accept a contract in period 1²¹. An intuitive idea of this kind of equilibrium is described below.

¹⁸Any other specification where one mistake is an order more likely than the other will also work

¹⁹Remember, worker can make mistakes. So even in a separating equilibrium the belief about the worker who forms his own firm goes close to 1 but is not equal to 1.

²⁰For a formal statement of this look at proposition 15 in the appendix.

²¹Proposition 16 in the appendix describes such an equilibrium.

Principal offers zero wage contract in period 1 and 2. p_g is low. Both type workers accept this contract in both periods. This is because if G type worker leaves to form his own firm, it will be considered a mistake and his reputation will be low (assuming $\alpha = 1 \Rightarrow$ equal probabilities of mistake). Forming his own firm with a low reputation is not IR for the G worker so he accepts the zero wage contract. However, with assumption B, if a worker forms his own firm in period one then the belief about that worker will become close to 1 (ratio of mistake probabilities is $\frac{1}{\epsilon}$ and $\epsilon \rightarrow 0$). This is because only the G worker can potentially gain from forming his own firm. The B type worker will always get negative payoffs. Now, this deviation becomes profitable to the G worker as his payoffs become approximately $V - R_w + \delta V$ (using assumption A) and therefore we can eliminate equilibria of this kind.

Assumption B is used to restrict beliefs about a worker who makes the costly investment and forms his own firm. Combined with $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$, this assumption implies that since only the G type worker can possibly gain by forming his own firm, the beliefs about the type of a worker who plays L in period 1 must be close to 1. This assumption is most natural when the difference between the expected future payoff of a G and B worker is large. If this difference is small, then this assumption becomes a strong one. The difference is inversely proportional to λ_b . Therefore, assumption B is mild if used with a small λ_b .

Proposition 3. *Let Assumption A and B hold. Let $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$. If $p_g < \frac{R_p - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p)}{R_w - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p)}$, there exists a separating equilibrium in which the G worker leaves to form own firm in period one and the B worker accepts a zero wage contract in period 1. Moreover, this gives the unique equilibrium outcome of the game.*

Proof. $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$ guarantees that the only equilibrium outcomes possible in period 1 are those in which either the G worker leaves and the B worker accepts a contract or those in which both type workers accept a contract. The above discussion eliminates the latter. Assumption A guarantees the play in period 2 is same as that in the separating equilibrium described before. \square

One might wonder if a separating equilibrium is the only way to get firm formation from the worker in equilibrium. In particular, what about a pooling equilibrium where the strategy for both type workers is to leave in period 1 to form their own firm? It can be shown that if the conditions required in proposition 2 hold and Assumption A also holds then there is no such pooling equilibrium. The details are in the appendix in section A.2.3.

4 Competitive Labour Market

In this section, I will consider an environment in which the market for labour is competitive. There will be two principals offering contracts to attract the worker. It will be interesting to contrast the results obtained in the previous section with the ones in this section. In particular, it is not obvious if it is easier or harder for the G worker to separate and signal his type in a competitive labour market as compared to the monopsonistic labour market analyzed in the previous section. On one hand, competition between principals will lead to the workers getting higher wage contracts which reduces the incentives to separate. On the other hand,

competition reduces the payoffs a principal may earn in equilibrium. This makes it less lucrative for any principal to prevent a worker from leaving by offering him high wage contracts.

The model remains the same as before except for the following change. There are two principals and one worker. In period 1, both principals offer competing contracts simultaneously. The worker decides between accepting either contract, forming his own firm or doing nothing. In period 2, both principals offer their contracts simultaneously again. Then, the worker decides between staying with his own firm (if he had formed his own firm in period 1), accepting a contract offered by one of the principals and doing nothing. If the worker had accepted a contract with a principal in period 1 then the worker chooses between the best contract, forming his own firm and the action N after observing the contracts offered by both principals. I will look for equilibria where the principals play a symmetric equilibrium.

4.1 Types are Common Knowledge

In this subsection, consider the case where there are two principals competing for one worker whose type is known by all players. I will assume that the principals offer just one contract each. Since there is no uncertainty about type, the only reason the principals may want to offer more than one contract is that they hope to catch the worker in a bad contract if he makes mistake. However, since mistake probabilities are almost zero, this is not be a big assumption and can be motivated by assuming a small cost to offering every additional contract.

The main result is discussed below. As in the complete information case with one principal, the worker never chooses to form his own firm in equilibrium. However, in this environment, competition between the principals causes their profits to go to zero in equilibrium and the workers get much more in wages.

I assume that when the worker type is G , the principals do not offer any reward for failure. Since the good type worker never fails, this is an innocuous assumption.

Proposition 4. *If the worker type is known and the worker is of type G then both principals play a symmetric mixed strategy in period 1: principals bid a contract which pays less than the contract $\{x, 0\}$ with probability $G(x)$ where $x \in [\underline{x}, \bar{x}]$. The G type worker accepts the better of the two realizations of contracts offered from the above distribution.*

Proof. Details (including a description of $G(x), \underline{x}, \bar{x}$) are in the appendix (proposition 19 in the appendix.). Once again, since the principals have a lower cost of firm formation, they are always able to offer a contract which he worker cannot refuse. To show that the worker will accept the contract it is sufficient to show that the worker accepts the lowest wage contract. The lower bound of the support of the distribution $G(x)$ is such that this holds. \square

Thus, if the worker type is G , then in the full information case, the worker will not form his own firm and the expected payoff to the worker will be approximately $V - R_p + \delta V$. The principals get an expected payoff of zero. These payoffs have been calculated as $\epsilon \rightarrow 0$.

Similarly it can be shown that if the worker type is B , then in the full information case, the worker will not form his own firm. The payoff to the worker will be $\lambda_b V - R_p + \delta \lambda_b V$ and competition between the principals will force their expected payoff to zero.

Thus payoffs in the equilibrium in the full information case can be summarized as follows. First coordinate is payoff for principal 1, second is payoff for principal 2, and third is payoff for worker.

Table 3: Expected Payoffs if Types are Known and Competitive Labour Market

<i>Principal 1, Principal 2, G</i>	<i>Principal 1, Principal 2, B</i>
$(0, 0, V - R_p + \delta V)$	$(0, 0, \lambda_b V(1 + \delta) - R_p)$

4.2 Types are Private Knowledge

As in the game with no competition in the labour market, when there is full information about the worker's type, the worker never forms his own firm. The worker's type is his private information in this subsection. The main results for this section are presented below. A detailed analysis can be found in the appendix.

The first result provides sufficient conditions under which we can have a separating equilibrium where the good type worker forms his own firm in period one while the bad type worker accepts a contract in period one.

4.2.1 Separating Equilibrium under Competitive Labour Market

Proposition 5. *Let $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$ and $R_p > \lambda_b R_w$. There exists a p_c such that if $p_g \in (0, p_c)$ then there exists a separating equilibrium where the G worker plays L in period 1 and the B worker accepts the best of the contracts offered by the two principals.*

Proof. The details of the proof are in the appendix (proposition 22 in appendix). The intuitive idea is the same as that in proposition 2. In the separating equilibrium, the play in period 2, following separation is the pooling on S equilibrium. The customers will pay according to their expected payoffs, if the belief about the worker is low then it may not be optimal for the principal to offer a high wage contract to stop the G worker from leaving and forming own firm. The B worker does not try to imitate the action of the G worker since he realizes that his expected future payoff is much lower than that of the G worker because of the possibility of failure leading to his type getting revealed. Note, however that we need an additional condition of $R_p > \lambda_b R_w$ here. This is because the outside option for the principals has changed. In the baseline principal-worker model, even if the G worker's strategy was to leave in period 1, the principal could get positive payoffs by extracting all rents if the worker happened to be B type. In a separating equilibrium, when the labour market is competitive, competition forces the principals to offer contracts which would give them zero rent even if the worker turns out to be B type. Thus, to deviate, all the principal needs is a contract which gives him positive payoffs (as opposed to the baseline case where, to get a deviation, the principal

would have required a deviation which pays more than the payoff from getting B type worker). Therefore, the principals are more likely to offer a higher pay contract in this environment and thus, to preclude the principals from offering any contract which would prevent G worker from leaving, we need the condition that the principal's cost of setting up the firm is not too low. \square

4.2.2 Uniqueness of Separating Equilibrium under Competitive Labour Market

We can get uniqueness of equilibrium outcome in much the same way as in the principal worker environment.

Proposition 6. *Let Assumption A and B hold. Let $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$. There exists a p_u such that if $p_g \in (0, p_u)$, there exists a separating equilibrium in which the G worker leaves to form own firm in period one and the B worker accepts a contract in period 1. Moreover, this gives the unique equilibrium outcome of the game.*

Proof. $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$ implies that there are no equilibria where the B type worker chooses to play L in period 1. Therefore, there are only two types of equilibrium plays are possible, either it is a separating equilibrium where the G worker leaves to form own firm or both type workers choose to accept contracts in period 1. We need to eliminate the latter type of equilibrium. Consider an equilibrium where both type workers choose to accept contracts in period 1. By assumption A and B, a G type worker can get a payoff of $V - R_w + \delta V$ by deviating and playing L . For this to be not incentive compatible, he must expect to get at least this payoff by accepting a contract in period 1. However, if $p_g < \frac{\lambda_b}{1-\lambda_b} \frac{R_p - \lambda_b V}{V - R_p + \lambda_b V}$, then the worker has to expect a payoff of zero tomorrow and neither principal can afford to offer such high wages ($V - R_w + \delta V$) in period 1. This is because they will get low prices from the customers on such contracts since the customers will pay according to their expected beliefs (which are low because p_g is low and both workers will accept any high wage offers). Therefore, we have a profitable deviation for the G type worker. Contradiction. \square

4.2.3 Separating in Competitive vs Non-Competitive Labour Market

In this subsection, I assume assumption A,B hold and the sufficient conditions needed for a separating equilibrium described in proposition 2 also hold i.e. $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$ and $p_g < \frac{R_p - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p)}{R_w - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p)}$. Therefore, by proposition 3, we know that the unique equilibrium outcome in the principal agent model is a separating equilibrium where the good type worker's strategy (along the equilibrium path) is to form his own firm in period 1 and stay with the new firm in period 2 and the bad type worker's strategy is to accept the best contract offered in either periods. The next proposition points out that under some conditions, there may exist a separating equilibrium under the principal worker environment but not when there are two principals competing for one worker. Thus, signalling one's ability via firm formation may be harder when the market for labour is competitive.

Proposition 7. *If $p_g \in (\frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}, \frac{R_p - \lambda_b R_w + \lambda_b V(1+\delta) - R_p}{R_w - \lambda_b R_w + \lambda_b V(1+\delta) - R_p})$, then there does not exist a separating equilibrium where the G worker forms his own firm and the B type worker accepts a contract.*

Proof. Detailed proof is in the appendix. A sketch of the proof is as follows.

Suppose not. Suppose there exists an equilibrium where the G type worker forms his own firm in period one and the B type worker accepts the best contract in period 1. Competition and symmetry of principals will drive the expected payoff of the principals to zero.

Given any equilibrium that will be played in period 2, we can show that if a principal deviates in period 1 and offers a contract which would give the G worker a payoff of $V - R_w + \delta V$ in total (over two periods), then that principal will make positive profits if $p_g > \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}$. This is a contradiction. \square

Thus, there are conditions under which separating is possible in the principal-worker model but not in the two principal one worker model.

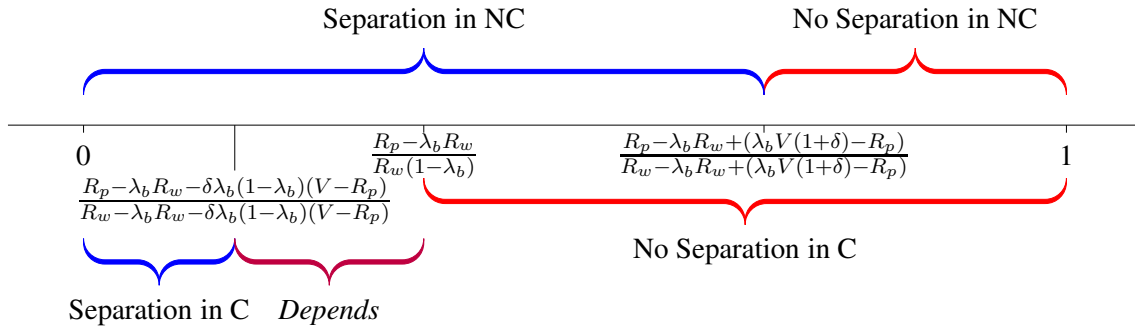
The following is a pictorial representation of the comparison between the principal-worker model with the model in which two principals compete for the same worker. The picture highlights when separating equilibria are possible. The labels in the figure have the following meaning:

Labels

C = Competition for labour - Two Principal- One worker model

NC = No Competition for labour - Principal-worker model

Figure 2: Prior Reputation (p_g) on x axis



5 Moral Hazard

I introduce moral hazard in this section in the baseline principal-agent model. There are two reasons for introducing moral hazard into this model. One, moral hazard is a ubiquitous feature of many real life relationships and it is important to understand how it affects outcomes. This serves as a sort of robustness check for my results. Two, a planner or a policy maker may be interested in knowing if the separating equilibrium actually generates higher incentives to put in effort (and therefore leads to higher probability of success for the job) as compared to other equilibria which may exist under the same conditions. In particular, this may be of interest from the standpoint of covenants not to compete.

The worker can put in costly effort $e \in \{0, 1\}$ to improve the probability of success of the project. Effort exerted is privately observed by the worker. Let $\beta \in [0, 1]$. Let $P(S/t, e)$ be the probability of success of the project given worker type t and effort e . I assume the following parametric specification of $P(S/t, e)$:

$$P(S/G, e) = \beta + (1 - \beta)e \quad (1)$$

$$P(S/B, e) = \beta\lambda_b + (1 - \beta)e \quad (2)$$

Thus, β is inversely related to how much effort matters in determining the success of a project. The higher β is, the more the success of the project depends upon the inherent abilities of the worker as opposed to the effort they put in. In particular, notice that as $\beta \rightarrow 1$, we obtain our original model. Also notice that if the good type worker puts in full effort then he will always succeed. This is not true for the bad type worker. Both type workers can fail if they put in zero effort.

Effort is costly to the worker. In particular, I assume that the cost of putting in effort e is e . The cost of effort is same for both worker types.

5.1 Principal Agent Problem

I will be considering the environment where there is one principal-one worker. I will also assume that $V < R_p$. This will guarantee that it is not individually rational for the principal to form the firm in period 2. This simplifies matters a lot. Therefore, if the worker leaves to form his own firm in period one, he will remain with his own firm in period two. Alternatively, I could have assumed Assumption A to hold.

First we deal with the case where the worker type is known.

5.1.1 Types are Common knowledge

The worker's type is common knowledge. Consider the case where the worker's type is G . The other case will follow. The result that the worker will not form his own firm when the type is known extends to the environment with moral hazard as well. First we need a lemma which point out that the worker will not put any effort if he leaves to form a spinoff in period 1.

Lemma 1. *Suppose the worker is type G . If the worker forms his own firm in period 1, then he will exert zero effort in both periods.*

Proof. In period 2, it is not IR for the principal to form the firm ($V < R_p$), therefore the only equilibrium outcome is that the worker will play S . Since the worker gets paid before he exerts effort, his effort choice in any equilibrium will be $e = 0$. Thus, the customers will bid βV in period 2.

In period 1, suppose the worker has already played L . The worker's payoff for tomorrow is fixed at βV . The worker puts in effort only after the customers have made their bid and paid the worker. Therefore, in any

equilibrium the worker has no incentive to put in any effort other than zero. \square

Corollary 2. *If the G worker plays L in period 1, his payoff is $\beta V - R_w + \delta\beta V$.*

Next, we describe the main result of this subsection. This upholds previous results which say that the worker never forms his own firm when the worker type is known. The reason, as before, is that the principal has a cost advantage in forming the firm. A more detailed analysis of this environment can be found in the appendix.

Proposition 8. *If the worker type is known to be G , the worker will never choose to form his own firm in any equilibrium.*

Proof. If the worker forms his firm in period 1, the payoff for the principal is zero. I will show that the principal can always offer a contract to the worker which satisfies the following two properties. The worker will accept and the principal will get positive payoffs. This rules out the separating equilibrium.

Consider the contract $\{\beta V - R_w + \delta\beta V, \beta V - R_w + \delta\beta V\}$ followed by the contract $\{0, 0\}$ in period 2 if the worker accepts in period 1. The worker cannot get more by leaving so he will accept the contract. Moreover, in both periods, the worker's payoff is independent of success so the worker always chooses zero effort. Therefore, the payoff for the principal from this contract is:

$$\begin{aligned} & \beta V - R_p + \delta\beta V - (\beta V - R_w + \delta\beta V) \\ & = R_w - R_p (> 0) \end{aligned}$$

\square

Similarly, we can show that a B type worker will also not choose to form his own firm in any equilibrium.

5.1.2 Types are Private Knowledge

I show that introducing moral hazard in the environment does not change the result that the worker never forms his own firm if the worker's type is known. The above result also highlights a key property of having moral hazard - namely, when the worker has his own firm, he finds it difficult to convince the customers that he will put in effort. On the other hand, if the worker signs a contract with the principal then the principal can get him to put effort by offering him contracts where he gets paid much more if he succeeds. This problem arises because the worker is unable to credibly offer himself outcome dependent contracts.

There are three primary results in this subsection. First, in any separating equilibrium, a G worker has to put in full effort after playing L in period 1. Two, under some conditions, there exists a separating equilibrium where the G worker's strategy (along the equilibrium path) is to leave and form his own firm in period 1 and the B worker's strategy (along the equilibrium path) is to accept a contract in period 1. Both these results are

in contrast to the full information case. The third result says that it is not clear if the separating equilibrium results in a higher or lower probability of success for the project as compared to other equilibria which may exist under the same parametric conditions. The answer depends upon whether the principal finds it worthwhile to pay higher wages to induce high effort from the worker who signs a contract with him. Under some conditions, the separating equilibrium result in a lower probability of success of the project. This result is driven by the workers inability to commit to effort in period 2 and the principal's ability to induce high effort with contracts which pay well for success. Under other conditions (including high β), the separating equilibrium may be the best equilibrium in terms of probability of success of the project. This is driven by the result that in any separating equilibrium, the first period effort of the G worker is high and if β is high then principal never wants to pay to induce high effort because output is not hugely dependent on effort.

All major proofs for this section are in the appendix in section A.5.

The first claim in this section describes effort put in by the good worker in any separating equilibrium.

Claim 1. *In any separating equilibrium where a G worker leaves to form his own firm in period 1, he puts in maximum effort in period 1.*

Proof. Suppose there exists a separating equilibrium where a G worker leaves to form own firm in period 1. Therefore, if a worker plays L , the belief about the worker is that the worker is almost surely G type (as $\epsilon \rightarrow 0$). Suppose the G worker puts in zero effort in equilibrium in period 1. Then payoff to G worker from playing L is approximately:

$$\beta V - R_w + \delta \beta V$$

In any separating equilibrium, it should not be incentive compatible for the B worker to imitate G worker. However, payoff to B worker from deviating and playing L is approximately:

$$\beta V - R_w + \delta \beta V$$

This is the same as G worker's payoff. This is because if the G worker puts $e = 0$, he can fail. Since the priors about the worker are arbitrarily close to 1, even if the worker fails the reputation of the worker remains extremely close to 1. Therefore, the success or failure of the project in period one leads to extremely small changes in the reputation of the worker and so the B worker can get almost exactly the same payoff as a G type worker from leaving. Since we assumed that a separating equilibrium exists, it must be the case that that B worker can get higher payoffs by accepting the contract. However, the G worker can also get this payoff by accepting the contract. This is because if the principal finds it incentive compatible to offer these wages to a B type worker, then he would definitely find it incentive compatible to offer the exact same wages to a G type worker. This means that the G worker would like to deviate to accepting a contract. Contradiction. \square

Thus, in any separating equilibrium, the G type worker's strategy asks him to put in maximum effort after separating. The G worker follows this strategy because if he puts in low effort and fails, then he will

be taken to be B type worker in period 2 which will severely reduce his payoff. Now, I give sufficient conditions under which there does, in fact, exist a separating equilibrium where along the equilibrium path, the G worker's strategy is to leave and form his own firm in period 1 and the B worker's strategy is to accept a contract in period 1.

Proposition 9. *There exists \bar{R} such that if $R_w > \bar{R}$ and the following holds*

$$\frac{\frac{1+\delta\beta}{1-\beta} + R_w}{1 + \delta\beta} < V < \frac{1 + R_w}{1 + \delta\beta(1 - \beta(1 - \lambda_b)^2)} < R_p$$

then there exists p' such that if $p_g < p'$, there exists a separating equilibrium where the G worker's strategy (along the equilibrium path) is to leave and form his own firm in period 1 and the B worker's strategy (along the equilibrium path) is to accept a contract in period 1. B type worker exerts $e = 1$ in both periods and G worker exerts $e = 1$ in period 1 and $e = 0$ in period 2. The principal offers the contract $\{\frac{1}{1-\beta}, 0\}$ to the worker in period 1 and the same in period two if the worker accepts in 1.

Proof. The details of the proof are in the appendix (section A.5). I present the basic ideas here.

Why doesn't the B type worker copy G type's strategy?

The B type worker does not try to imitate the G type worker and play L because of the following reason: the G type worker is supposed to exert $e = 1$ after leaving. This means that the G type worker will succeed in period 1. Thus, if a worker fails (and the B type worker can fail even if he puts in maximum effort), he will be recognized as a B type worker and get paid less tomorrow. This reduces the expected future wages of the B type worker enough for him to not copy the G type worker's strategy.

Why does the G type worker put in maximum effort in period 1?

The G type worker does not deviate and put in less effort for fear of failing and being thought of as a B type worker (which would reduce his payoff tomorrow).

Why can't the principal stop the G type worker from leaving?

If the belief about the worker type is very low then the principal cannot expect huge prices from the customers (even if he offers lucrative contracts which makes the worker put in maximum effort). This means that the principal is unwilling to offer the high wage contract needed to stop the G type worker from leaving.

Why does the B type worker put in $e = 1$ in both periods?

Given the conditions the principal finds it worthwhile to offer wages high enough to induce $e = 1$ from a B type worker. □

Note that the above describes sufficient conditions for one separating equilibrium. There could be others where the worker strategies involves different choice of efforts. Also, under the same conditions there might be pooling equilibria. I describe one such equilibrium next.

Proposition 10. *Suppose the conditions required for the previous proposition hold. Let mistake probability be equal for both type of workers. Then there exists a p'' such that if $p_g \in [0, p'']$ then there exists a pooling*

equilibrium in which a worker of any type chooses to accept a contract in period 1 and 2 and puts in effort=1 in both periods.

Proof. The details of the proof are in the appendix (section A.5). The intuitive idea here is the following: if the belief about the reputation of the worker is really low and both type workers are expected to accept a contract, then no type worker wants to deviate and play L instead. This is because mistake probability being equal guarantees that a worker who chooses to play L will have a low reputation which would give negative payoffs to any worker who invests money to form his own firm. \square

Corollary 3. *Given worker type, the above pooling equilibrium is weakly better than the separating equilibrium described in proposition 9 since it gives a higher probability of success of the project in each period.*

Proof. For any type of worker, the pooling equilibrium guarantees full effort in both periods. Whereas, the separating equilibrium does not give us full effort from the G type worker in period 2. \square

Next, I investigate to see if the separating equilibrium can generate higher effort and therefore higher probability of success as compared to other equilibria which exist under the same parametric conditions. This may be of interest to policy makers and planners who may be more concerned with the project succeeding than the actual division of surplus between principal and agent. It is possible to find conditions under which there is a separating equilibrium and the principal does not want to induce high effort in the pooling equilibrium (V is high but not too high). For example, suppose β is high so that the success of the project is largely dependent upon the inherent ability of the worker rather than on the effort they put in. In this case, the principal may not find it worthwhile to pay high wages to get high effort in equilibrium. In this case separating equilibrium will give a higher likelihood of project success. This follows from claim 1. A formalization of this idea follows.

The next two propositions highlight conditions under which the separating equilibrium generates a higher probability of success than other equilibria which exists under the same conditions. Proposition 11 describes conditions under which there exists a separating equilibrium where the bad type worker accepts a contract with the principal in both periods and exerts zero effort. The good type worker on the other hand, forms his own firm and exert full effort in period 1 and zero effort in period 2.

Proposition 11. *There exists β' such that if $\beta > \beta'$ and the following hold.*

1. $\frac{1+\delta\beta}{\delta\beta(1-\beta)(1-\delta)} - 1 < R_w < \frac{(1+\delta\beta(1-\beta(1-\lambda_b)^2))(1-\beta+\beta\lambda_b)}{(1-\beta)^2} - 1$
2. $\frac{1+R_w}{1+\delta\beta} < V < \frac{1+R_w}{1+\delta\beta(1-\beta(1-\lambda_b)^2)} < R_p$.

Then there exists a p_1 such that if $p_g \in [0, p_1)$, there exists a separating equilibrium where G type worker's strategy is to play L in period 1 and then put in effort=1 in period 1 and effort=0 in period 2. The B worker's accepts the zero wage contract offered in period 1 and 2 and puts in zero effort in both periods.

Proof. The details of the proof are in the appendix. I present the idea behind the proof here.

The intuitive idea is that under these conditions, G type worker will play $e = 1$ after playing L for fear of failing and receiving low payoffs in period 2 because the customers believe he is B type. However, since only the B worker accepts the contract, the principal does not want to pay high wages to induce high effort. This is because β is high which means that the worker's output is largely dependent on his inherent talents rather than on the effort he puts in. Thus there are small immediate gains to extracting high effort. Furthermore, the future gains of having a worker succeed are also small. This is because even if the worker succeeds after accepting a contract his reputation (and therefore the price offered by the customers) will rise only marginally since it was almost zero before. In contrast, if the worker forms his own firm and then fails his reputation goes from close to 1 to almost zero.

The B type worker does not try to imitate the G type worker and play L because of the following reason: the G type worker is supposed to exert $e = 1$ after leaving. This means that the G type worker will succeed in period 1. Thus, if a worker fails (and the B type worker can fail even if he puts in maximum effort), he will be recognized as a B type worker and get paid less tomorrow. This reduces the expected future wages of the B type worker enough for him to not copy the G type worker's strategy.

If the belief about the worker type is very low ($p_g < p_1$) then the principal cannot expect huge prices from the customers (even if he offers lucrative contracts which makes the worker put in maximum effort). This means that the principal is unwilling to offer the high wage contract needed to stop the G type worker from leaving.

□

Next I will argue that under the conditions required in the proposition above, the separating equilibrium is the equilibrium which generates maximum incentives to work. To do this, first, I will demonstrate that there exists another equilibrium which generates a lower probability of success. Subsequently, it can be argued from the proof of that result that under those conditions, any equilibrium outcome different from the outcome of the separating equilibrium in which the G type worker's strategy is to play L in period 1, generates a lower probability of success for the project in period 1 and the same probability of success for the project in period 2.

Proposition 12. *Suppose the conditions needed to guarantee the separating equilibrium in proposition 11 hold. Let mistake probability be equal for both type of workers. There exists a β' such that if $\beta > \beta'$ then there exists a p_2 such that if $p_g < \min\{p_1, p_2\}$, there exists a pooling equilibrium where the strategy of both types of workers is to accept the best contract in period 1 and 2. The principal offers a zero wage contract in both periods. The G/B type worker's strategy is to accept the contract in each period and put in zero effort in both periods.*

Proof. The details of the proof are in the appendix. The intuitive idea here is that if the belief about the reputation of the worker is really low and both type workers are expected to accept a contract, then no type worker wants to deviate and play L instead. This is because mistake probability is equal which guarantees

that a worker who chooses to play L will have a low reputation which would lead to negative payoffs if he invests money to form his own firm. The value of V is low enough to ensure that the principal does not want to pay to induce high effort from the worker. The principal does not want to pay for high effort because effort is not a big factor in determining outcomes (high β implies that the impact of higher effort on probability of success is small). Also, since the prior reputation of the worker who accepts the contract is really low, the increase in reputation (and therefore future gains) from a success is low. Thus, the principal does not have strong incentives to offer a high wage contract to extract high effort. \square

Corollary 4. *There exists β'' such that if $\beta > \beta''$ and the following hold.*

1. $\frac{1+\delta\beta}{\delta\beta(1-\beta)(1-\delta)} - 1 < R_w < \frac{(1+\delta\beta(1-\beta(1-\lambda_b)^2))(1-\beta+\beta\lambda_b)}{(1-\beta)^2} - 1$
2. $\frac{1+R_w}{1+\delta\beta} < V < \frac{1+R_w}{1+\delta\beta(1-\beta(1-\lambda_b)^2)} < R_p$.

Then there exists a p_3 such that if $p_g \in [0, p_3)$, then:

1. *There exists a separating equilibrium where G type worker's strategy is to play L in period 1 and then put in effort=1 in period 1 and $e=0$ in period 2. The B type worker's strategy dictates accepting the zero wage contract offered in period 1 and 2 and putting in zero effort in both periods.*
2. *In any other equilibrium, the strategy for either type of worker involves putting in no effort in both periods along the equilibrium path.*

Proof. The intuitive idea is the following. First of all notice that the above condition ($V < \frac{1+R_w}{1+\delta\beta(1-\beta(1-\lambda_b)^2)}$) imply that it is not IC for the B type worker to form his own firm. Therefore, there are only two kinds of equilibrium outcomes possible. B type worker accepts a contract in period 1, G type worker plays L or accepts a contract in period 1. In period 2, if a worker had accepted a contract in period 1, then he always accepts a contract and if the worker had formed a firm in period 1 then he always stays with his own firm (since $V < R_p$, it is not IR for the principal to form a new firm in period 2). Therefore, all we need to show is that in any equilibrium where the G type worker accepts a contract in period 1, both type workers don't put in any effort in either periods. Suppose both workers accept a contract in equilibrium in period 1. If $p_g \approx 0$ and β is high enough, then, in any equilibrium, it is not IC for the principal to offer a contract which would make it IC for the worker to put in any more than zero effort. This is because the value of the prize (V) is not high enough and effort barely influences the outcome (high β implies that the increase in probability of success when effort goes from zero to one is very small). So the equilibrium strategy for either type worker will be to not put in any effort in both periods after signing a contract in period 1. \square

Thus, under these conditions, the separating equilibrium generates higher incentives to work compared to other equilibria which exist under the same conditions.

6 Conclusion

This paper presents a theory of new firm formation based on signalling and reputation concerns. I show that in the presence of asymmetric information about the worker's type, there may exist separating equilibria where the good type worker can signal his ability by forming his own firm. This is true even if the principal can offer contracts to try and stop the worker from leaving. Such signalling behaviour is restricted when the market for labour is competitive as competition between the principals leads to high wages for the worker which reduces his incentives to signal using a costly action. If the outcome of the worker's job depends upon unobserved effort as well as inherent ability then, under some conditions, the separating equilibrium (in which the good type worker forms his own firm) provides the highest incentive to put in effort.

In view of its impact on the welfare of the high type workers and on effort efficiency, the signalling aspect of new firm formation is important to understand. As I point out, these issues may have policy implications in the areas of brain drain and non compete clauses. Moreover, entrepreneurship is crucial for the economic progress of a country and we must try and understand all possible causes behind new firm formation. This paper highlights these issues with a simple principal agent model. In the future, I hope to use the intuition developed in this paper to deal with more specific problems like firm formation in teams and general optimal contracts in such environments.

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A Appendix

A.1 Types are common knowledge and Principal Agent Model

I will solve the one period game beginning in period 2 first and then proceed by backwards induction. The relevant state variables ²² which affect decisions in period 2 are: One, worker formed own firm in period 1 (f) or accepted contract (nf) or play N and two, reputation of worker at the end of period 1. Let this be denoted by p_g^2 .

Therefore, equilibrium strategies in period 2 are functions of the state variables augmented history. In particular, the strategy for principal in period 2 depends only on the value taken by the state variables:

$$s_{p,2} : (f/nf/N, p_g^2) \rightarrow \{x \in 2^{\{(s,f); s,f \geq 0\}}; |x| < 3\}$$

Strategy for worker in period 2 depends upon the state variables, action taken by the principal in period 2 and own type:

$$\begin{aligned} s_{w,2} : (nf/N, p_g^2, a_p(2), T) &\rightarrow \{N, acc_1, acc_2, L\} ; T \in \{G, B\} \\ s_{w,2} : (f, p_g^2, a_p(2), T) &\rightarrow \{N, acc_1, acc_2, S\} ; T \in \{G, B\} \end{aligned}$$

Strategy for customer in period 2:

$$s_{c,2} : (f/nf/N, p_g^2, a_p(2), a_w(2)) \rightarrow \{b; b \in \mathbb{R}_+\}$$

Since I have mistakes in the model and mistake probabilities go to zero, the relevant equilibrium concept is trembling hand perfect equilibrium.

A.1.1 In Period 2

Claim 2. *Suppose the worker formed a firm in period 1. Then, in period 2, any contract which is IC for the worker to accept is not IR for the principal.*

Proof. Let the worker be G type. Then the worker gets V as project price from the customers²³. To make it incentive compatible for the worker to accept his contract, the principal will have to pay atleast V as wages. The customers pay the same price (V) for the project when the firm is owned by the principal because their

²²Technically, a strategy can be based on the entire history of play. However, if the customers bid something off the equilibrium path (i.e. not the expected payoff) then it does not affect the beliefs of the players about the worker type. This can be justified using Fudenberg and Tirole (1991)'s "no signalling what you don't know" condition which is built into the definition of perfect Bayesian equilibrium.

²³The customers are risk neutral and are competing with one another to get the services of the firm. So they will both bid their expected value (V) of the project and one will be randomly selected as the winner.

bidding simply depends upon the worker type. Thus, if the worker accepts his contract, the principal's payoff will be at most $V - V - R_p < 0$. If the worker type is B , then it is not worthwhile for the principal to form the firm even if he can pay the worker zero wages. This follows from the assumption $\lambda_b V \leq R_p$. \square

Corollary 5. *In any SPNE, if the worker type is G , the principal will offer the contract $\{0, 0\}$ in period 2 if the worker did not accept a contract in period 1.*

Proof. If the worker formed his own firm in period one then the principal forms his firm in period 2 only if the worker makes a mistake. Therefore, he offers the lowest paying contract. If the worker had chosen N in period 1, then we know that the worker can only get a negative payoff by forming his own firm in period 2 ($R_w > V$). Therefore, he will accept a zero wage contract.²⁴ It is IR for the principal to offer the zero wage contract because $R_p < V$. Obviously, it is not optimal for the principal to offer higher wages. \square

Corollary 6. *In any SPNE, if the worker type is B , the principal will offer no contract in period 2 if the worker did not accept a contract in period 1.*

Proof. The highest payoff the principal could get is when he does not have to pay any wages. This payoff is $(\lambda_b V)$. This does not compensate for the cost of setting up the firm R_p by assumption $(\lambda_b V < R_p)$. \square

Corollary 7. *In any SPNE, the principal will offer the contract $\{0, 0\}$ in period 2 if the worker accepted a contract in period 1.*

Proof. Since the payoff from forming a firm in period 2 is negative for worker he is willing to accept zero wages. The principal is willing to offer this contract since he has already invested R_p in period 1 and therefore faces no additional cost in offering this zero wage contract. \square

Corollary 8. *If worker type is known and the worker has formed his firm at period 1, then the principal can never form a principal-worker firm in period 2.²⁵*

Thus the payoffs in period 2 can be summarized as follows (first coordinate is principal's payoffs and the second is worker's payoff.):

Table 4: Payoffs in Period 2

Table 5: Payoff if contract accepted in Period 1

$(Principal, G)$	$(Principal, B)$
$(V, 0)$	$(\lambda_b V, 0)$

Table 6: Payoff if Spinoff in Period 1

$(Principal, G)$	$(Principal, B)$
$(0, V)$	$(0, \lambda_b V)$

Table 7: Payoff if N in Period 1

$(Principal, G)$	$(Principal, B)$
$(V - R_p, 0)$	$(0, 0)$

²⁴The worker could also choose N . However, we have assumed that if the worker is getting atleast zero from any other action choice he will choose that action.

²⁵Unless there is a mistake.

A.1.2 In Period 1

Claim 3. *In the unique subgame perfect equilibrium of the game, when the worker type is B , the principal offers the worker a contract with zero wages $(\{0, 0\})$ in period 1 and the worker accepts. If the worker type is G then the principal offers the contract $\{V - R_w + \delta V, 0\}$ in period 1 and it is accepted by the worker. In period 2, (along the equilibrium path) the principal offers a zero wage contract which is accepted by the worker.*

Proof. By assumption, it is not IR for B worker to invest in forming own firm at period 1. Therefore, the principal will offer zero wage contract which will be accepted. Note that it is IR for the principal to offer this contract by assumption that $\lambda_b V(1 + \delta) > R_p$. When the worker type is G the worker gets the following payoff by investing R_w and starting his own firm:

$$V - R_w + \delta V$$

The principal must offer him this amount to dissuade him from leaving. Offering more will not be optimal. Moreover, it is known that the principal cannot commit to offering more than zero wages in period 2 if the worker does not form a firm in period 1. Therefore, the principal offers him a contract which pays him $V - R_w + \delta V$ upon success ²⁶ :

Offering this contract is also IR for the principal as the principal's payoff is $V - (V - R_w + \delta V) - R_p + \delta V = R_w - R_p (> 0)$. \square

Corollary 9. *The worker never forms his own firm if worker type is known.*

Thus equilibrium payoffs to the principal and worker in the game where types are known are given by the following matrix where (the first coordinate refers to the principal's payoff)

Table 8: Payoffs if Types are Known

$(Principal, G)$	$(Principal, B)$
$(R_w - R_p, V - R_w + \delta V)$	$(\lambda_b V(1 + \delta) - R_p, 0)$

A.2 Types are Private knowledge and Principal Agent Problem

The previous subsection tells us that when the worker type is known then the worker never forms his own firm. This result does not hold true when there is asymmetric information about the worker type. In this subsection, I will illustrate this case and some other equilibria which may occur when the worker type is private knowledge. We will begin by analysing subgame perfect outcomes in period 2.

²⁶Since G type always succeeds at V , there is no point in offering him any payoff for failure

A.2.1 Period 2

Case 1 - Worker forms own firm in period 1.

If the worker failed in period one, then the worker is revealed to be B type. We know the payoffs from the previous subsection. Consider the case where the worker formed his own firm in period 1 and the worker succeeded. If the worker formed his firm in period 1, then there are 5 possible ‘pure’²⁷ equilibrium outcomes in period 2:

1. Pooling on S by worker types.
2. Pooling on a contract by worker types.
3. Separating equilibrium where G worker accepts contract in period 2 and B worker plays S .
4. Separating equilibrium where B worker accepts contract in period 2 and G worker plays S .
5. Separating equilibrium where G worker accepts contract 1 in period 2 and B worker accepts a contract 2.

I will describe conditions under which the only possible ‘pure’ equilibria in period 2 are the pooling on S equilibrium and the separating equilibrium where the G worker accepts a contract and B worker plays S .

We can immediately eliminate equilibria of the 4th kind. This is because in such an equilibrium, the principal would have to offer the B worker at least $\lambda_b V$ as expected payment and this would also be the price that the risk neutral customers would pay in equilibrium to the principal if a worker accepts a contract. Since the principal has positive cost of forming the firm, he will get negative payoffs. He can easily avoid these by offering no contract instead.

²⁷No mixing on the equilibrium path.

The following strategies describe the pooling on S equilibrium in period 2.

For Principal :

$$\begin{aligned}
s_{p,2}(nf, p_g^2) &= \{\{0, 0\}\} \\
s_{p,2}(N, p_g^2) &= \{\{0, 0\}\} ; p_g^2 \geq \frac{R_p - \lambda_b V}{V(1 - \lambda_b)} \\
s_{p,2}(N, p_g^2) &= \phi ; p_g^2 < \frac{R_p - \lambda_b V}{V(1 - \lambda_b)} \\
s_{p,2}(f, p_g^2) &= \{\{0, 0\}\} ; \alpha \geq \frac{1 - p_g^2}{p_g^2} \frac{R_p - \lambda_b V}{V - R_p} \\
s_{p,2}(f, p_g^2) &= \phi ; \alpha < \frac{1 - p_g^2}{p_g^2} \frac{R_p - \lambda_b V}{V - R_p}
\end{aligned}$$

For Worker :

$$(WLOG \ s_1^2 \geq s_2^2 \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 \geq \lambda_b s_1^2 + (1 - \lambda_b) f_1^2)$$

If worker had formed firm in period 1 :

$$\begin{aligned}
s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) &= acc_1 ; s_1^2 \geq V \\
s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) &= acc_1 ; V > s_1^2 \geq p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 < \lambda_b V \\
s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) &= Any \ play \\
; V > s_1^2 \geq p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } p_g^2 V + (1 - p_g^2) \lambda_b V &\geq \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 \geq \lambda_b V \\
s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) &= S ; V > s_1^2 \geq p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 > p_g^2 V + (1 - p_g^2) \lambda_b V \\
s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) &= S ; s_1^2 < p_g^2 V + (1 - p_g^2) \lambda_b V \\
s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) &= acc_2 ; s_1^2 \geq V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 \geq \lambda_b V \\
s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) &= S ; s_1^2 \geq V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 < \lambda_b V \\
s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) &= S ; V > s_1^2 \geq p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 < \lambda_b V \\
s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) &= Any \ play \\
; V > s_1^2 \geq p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } p_g^2 V + (1 - p_g^2) \lambda_b V &\geq \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 \geq \lambda_b V \\
s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) &= acc_2 ; V > s_1^2 \geq p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 > p_g^2 V + (1 - p_g^2) \lambda_b V \\
s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) &= acc_2 ; s_1 < p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 \geq p_g^2 V + (1 - p_g^2) \lambda_b V \\
s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) &= S ; s_1 < p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 < p_g^2 V + (1 - p_g^2) \lambda_b V \\
s_{w,2}(f, p_g^2, \phi, G/B) &= S
\end{aligned}$$

If worker had not formed firm in period 1 :

$$\begin{aligned}
s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) &= acc_1 ; s_1^2 \geq s_2^2 \\
s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) &= acc_2 ; s_1^2 < s_2^2 \\
s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) &= acc_1 ; \lambda_b s_1^2 + (1 - \lambda_b) f_1^2 > \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 \\
s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) &= acc_2 ; \lambda_b s_1^2 + (1 - \lambda_b) f_1^2 \leq \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 \\
s_{w,2}(nf/N, p_g^2, \phi, G/B) &= N
\end{aligned}$$

For Customers (both) :

$$\begin{aligned}
s_{c,2}(f/nf, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, a_w(2)) &= p_g^{2'} V + (1 - p_g^{2'}) \lambda_b V \\
; \text{ where } p_g^{2'} &\text{ has been obtained by bayesian updating based on strategies of players.}
\end{aligned}$$

Proposition 13. *If $p_g^2 \geq \frac{V(1-\lambda_b)-R_p}{V(1-\lambda_b)}$, then the above strategies constitute a PBE for the game starting period 2 with the following beliefs on the equilibrium path:*

1. If the worker had formed his own firm in period 1:

$$\begin{aligned} \text{If } a_w(2) = acc_i &\rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)} \\ \text{If } a_w(2) = N &\rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)} \\ \text{If } a_w(2) = S &\rightarrow P(G) = \frac{(1 - \alpha\epsilon)p_g^2}{(1 - \alpha\epsilon)p_g^2 + (1 - \epsilon)(1 - p_g^2)} \approx p_g^2 \end{aligned}$$

2. If the worker had accepted a contract in period 1:

$$\begin{aligned} \text{If } a_w(2) = acc_i &\rightarrow P(G) = \frac{(1 - \alpha\epsilon)p_g^2}{(1 - \alpha\epsilon)p_g^2 + (1 - \epsilon)(1 - p_g^2)} \approx p_g^2 \\ \text{If } a_w(2) = L &\rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)} \\ \text{If } a_w(2) = N &\rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)} \end{aligned}$$

3. If the worker had took action N in period 1 and principal offers a contract:

$$\begin{aligned} \text{If } a_w(2) = acc_i &\rightarrow P(G) = \frac{(1 - \alpha\epsilon)p_g^2}{(1 - \alpha\epsilon)p_g^2 + (1 - \epsilon)(1 - p_g^2)} \approx p_g^2 \\ \text{If } a_w(2) = L &\rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)} \\ \text{If } a_w(2) = N &\rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)} \end{aligned}$$

Proof. It is obvious that a worker of any type will accept any contract in period two if he had accepted a contract in period one. This is because $R_w > V$, so the worker will get negative payoffs if he chooses to form his firm in period 2. Thus, both type workers will accept the zero wage contract. If the worker had taken the action N in period 1 then he will accept any contract he is offered. It needs to be shown that if the worker had formed his firm in period 1, then he will play according to the strategies given above. This can be checked easily from the strategies. \square

Thus, if the state variables at the end of period one are $\{f, p_g^2\}$ and $p_g^2 > \frac{V(1-\lambda_b)-R_p}{V(1-\lambda_b)}$ then an equilibrium outcome in period 2 is that the worker, irrespective of type, chooses the action S . Therefore the payoffs in period 2 under these conditions are given by :

- Principal ≈ 0 ²⁸
- Worker of any type $\approx p_g^2 V + (1 - p_g^2) \lambda_b V$.

²⁸Not exactly zero because of the possibility of mistakes.

If the worker has formed his own firm in period 1 and $p_g^2 \geq \frac{V(1-\lambda_b)-R_p}{V(1-\lambda_b)}$, then, in period 2 there is an equilibrium where the G worker accepts a contract and the B worker chooses S . The next claim makes clear some of the intuitive ideas behind proposition 13.

Claim 4. *Suppose the worker formed his own firm in period 1 and reputation of worker is above $\frac{V(1-\lambda_b)-R_p}{V(1-\lambda_b)}$ at the end of period 1. Then there exists a PBE such that, on the equilibrium path, a worker of any type will choose the action S . If additionally $\alpha = 1$, then there does not exist an equilibria where both type workers choose to accept contracts.*

Proof. Suppose the worker formed his firm in period 1 and the reputation of the worker at the end of period 1 is q . In the pooling on S equilibrium, if q is high then there is no contract which the principal can offer which will be incentive compatible for the worker to accept and individually rational for the principal. Consider a pooling on S equilibrium. I will check to see if the principal can offer a contract which will make any worker deviate. Clearly, it will not be individually rational of the principal to offer a contract which only the B worker accepts. Can he offer a contract to which the G type worker deviates?

Given B worker's strategy, if the G worker chooses to stay then he can get a payoff of $qV + (1-q)\lambda_b V^{29}$. Therefore the principal will have to offer him atleast this in expected wages for him to accept any contract. Expected payoff to principal if he offers these wages = $q(V - (qV + (1-q)\lambda_b V) - R_p)$. Clearly if $q > \frac{V(1-\lambda_b)-R_p}{V(1-\lambda_b)}$, then such a contract would not be IR for the principal.

$\alpha = 1$ implies that mistake probabilities are the same. Therefore, if there is an equilibrium where workers of both types accept contracts then they must get atleast their outside option which is $qV + (1-q)\lambda_b V$. We can use the above argument to rule out these equilibria. \square

Next, I will describe the possible separating equilibrium in period 2.

Let $a \leq V(1-\lambda_b) - R_p$ and $\alpha \approx 1$. The equilibrium outcome cannot be that both type workers choose to accept contracts. This is because if both workers choose to accept contracts then the principal gets an expected payoff of $p_g^2 V + (1-p_g^2)\lambda_b V - E(w) - R_p$ in period 2 where $E(w)$ is the expected wages paid to the worker. If $\alpha \approx 1$, then in any equilibrium where both type workers are supposed to accept contracts, a worker can get $p_g^2 V + (1-p_g^2)\lambda_b V$ by deviating and playing S . This implies that a worker of any type will have to be paid at least this for them to accept contracts. Thus, the expected wages are bounded below by $p_g^2 V + (1-p_g^2)\lambda_b V$. However, in this case, the principal's payoff in period 2 is negative. Therefore, both type workers choosing to accept contracts will not be an equilibrium outcome.

Take any strategy profile where the following holds and the workers respond optimally to any other

²⁹Both type workers choose the same action, therefore reputation is unchanged at $q \Rightarrow$ customers pay $qV + (1-q)\lambda_b V$.

contract as well.

For Principal :

$$s_{p,2}(f, p_g^2) = \{\{\lambda_b V + a, 0\}\}$$

For Worker :

$$(WLOG \ s_1^2 \geq s_2^2 \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 \geq \lambda_b s_1^2 + (1 - \lambda_b) f_1^2)$$

If worker had formed firm in period 1 :

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = acc_1 ; s_1^2 \geq \lambda_b V + a \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 < \lambda_b V$$

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = S ; s_1^2 < \lambda_b V + a \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 < p_g^2 V + (1 - p_g^2) \lambda_b V$$

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = S ; s_1^2 \geq \lambda_b V + a \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 < \lambda_b V$$

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = S ; s_1^2 < \lambda_b V + a \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 < p_g^2 V + (1 - p_g^2) \lambda_b V$$

$$s_{w,2}(f, p_g^2, \phi, G/B) = S$$

For Customers (both) :

$$s_{c,2}(f/nf, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, a_w(2)) = p_g^{2'} V + (1 - p_g^{2'}) \lambda_b V$$

; where $p_g^{2'}$ has been obtained by bayesian updating based on strategies of players.

Claim 5. *If the worker forms his own firm in period 1 and $p_g^2 \geq \frac{V(1-\lambda_b)-R_p}{V(1-\lambda_b)}$, then in period 2, there is an equilibrium where the equilibrium path play is given by the strategies above. The beliefs on equilibrium path are given by:*

- *If worker accepts contract - $P(G) = \frac{(1-\alpha\epsilon)p_g^2}{(1-\alpha\epsilon)p_g^2 + (1-p_g^2)\frac{\epsilon}{2}} \approx 1$*
- *If worker chooses S - $P(G) = \frac{\frac{\alpha\epsilon}{2}p_g^2}{\frac{\alpha\epsilon}{2}p_g^2 + (1-p_g^2)(1-\epsilon)} \approx 0$*
- *If worker chooses N - $P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1-p_g^2)}$*

Proof. Consider the case $a = 0$. Note that the principal has to offer at least $\lambda_b V$ as expected payoff to attract any worker. This is because worker can always choose S and get at least this wage. If the workers play according to the strategies above then there cannot be any other contract which gives the principal a higher payoff. Thus, we do not need to explicitly state the response of workers to other contracts (they just have to be best responses.) It is clear also that given the principal's contract of $\{\lambda_b V, 0\}$ and the beliefs, the workers respond optimally and given the strategies and contract, the beliefs are correct on the equilibrium path.

We can show that the above strategies will be the equilibrium strategies for any other $a \leq V(1 - \lambda_b) - R_p$ as well. $a > V(1 - \lambda_b) - R_p$ would violate the IR condition for the principal. \square

Note that the $a = 0$ case need not be an equilibrium of the entire game since it may not be optimal for the worker to form his own firm if the play in period 2 is expected to be the separating equilibrium corresponding to $a = 0$. I only point this case out since it describes a possible equilibria in the one period game beginning period 2 and it tells us about a possible equilibria of the entire game where the worker forms a firm in period 1 and then signs a contract with the principal in period 2.

Corollary 10. *Any worker earns a lower payoff in this equilibrium as compared to the pooling on S equilibrium.*

Proof. Follows from $a \leq V(1 - \lambda_b) - R_p \Rightarrow a \leq p_g^2 V(1 - \lambda_b)$. □

Thus we have two types of possible period 2 equilibria if $p_g^2 \geq \frac{V(1-\lambda_b)-R_p}{V(1-\lambda_b)}$ and $\alpha \approx 1$ and the worker forms his own firm in period 1. In one, both workers play S and in the other the G worker accepts a contract and the B worker plays S .

If $0 < p_g^2 < \frac{V(1-\lambda_b)-R_p}{V(1-\lambda_b)}$ and $\alpha \approx 1$, then the principal can get positive payoffs by attracting the G type worker. However, this case will not be interesting from the point of view of my analysis in the paper. This is because in all propositions I am interested in separating equilibria where the G worker leaves to form own firm in period 1. Thus p_g^2 will be close to 1 in all cases analyzed in the main part of the paper.

Case 2 - Worker accepts a contract in period 1.

Claim 6. *If the worker did not form his firm in period 1 and accepted a contract, then, in any PBE, in period 2, the principal offers the contract $\{0, 0\}$ and worker of any type accepts this contract.*

Proof. Suppose the worker accepted a contract in period 1. Since $V < R_w$, no worker can get a positive payoff by forming a firm in period 2. Therefore, the worker's optimal response to any contract offer is to simply choose the best contract. Given this, the principal can offer one contract ($\{0, 0\}$) or two such contracts (to separate the types). Since expected payoff is the same in either case, it is optimal for him to offer the former. Since the principal has already invested R_p in period one, offering the zero wage contract is IR for the principal in period 2. □

Case 3 - Worker plays N in period 1.

Claim 7. *If the worker chose N in period one, then it is optimal for the principal to offer the zero wage contract in period 2 if $p_g^2 > \frac{R_p - \lambda_b V}{V - R_p}$. Else he offers no contract in period 2. In the former case, the worker will accept the contract. In the latter case worker will choose N in period 2.*

Proof. Since we have the condition that $R_p > \lambda_b V$, we can show that unless the reputation of the worker is high enough, it is not worthwhile for the principal to form the firm. □

Corollary 11. *If the worker did not form his firm in period 1, he will not do so in period 2.*

A.2.2 Period 1

In period 1 of the game the following equilibrium outcomes are possible:

1. Separating equilibrium where G leaves and forms firm and B accepts contract.
2. Separating equilibrium where B leaves and forms firm and G accepts contract.
3. Separating equilibrium where G and B accept different contracts.
4. Pooling on L equilibrium.
5. Pooling on a contract equilibrium.

We can immediately eliminate the separating equilibrium where B worker leaves and G worker accepts contract. This is because the payoff in such an equilibrium will be negative for B worker. He can easily avoid this by choosing N in both periods. We will be most interested in the equilibrium described next.

Separating Equilibrium where G worker plays L

If there is a PBE of the game which is a separating equilibrium where the G type worker leaves in period one to form his own firm, then we know the payoffs and actions chosen in period 2 of that PBE (assuming the workers play the equilibrium - pooling on S in period 2). This is because on the equilibrium path, p_g^2 will satisfy the condition $p_g^2 > \frac{V(1-\lambda_b)-R_p}{V(1-\lambda_b)}$.

Consider the following strategy profile for play in period 2:

For Principal :

$$s_{p,2}(nf, p_g^2) = \{\{0, 0\}\}$$

$$s_{p,2}(N, p_g^2) = \{\{0, 0\}\} ; p_g^2 \geq \frac{R_p - \lambda_b V}{V(1 - \lambda_b)}$$

$$s_{p,2}(N, p_g^2) = \phi ; p_g^2 < \frac{R_p - \lambda_b V}{V(1 - \lambda_b)}$$

$$s_{p,2}(f, p_g^2) = \{\{0, 0\}\} ; \alpha \geq \frac{1 - p_g^2}{p_g^2} \frac{R_p - \lambda_b V}{V - R_p}$$

$$s_{p,2}(f, p_g^2) = \phi ; \alpha < \frac{1 - p_g^2}{p_g^2} \frac{R_p - \lambda_b V}{V - R_p}$$

For Worker :

$$(WLOG s_1^2 \geq s_2^2 \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 \geq \lambda_b s_1^2 + (1 - \lambda_b) f_1^2)$$

If worker had formed firm in period 1 :

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = acc_1 ; s_1^2 \geq V$$

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = acc_1 ; V > s_1^2 \geq p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 < \lambda_b V$$

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = Any \text{ play}$$

$$; V > s_1^2 \geq p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } p_g^2 V + (1 - p_g^2) \lambda_b V \geq \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 \geq \lambda_b V$$

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = S ; V > s_1^2 \geq p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 > p_g^2 V + (1 - p_g^2) \lambda_b V$$

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = S ; s_1^2 < p_g^2 V + (1 - p_g^2) \lambda_b V$$

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = acc_2 ; s_1^2 \geq V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 \geq \lambda_b V$$

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = S ; s_1^2 \geq V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 < \lambda_b V$$

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = S ; V > s_1^2 \geq p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 < \lambda_b V$$

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = Any \text{ play}$$

$$; V > s_1^2 \geq p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } p_g^2 V + (1 - p_g^2) \lambda_b V \geq \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 \geq \lambda_b V$$

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = acc_2 ; V > s_1^2 \geq p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 > p_g^2 V + (1 - p_g^2) \lambda_b V$$

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = acc_2 ; s_1 < p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 \geq p_g^2 V + (1 - p_g^2) \lambda_b V$$

$$s_{w,2}(f, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = S ; s_1 < p_g^2 V + (1 - p_g^2) \lambda_b V \text{ and } \lambda_b s_2^2 + (1 - \lambda_b) f_2^2 < p_g^2 V + (1 - p_g^2) \lambda_b V$$

$$s_{w,2}(f, p_g^2, \phi, G/B) = S$$

If worker had not formed firm in period 1 :

$$s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = acc_1 ; s_1^2 \geq s_2^2$$

$$s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = acc_2 ; s_1^2 < s_2^2$$

$$s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = acc_1 ; \lambda_b s_1^2 + (1 - \lambda_b) f_1^2 > \lambda_b s_2^2 + (1 - \lambda_b) f_2^2$$

$$s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = acc_2 ; \lambda_b s_1^2 + (1 - \lambda_b) f_1^2 \leq \lambda_b s_2^2 + (1 - \lambda_b) f_2^2$$

$$s_{w,2}(nf/N, p_g^2, \phi, G/B) = N$$

For Customers (both) :

$$s_{c,2}(f/nf, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, a_w(2)) = p_g^{2'} V + (1 - p_g^{2'}) \lambda_b V$$

; where $p_g^{2'}$ has been obtained by bayesian updating based on strategies of players.

Consider the following strategy profile for play in period 1:

For Principal :

$$s_p(\phi) = \{\{0, 0\}\}$$

For Worker :

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_1 ; s_1^1 \geq s_2^1 \text{ and } s_1^1 \geq V - R_w + \delta V$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_2 ; s_1^1 < s_2^1 \text{ and } s_2^1 \geq V - R_w + \delta V$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = L ; \text{ else.}$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, B) = acc_1 ; \lambda_b s_1^1 + (1 - \lambda_b) f_1^1 > \lambda_b s_2^1 + (1 - \lambda_b) f_2^1$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, B) = acc_2 ; \lambda_b s_2^1 + (1 - \lambda_b) f_2^1 \leq \lambda_b s_1^1 + (1 - \lambda_b) f_1^1$$

$$s_w(\phi, B) = N$$

$$s_w(\phi, G) = L$$

For Customers :

$$s_c(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, L) = V ; s_i^1 < V - R_w + \delta V \forall i$$

$$s_c(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, acc_1) = \lambda_b V ; s_i^1 < V - R_w + \delta V \forall i \text{ and } \lambda_b s_1^1 + (1 - \lambda_b) f_1^1 > \lambda_b s_2^1 + (1 - \lambda_b) f_2^1$$

$$s_c(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, acc_2) = \lambda_b V ; s_i^1 < V - R_w + \delta V \forall i \text{ and } \lambda_b s_2^1 + (1 - \lambda_b) f_2^1 \geq \lambda_b s_1^1 + (1 - \lambda_b) f_1^1$$

$$s_c(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, acc_1) = \frac{\alpha p_g}{\alpha p_g + (1 - p_g)} V + \frac{1 - p_g}{\alpha p_g + (1 - p_g)} \lambda_b V ; s_i^1 < V - R_w + \delta V \forall i \text{ and } \lambda_b s_1^1 + (1 - \lambda_b) f_1^1 \leq \lambda_b s_2^1 + (1 - \lambda_b) f_2^1$$

$$s_c(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, acc_2) = \frac{\alpha p_g}{\alpha p_g + (1 - p_g)} V + \frac{1 - p_g}{\alpha p_g + (1 - p_g)} \lambda_b V ; s_i^1 < V - R_w + \delta V \forall i \text{ and } \lambda_b s_2^1 + (1 - \lambda_b) f_2^1 < \lambda_b s_1^1 + (1 - \lambda_b) f_1^1$$

Similarly when $s_i^1 \geq V - R_w + \delta V$ for some i ...

Proposition 14. *If $V < \frac{R_w}{1 + \delta \lambda_b (2 - \lambda_b)}$. Then, if $p_g < \frac{R_p - \lambda_b R_w + (\lambda_b V (1 + \delta) - R_p)}{R_w - \lambda_b R_w + (\lambda_b V (1 + \delta) - R_p)}$, there is a separating equilibrium where the strategies in period 1 and 2 are given as above.*

Beliefs in the PBE in period 1 are given by:

$$\text{If } a_w(1) = acc_1 \rightarrow P(G) = \frac{\alpha \epsilon p_g}{\alpha \epsilon p_g + (1 - p_g)(1 - \epsilon)} \approx 0$$

$$\text{If } a_w(1) = N \rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)}$$

$$\text{If } a_w(1) = S \rightarrow P(G) = \frac{(1 - \alpha \epsilon) p_g}{(1 - \alpha \epsilon) p_g + (1 - p_g) \epsilon} \approx 1$$

Beliefs in the PBE in period 2 are given by:

1. If the worker had formed his own firm in period 1:

$$\begin{aligned} \text{If } a_w(2) = acc_i &\rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)} \\ \text{If } a_w(2) = N &\rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)} \\ \text{If } a_w(2) = S &\rightarrow P(G) = \frac{(1 - \alpha\epsilon)p_g^2}{(1 - \alpha\epsilon)p_g^2 + (1 - \epsilon)(1 - p_g^2)} \approx p_g^2 \end{aligned}$$

2. If the worker had accepted a contract in period 1:

$$\begin{aligned} \text{If } a_w(2) = acc_i &\rightarrow P(G) = \frac{(1 - \alpha\epsilon)p_g^2}{(1 - \alpha\epsilon)p_g^2 + (1 - \epsilon)(1 - p_g^2)} \approx p_g^2 \\ \text{If } a_w(2) = L &\rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)} \\ \text{If } a_w(2) = N &\rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)} \end{aligned}$$

3. If the worker had took action N in period 1 and principal offers a contract:

$$\begin{aligned} \text{If } a_w(2) = acc_i &\rightarrow P(G) = \frac{(1 - \alpha\epsilon)p_g^2}{(1 - \alpha\epsilon)p_g^2 + (1 - \epsilon)(1 - p_g^2)} \approx p_g^2 \\ \text{If } a_w(2) = L &\rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)} \\ \text{If } a_w(2) = N &\rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)} \end{aligned}$$

Proof.

Lemma 2. If $p_g^2 \geq \frac{V(1-\lambda_b)-R_p}{V(1-\lambda_b)}$, then the above strategies for play in period 2 constitute a PBE for the game starting period 2 with the following beliefs on the equilibrium path:

1. If the worker had formed his own firm in period 1:

$$\begin{aligned} \text{If } a_w(2) = acc_i &\rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)} \\ \text{If } a_w(2) = N &\rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)} \\ \text{If } a_w(2) = S &\rightarrow P(G) = \frac{(1 - \alpha\epsilon)p_g^2}{(1 - \alpha\epsilon)p_g^2 + (1 - \epsilon)(1 - p_g^2)} \approx p_g^2 \end{aligned}$$

2. If the worker had accepted a contract in period 1:

$$\text{If } a_w(2) = acc_i \rightarrow P(G) = \frac{(1 - \alpha\epsilon)p_g^2}{(1 - \alpha\epsilon)p_g^2 + (1 - \epsilon)(1 - p_g^2)} \approx p_g^2$$

$$\text{If } a_w(2) = L \rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)}$$

$$\text{If } a_w(2) = N \rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)}$$

3. If the worker had took action N in period 1 and principal offers a contract:

$$\text{If } a_w(2) = acc_i \rightarrow P(G) = \frac{(1 - \alpha\epsilon)p_g^2}{(1 - \alpha\epsilon)p_g^2 + (1 - \epsilon)(1 - p_g^2)} \approx p_g^2$$

$$\text{If } a_w(2) = L \rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)}$$

$$\text{If } a_w(2) = N \rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)}$$

Proof. The principal's strategy is easy to check for optimality. If the worker had already accepted a contract in period one then the principal has already made his investment of R_p and also knows that the worker gets negative payoff from forming own firm in period 2. Therefore, it is optimal for the principal to offer zero wage contract. If the worker had played N in period one, then the principal can only get positive payoffs from hiring the worker and making his investment of R_p if there is a high enough probability that the worker is of type G . This is because $R_p > \lambda_b V, R_p < V$. Lastly, if the worker had formed his own firm in period one, then according to worker's strategies, the principal can only attract the G worker if he is willing to pay more than $p_g^2 V + (1 - p_g^2) \lambda_b V$ as wage for success. It can be easily checked that if $p_g^2 \geq \frac{V(1 - \lambda_b) - R_p}{V(1 - \lambda_b)}$, then offering these wages are not IR for the the principal. Thus, the principal can only hope to get the (with high probability) G worker if he makes a mistake. In this case, it is best for the principal to offer a zero wage contract.

It is obvious that a worker of any type will accept any contract in period two if he had accepted a contract in period one. This is because $R_w > V$, so the worker will get negative payoffs if he chooses to form his firm in period 2. Thus, both type workers will accept the zero wage contract. If the worker had taken the action N in period 1 then he will accept any contract he is offered. It needs to be shown that if the worker had formed his firm in period 1, then he will play according to the strategies given above.

Suppose the worker formed his firm in period one. If the principal offers no contract in period 2, then it is optimal for the worker to play S . Suppose the principal offers the contracts $\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}$ in period 2. WLOG, let $s_1^2 \geq s_2^2$ and $\lambda_b s_2^2 + (1 - \lambda_b) f_2^2 \geq \lambda_b s_1^2 + (1 - \lambda_b) f_1^2$.

The rest of the proof is just checking that the worker of any type responds optimally, given the contracts and strategy of worker of other type. \square

Since we are talking about a separating equilibrium where the G worker plays L and the B worker

accepts a contract, we know the play of the game in period 2 (assuming pooling on S equilibrium is played after worker plays L in 1). If the worker forms his firm then his reputation is almost 1³⁰ and therefore the beliefs about the player being G type at the end of period one are either almost one³¹ or zero³². In the first case lemma 2 applies and the worker will choose the action S in period 2. In the latter case, the type of the worker is revealed to be B . The customers will bid $\lambda_b V$ in period two. The worker can get this by choosing S . If the principal offers this amount as wages to worker then the worker will accept but since the principal will also have to invest R_p to set up his firm, this contract offer will not be individually rational for the principal. Therefore, in this case as well, the worker will choose S in period 2. If the worker does not form firm in period one, then claim 2 applies and we know that the principal will offer a zero pay contract which the worker will accept.

Consider the strategies given above as strategies to be played in period one. Let's look at G type worker's strategy, given the strategy of others. Clearly, he has no incentive to deviate if $V - R_w + \delta V > 0$.

Consider the strategy of the B type worker now. To make sure accepting a contract is incentive compatible for him we need:

$$0 > V - R_w + \delta(\lambda_b V + (1 - \lambda_b)\lambda_b V)$$

Thus we need the condition $V < \frac{R_w}{1 + \delta\lambda_b(2 - \lambda_b)}$.

Consider the strategy for principal now. According to the strategies, B type worker will accept the contract and if the worker is G type, then he will leave to form his own firm. We will have to check if the principal can deviate and do better. The only profitable deviation may be if he can attract the G type worker with a contract. To do so he will have to offer a contract in which the reward for success is $V - R_w + \delta V$. This is the payoff that the G type player expects by leaving.³³ The principal will not deviate if he cannot offer a contract which gives him higher payoff than the strategy above. Note that the B type player will also have to get an expected wage of at least $\lambda_b(V - R_w + \delta V)$ in period one in any equilibrium (since he can always accept the contract offered (s_i, f_i)). Therefore, expected payoff for principal is bounded above by:

$$p_g V + (1 - p_g)\lambda_b V - (p_g(V - R_w + \delta V) + (1 - p_g)\lambda_b(V - R_w + \delta V)) - R_p + \\ \delta(p_g(\frac{p_g}{p_g + (1 - p_g)\lambda_b}V + \frac{(1 - p_g)\lambda_b}{p_g + (1 - p_g)\lambda_b}\lambda_b V) + (1 - p)(\lambda_b(\frac{p_g}{p_g + (1 - p_g)\lambda_b}V + \frac{(1 - p_g)\lambda_b}{p_g + (1 - p_g)\lambda_b}\lambda_b V) + (1 - \lambda_b)\lambda_b V))$$

Under the strategy proposed, principal's payoffs are $(1 - p_g)(\lambda_b V - R_w + \delta\lambda_b V)$. The above expression is less than these payoffs if $p_g < \frac{R_p - \lambda_b R_w + (\lambda_b V(1 + \delta) - R_p)}{R_w - \lambda_b R_w + (\lambda_b V(1 + \delta) - R_p)}$. Since $V(1 + \delta) > R_w$, $\lambda_b V(1 + \delta) > R_p$ and $R_w > R_p$, therefore the expression on the RHS is between zero and one. \square

Expected payoffs in this equilibrium are as follows:

³⁰Not one because there is a small probability of mistake.

³¹If job is successful.

³²Job fails.

³³Today's payoff is $V - R_w$. This is because he will have to invest R_w today and the customers will bid V if firm is formed since they expect only the G type to form it. The G type player will be successful and his reputation will remain close to one (not one because there is a small probability of mistake). Therefore, as analysed before, in period 2 he will remain with his new firm and get a payoff of approximately V .

- Principal $\approx (1 - p_g)(\lambda_b V(1 + \delta) - R_p)$.
- Worker type $G \approx V - R_w + \delta V$.
- Worker type $B \approx 0$.

In words, if the gains from the project is high enough only for the G worker ($\frac{R_w}{1+\delta} < V < \frac{R_w}{1+\delta\lambda_b(2-\lambda)}$) then the G worker can signal his type by separating in equilibrium. For the principal to be unable to stop the G worker from moving, we need the condition that prior reputation of the worker is low.

The strategy profile describes above uses the assumption that the second period equilibrium is one in which worker types pool on S . We can get a separating equilibrium in period 1 where the second period play is directed by the other equilibrium (G accepts contract and B plays S) as well.

Proposition 15. *Let $\frac{R_w + \delta R_p}{1+\delta} < V < \frac{R_w}{1+\delta\lambda_b}$ and $\lambda_b R_w + \delta V \leq R_w + \delta R_p$. Then, there exists $a' \in (0, V(1 - \lambda_b) - R_p)$ such that if $a > a'$ and $p_g < \frac{\lambda_b[V - R_w + \delta V + \delta(\lambda_b V + a)]}{R_w(1 - \lambda_b) + \lambda_b V(1 + \delta) + \delta\lambda_b(\lambda_b V + a) - \delta V - R_p(1 - \delta)}$, there is a separating equilibrium where the strategies in period 1 are such that principal offers zero wage contract which worker of B type accepts and worker of G type chooses L . In period 2, if worker had formed firm in period one, then principal offers the contract $\{\lambda_b V + a, 0\}$ and worker of type G accepts it while worker of type B plays S . Beliefs in the PBE in period 1 are given by:*

$$\begin{aligned} \text{If } a_w(1) = acc_1 &\rightarrow P(G) = \frac{\alpha \epsilon p_g}{\alpha \epsilon p_g + (1 - p_g)(1 - \epsilon)} \approx 0 \\ \text{If } a_w(1) = N &\rightarrow P(G) = \frac{\alpha p_g^2}{\alpha p_g^2 + (1 - p_g^2)} \\ \text{If } a_w(1) = S &\rightarrow P(G) = \frac{(1 - \alpha \epsilon) p_g}{(1 - \alpha \epsilon) p_g + (1 - p_g) \epsilon} \approx 1 \end{aligned}$$

Proof. The proof works in the same way as the proof for proposition 2. □

Now I will outline some of the other equilibria possible in the principal agent model

Both types accept Contract Equilibrium

In an equilibrium where both types choose contracts, the payoffs in the contract must depend on the beliefs in the market about the worker type and each worker's payoff from forming own firm. In an equilibrium in which both workers choose contracts, if a worker forms his own firm, it must be judged as a mistake. Therefore, the reputation of a worker who forms his own firm in period one when the equilibrium play was to accept a contract would have a reputation equal to:

$$\frac{\alpha p_g}{\alpha p_g + (1 - p_g)}$$

If $\alpha \approx 1$, i.e. the probability of mistakes from either side is the same then the belief about a worker who forms his own firm in period one must be equal to the prior p_g . Let $\alpha = 1$. Consider an equilibrium where

both type workers are expected to sign a contract in period 1. Then, the payoff from forming own firm for G worker is:

$$p_g V + (1 - p_g) \lambda_b V - R_w + \delta \left(\frac{p_g V}{p_g + (1 - p_g) \lambda_b} + \frac{(1 - p_g) \lambda_b \lambda_b V}{p_g + (1 - p_g) \lambda_b} \right)$$

It is clear that the above function is increasing in p_g and is negative as $p_g \rightarrow 0$ and positive as $p_g \rightarrow 1$. Thus there exists a cut off p such that the payoff from forming own firm is negative if $p_g < p$. Formally we have that the following strategies constitute an equilibrium if $p_g < p$ and $\alpha = 1$.

For Principal :

$$s_p(\phi) = \{\{0, 0\}\}$$

For Worker :

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_1 ; s_1^1 \geq s_2^1$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_2 ; s_1^1 < s_2^1$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, B) = acc_1 ; \lambda_b s_1^1 + (1 - \lambda_b) f_1^1 > \lambda_b s_2^1 + (1 - \lambda_b) f_2^1$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, B) = acc_2 ; \lambda_b s_2^1 + (1 - \lambda_b) f_2^1 \leq \lambda_b s_1^1 + (1 - \lambda_b) f_1^1$$

$$s_w(\phi, B) = N$$

$$s_w(\phi, G) = N$$

For Customers (on equilibrium path) :

$$s_c(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, L) = p_g V + (1 - p_g) \lambda_b V$$

$$s_c(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, acc_1) = p_g V + (1 - p_g) \lambda_b V$$

Proposition 16. *Let $\alpha = 1$ and $p_g < p$, then the above strategies constitute a PBE with the beliefs being p_g for any history.*

Proof. Given $\alpha = 1$ and $p_g < p$, the outside option for a worker of any type is negative³⁴. Therefore, any worker would accept a zero wage contract. Since offering a zero wage contract is IR for the principal and equivalent in expected payoff to offering two zero wage contracts (and separating the types), the principal will make this offer and the worker will accept it. \square

Equilibria with mixing on the Equilibrium path

It is possible to have equilibria where players mix on the equilibrium path as well. Consider the one period game in period 2. We can show that there are equilibria in which, along the equilibrium path, the G worker

³⁴Outside option for G worker is negative implies outside option for B worker is negative.

mixes between accepting contract and playing S and the B worker's strategy is to play S . The proofs would be very similar to the ones discussed before.

A.2.3 Pooling Equilibrium

In this subsection, I am interested in understanding when worker strategies can lead to a pooling equilibrium where both type workers choose to leave to form their own firm. I show that if the conditions needed to get a separating equilibrium in proposition 2 hold, then there cannot exist a pooling equilibrium. For this section I will assume that:

$$V - R_w + \delta(\lambda_b V + (1 - \lambda_b)\lambda_b V) > 0 \quad (3)$$

This condition requires that the maximum payoff that a B type worker may get from a pooling equilibrium be positive. If this condition is not met then it is not IR for the B worker to form the firm in period one. Therefore, without this condition, there does not exist any pooling PBE of the game. Also, I will keep the two assumptions introduced at the end of the previous subsection i.e. I will assume that assumption A and B hold.

Since $V - R_w + \delta(\lambda_b V + (1 - \lambda_b)\lambda_b V) > 0$, assumption B loses its bite for this subsection. Mistake probabilities will always be the same for G and B type workers. However, assumption A will simplify matters. We will need that the prior reputation is high enough to have a chance of getting pooling on L equilibrium in period 1. This is formalized in the following claim:

Claim 8. *There exists a p' such that if $p_g \in [0, p']$, there does not exist a pooling equilibrium.*

Proof. We know that:

$\lambda_b V - R_w + \delta(\lambda_b V) < 0$ and $V - R_w + \delta(\lambda_b V + (1 - \lambda_b)\lambda_b V) > 0$. By intermediate value theorem there exists a p' such that it is IR for B worker to play L in period 1 as part of a pooling equilibrium only if $p_g > p'$:

$$p_g V + (1 - p_g)\lambda_b V - R_w + \delta(\lambda_b [\frac{p_g V + (1 - p_g)\lambda_b \lambda_b V}{p_g + (1 - p_g)\lambda_b}] + (1 - \lambda_b)\lambda_b V) > 0 \Leftrightarrow p_g > p'$$

To find value of p' :

$$p_g V + (1 - p_g) \lambda_b V - R_w + \delta [\lambda_b (\frac{p_g}{p_g + (1 - p_g) \lambda_b} V + \frac{(1 - p_g) \lambda_b}{p_g + (1 - p_g) \lambda_b} \lambda_b V) + (1 - \lambda_b) \lambda_b V] > 0$$

$$\text{Let } x = p_g + (1 - p_g) \lambda_b$$

Then above reduces to :

$$V x^2 + x(\delta \lambda_b V(1 - \lambda_b) - R_w) + \delta \lambda_b V(p_g + (1 - p_g) \lambda_b^2) > 0$$

$$\text{Now } p_g + (1 - p_g) \lambda_b^2 = x \lambda_b + p_g(1 - \lambda_b)$$

$$\text{And } p_g(1 - \lambda_b) = x - \lambda_b \Rightarrow p_g + (1 - p_g) \lambda_b^2 = x \lambda_b + x - \lambda_b$$

Therefore we have

$$x^2 V + x(\delta \lambda_b V(1 - \lambda_b) - R_w) + \delta \lambda_b V(x(1 + \lambda_b) - \lambda_b) > 0$$

$$\Leftrightarrow x^2 V + x(2\delta \lambda_b V - R_w) - \delta \lambda_b^2 V > 0$$

$$\Leftrightarrow (x\sqrt{V} + \frac{2\delta \lambda_b V - R_w}{2\sqrt{V}})^2 - (\sqrt{(\frac{2\delta \lambda_b V - R_w}{2\sqrt{V}})^2 + \delta \lambda_b^2 V})^2 > 0$$

$$\Leftrightarrow x > \frac{\sqrt{(\frac{2\delta \lambda_b V - R_w}{2\sqrt{V}})^2 + \delta \lambda_b^2 V} + \frac{R_w - 2\delta \lambda_b V}{2\sqrt{V}}}{\sqrt{V}}$$

$$\Leftrightarrow p_g > \frac{\sqrt{(\frac{2\delta \lambda_b V - R_w}{2\sqrt{V}})^2 + \delta \lambda_b^2 V} + \frac{R_w - 2\delta \lambda_b V}{2\sqrt{V}} - \lambda_b \sqrt{V}}{\sqrt{V}(1 - \lambda_b)}$$

We can show that the RHS is above zero and less than one. The former uses $R_w > \lambda_b V(1 + \delta)$ (assumed in the model) and the latter uses the condition we assumed in the beginning of the subsection $V - R_w + \delta(\lambda_b V + (1 - \lambda_b) \lambda_b V) > 0$.

$$\text{Therefore, the exact expression for } p' \text{ is } p' = \frac{\sqrt{(\frac{2\delta \lambda_b V - R_w}{2\sqrt{V}})^2 + \delta \lambda_b^2 V} + \frac{R_w - 2\delta \lambda_b V}{2\sqrt{V}} - \lambda_b \sqrt{V}}{\sqrt{V}(1 - \lambda_b)}$$

□

For the rest of this subsection, I will assume that $p_g > p'$.

Proposition 17. *If $p_g \geq \max\{p', \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}\}$, then there does not exist a PBE pooling equilibrium where both types of workers choose to play L in period 1.*

Proof. Suppose there is a pooling on L equilibrium. Then, by assumption A, in a pooling equilibrium where the worker forms his firm in period 1, the principal gets a payoff of zero. Therefore, it must be the case that to any contract that the principal offers, either the optimal response for the G, B worker is to play L or the optimal response is such that it gives the principal a negative payoff (not IR for principal).

I will show that there is a deviation for the principal, where, if the two types of workers respond optimally, the principal gets positive payoff. Consider the contract $\{V - R_w + \delta V, 0\}$. Since this is the best

payoff a G player can achieve by L , he will accept it. Given the contract and G worker's best response, it is optimal for the B worker to accept it too. Payoff to principal in this case is positive if:

$$p_g V + (1 - p_g)\lambda_b V - (p_g(V - R_w + \delta V) + (1 - p_g)\lambda_b(V - R_w + \delta V)) - R_p + \\ \delta[p_g(\frac{p_g}{p_g + (1 - p_g)\lambda_b}V + \frac{(1 - p_g)\lambda_b}{p_g + (1 - p_g)\lambda_b}\lambda_b V) + (1 - p_g)(\lambda_b(\frac{p_g}{p_g + (1 - p_g)\lambda_b}V + \frac{(1 - p_g)\lambda_b}{p_g + (1 - p_g)\lambda_b}\lambda_b V) + (1 - \lambda_b)\lambda_b V)] > 0 \\ \Leftrightarrow p_g > \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}$$

Now $p_g \geq \max\{p', \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}\}$. Therefore, principal has a deviation which gives him positive payoffs i.e. a higher payoff than what he would receive in a pooling on L equilibrium. Thus, the principal will deviate and so there is no pooling on L equilibrium if $p_g \geq \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}$. \square

Corollary 12. *If $R_p < \lambda_b R_w$, then there is no pooling on L perfect Bayesian equilibrium.*

Corollary 13. *Let $\max\{p', \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}\} = \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}$.*

A necessary condition for pooling on L PBE to exist is:

$$p' \leq p_g \leq \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}.$$

If $\max\{p', \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}\} = p'$ then there is no pooling on L equilibrium.

Proposition 18. *There is no pooling on L equilibrium.*

Proof. Suppose there is a pooling equilibrium. Now, by assumption A, if there is pooling on L , then the principal gets a payoff of zero. Therefore, it must be the case, that the principal cannot offer any IR contract which any worker would accept. Remember also that the second period payoff for a worker who accepts a contract in period one has to be zero in any equilibrium.

Suppose the principal offers a contract of the following form: $\{s, f\}$ such that the following hold:

$$s > p_g V + (1 - p_g)\lambda_b V - R_w + \delta(\frac{p_g V + (1 - p_g)\lambda_b^2 V}{p_g + (1 - p_g)\lambda_b}) \quad (4)$$

$$\lambda_b s + (1 - \lambda_b)f < p_g V + (1 - p_g)\lambda_b V - R_w + \delta(\lambda_b[\frac{p_g V + (1 - p_g)\lambda_b^2 V}{p_g + (1 - p_g)\lambda_b}] + (1 - \lambda_b)\lambda_b V) \quad (5)$$

To show - there is no equilibrium in the game between the worker types. The following are all the possible strategy choice combinations that the G, B workers can make

1. $G \rightarrow L, B \rightarrow L$.
2. $G \rightarrow L, B \rightarrow A$.
3. $G \rightarrow A, B \rightarrow A$.
4. $G \rightarrow A, B \rightarrow L$.

5. $G \rightarrow \text{mix}\{L, A\}, B \rightarrow A$.
6. $G \rightarrow \text{mix}\{L, A\}, B \rightarrow L$.
7. $G \rightarrow L, B \rightarrow \text{mix}\{L, A\}$.
8. $G \rightarrow A, B \rightarrow \text{mix}\{L, A\}$.
9. $G \rightarrow \text{mix}\{L, A\}, B \rightarrow \text{mix}\{L, A\}$.

I will eliminate each one by one.

1 can't be equilibrium responses because G would want to deviate and accept the contract.

2 can't be equilibrium responses because B would want to deviate and imitate G .

3 can't be equilibrium responses because B would want to deviate. Probability of mistakes are equal so B would get a payoff of $p_g V + (1 - p_g) \lambda_b V - R_w + \delta(\lambda_b [\frac{p_g V + (1 - p_g) \lambda_b^2 V}{p_g + (1 - p_g) \lambda_b}] + (1 - \lambda_b) \lambda_b V) (> \lambda_b s + (1 - \lambda_b) f)$.

4 can't be equilibrium responses because it is not IR for B worker

5 can't be equilibrium responses because B worker would want to deviate. This is because payoff from deviate is $V - R_w + \delta(\lambda_b V + (1 - \lambda_b) \lambda_b V) (> p_g V + (1 - p_g) \lambda_b V - R_w + \delta(\lambda_b [\frac{p_g V + (1 - p_g) \lambda_b^2 V}{p_g + (1 - p_g) \lambda_b}] + (1 - \lambda_b) \lambda_b V) > \lambda_b s + (1 - \lambda_b) f)$

6 can't be equilibrium responses because G worker cannot be indifferent between A and L . This is because payoff from L in equilibrium is at most $p_g V + (1 - p_g) \lambda_b V - R_w + \delta(\frac{p_g V + (1 - p_g) \lambda_b^2 V}{p_g + (1 - p_g) \lambda_b})$ which is less than s .

7 can't be equilibrium responses because B worker can't be indifferent between A and L . This is because payoff from L in equilibrium will be at least $p_g V + (1 - p_g) \lambda_b V - R_w + \delta(\lambda_b [\frac{p_g V + (1 - p_g) \lambda_b^2 V}{p_g + (1 - p_g) \lambda_b}] + (1 - \lambda_b) \lambda_b V) (> \lambda_b s + (1 - \lambda_b) f)$

8 can't be equilibrium responses because B worker can't be indifferent between A and L . This is because payoff from L is $\lambda_b V(1 + \delta) - R_w (< 0 < \lambda_b s + (1 - \lambda_b) f)$

9 can't be equilibrium responses because both G and B can't be indifferent between L and A . Suppose G plays L with a lower probability than B , then G would strictly prefer A to L . Suppose G plays L with a higher probability than B , then B would strictly prefer L to A .

Still, it is possible to have a pooling equilibrium if a contract of the sort described above is never offered in equilibrium. Since then, it would not matter how the workers respond to this contract. For this to be true, it should not be IR for the principal to offer this contract for every response possible by the workers. This is not true. In particular if G worker accepted, B formed his own firm and $s = p_g V + (1 - p_g) \lambda_b V - R_w + \delta(\frac{p_g V + (1 - p_g) \lambda_b^2 V}{p_g + (1 - p_g) \lambda_b}) + \nu$ for a very small positive ν . then it would be IR for the principal to offer this contract in equilibrium. \square

A.3 Competitive Labour market with types being Common knowledge

When two principals compete for one worker, the worker gets offered high wage contracts and competition between principals erode their payoff to zero. In this subsection, I establish these results in the baseline case where worker type is known. Once again, I look at possible equilibria in period 2 first and then discuss equilibria of the entire game.

A.3.1 Period 2

Claim 9. *If the worker had already formed his firm in period 1 then the principals cannot offer a contract which is IR for the principal to offer and which the worker will accept in period 2.*

Proof. If the worker had already formed his firm in period one and his type is known then he expects to get his expected product in period 2 by choosing to stay with his own firm i.e. G worker gets price V and B worker will get price $\lambda_b V$. Whichever firm the worker works with the customers will pay these prices. Neither principal can afford to pay the worker all of what they receive from the customer since they have a positive cost of firm formation. \square

Corollary 14. *Suppose the worker formed his own firm in period 1. If the worker type is G , then the principals will offer a zero wage contract in period 2 which will only be accepted upon a mistake. If the worker type is B , the principals will offer no contracts*

Proof. Follows from $V > R_p$ and $\lambda_b V < R_p$ \square

Claim 10. *Suppose the worker did not form his own firm in period one. Suppose the worker accepted a contract with principal i in period 1. If the worker is of type G , then in period 2, principal i will offer contracts of the form $\{x, y\}$ where the probability of offering a contract with $x \leq x_0$ is given by $F_i(x_0) = \frac{Ax_0}{V - R_p - x_0}$; $A = \frac{\alpha\epsilon}{3-4\alpha\epsilon}$ and the support of the distribution is $x \in (0, (V - R_p) \frac{3-4\alpha\epsilon}{3(1-\alpha\epsilon)})$, $y \in \mathbb{R}^+$. Principal j ($\neq i$) will offer contracts of the form $\{a, b\}$ where the probability of offering a contract with $a \leq a_0$ is given by $F_j(a_0) = \frac{Ba_0}{V - a_0}$; $B = \frac{R_p}{V} + A$ and the support of this distribution is $a \in [0, (V - R_p) \frac{3-4\alpha\epsilon}{3(1-\alpha\epsilon)}]$, $b \in \mathbb{R}^+$ with a mass point at $a = 0$ where the mass at zero is $\frac{R_p}{V}$.*

Proof. It is clear that the principal who signs the worker in period one has a strong advantage in period 2. This is because that principal has already invested R_p in period 2. So, while principal j can offer a maximum wage of $V - R_p$ in period 2 for a G worker, principal i can offer a higher wage. This is why the payoff for the principals according to the strategy above is (as $\epsilon \rightarrow 0$) R_p for principal i , 0 for principal j and $V - R_p$ for the good type worker. The proof is similar to the proof in claim 13 \square

Claim 11. *Suppose the worker did not form his own firm in period one. Suppose the worker accepted a contract with principal i in period 1. If the worker is of type B , then in period 2, principal i will offer the contract $\{0, 0\}$ and the other principal will offer no contract.*

Proof. Follows from $R_p > \lambda_b V$. □

Corollary 15. *If the type B worker accepted a contract with principal i in period 1, then in any PBE, the expected payoffs for the B worker is 0 in period 2, the expected payoffs for principal i is approximately³⁵ $\lambda_b V$ and the expected payoff for principal j ($j \neq i$) is approximately 0.*

Claim 12. *Suppose the worker did not form his own firm in period one. Suppose the worker played N in period 1. If the worker is of type B, then in period 2, both principals will offer no contract.*

Proof. Follows from $R_p > \lambda_b V$. □

Claim 13. *Suppose the worker did not form his own firm in period one. Suppose the worker played N in period 1. If the worker is of type G, then in period 2, the two principals will play a symmetric mixed strategy where they will offer contracts paying less than $\{x, 0\}$ with probability $\frac{Ax}{V-R_p-x}$ where $A = \frac{\alpha\epsilon}{3-4\alpha\epsilon}$ and support of the distribution is $x \in [0, \frac{(V-R_p)(3-4\alpha\epsilon)}{3(1-\alpha\epsilon)}]$.*

Proof. It is easy to show that there is no pure strategy equilibrium for the game between the two principals. I will confine my investigation into symmetric mixed strategy equilibrium into contracts of the form $\{y, 0\}$. Since the G type worker never fails, any contract which has a positive second contract is equivalent to a contract with zero wages for failure. Suppose principal 1 chooses the contract $\{x, 0\}$ and principal 2 chooses the contract $\{y, 0\}$. Then utility function for principal 1 is given by:

$$\begin{aligned} u(x, y) &= (1 - \alpha\epsilon)(V - R_p - x) ; x > y \\ &= \frac{1 - \alpha\epsilon}{2}(V - R_p - x) ; x = y \\ &= \frac{\alpha\epsilon}{3}(V - R_p - x) ; x < y \end{aligned}$$

If both principals are mixing according to the same distribution F then expected utility from $\{x, 0\}$:

$$F(x)(1 - \alpha\epsilon)(V - R_p - x) + (1 - F(x))\frac{\alpha\epsilon}{3}(V - R_p - x)$$

The first order condition to the above equation gives us a differential equation the solution to which (using boundary condition $F(0) = 0$) reveals $F(x) = \frac{Ax}{V-R_p-x}$ where $A = \frac{\alpha\epsilon}{3-4\alpha\epsilon}$ □

Thus payoffs to the players in period 2 are given by the following matrices. The first coordinate is the payoff to principal 1, second is payoff to principal 2, third is worker. The payoffs are limits of when $\epsilon \rightarrow 0$.

A.3.2 Period 1

Suppose the worker is of type G .

³⁵ as $\epsilon \rightarrow 0$

Table 9: Payoffs in Period 2

Table 10: Payoff if principal 1's contract accepted in Period 1

$(P\ 1, P\ 2, G)$	$(P\ 1, P\ 2, B)$
$(R_p, 0, V - R_p)$	$(\lambda_b V, 0, 0)$

Table 11: Payoff if Spinoff in Period 1

$(P\ 1, P\ 2, G)$	$(P\ 1, P\ 2, B)$
$(0, 0, V)$	$(0, 0, \lambda_b V)$

Table 12: Payoff if N in Period 1

$(P\ 1, P\ 2, G)$	$(P\ 1, P\ 2, B)$
$(0, 0, V - R_p)$	$(0, 0, 0)$

Suppose the two principals offer the contracts $\{x, 0\}, \{y, 0\}$ in period 1. Then payoff to G worker from his action choice accept $\{x, 0\}$ is:

$$\begin{aligned} \{x, 0\} \rightarrow & (1 - \alpha\epsilon)(x + \delta[(1 - \alpha\epsilon)E(\max\{F1, F2\}) + \frac{\alpha\epsilon}{3}E(\min\{F1, F2\}) + \frac{\alpha\epsilon}{3}(V - R_w) + \frac{\alpha\epsilon}{3}.0]) \\ & + \frac{\alpha\epsilon}{3}(y + \delta[(1 - \alpha\epsilon)E(\max\{F1, F2\}) + \frac{\alpha\epsilon}{3}E(\min\{F1, F2\}) + \frac{\alpha\epsilon}{3}(V - R_w) + \frac{\alpha\epsilon}{3}.0]) \\ & + \frac{\alpha\epsilon}{3}(V - R_w + \delta(1 - \alpha\epsilon)V) \\ & + \frac{\alpha\epsilon}{3}(0 + \delta[(1 - \alpha\epsilon)E(\max\{F, F\}) + \frac{\alpha\epsilon}{3}E(\min\{F, F\}) + \frac{\alpha\epsilon}{3}(V - R_w) + \frac{\alpha\epsilon}{3}.0]) \end{aligned}$$

Similarly expected payoff from accepting $\{y, 0\}$. Payoff from leaving and forming own firm is given by:

$$\begin{aligned} L \rightarrow & (1 - \alpha\epsilon)(V - R_w + \delta(1 - \alpha\epsilon)V) \\ & + \frac{\alpha\epsilon}{3}(x + \delta[(1 - \alpha\epsilon)E(\max\{F1, F2\}) + \frac{\alpha\epsilon}{3}E(\min\{F1, F2\}) + \frac{\alpha\epsilon}{3}(V - R_w) + \frac{\alpha\epsilon}{3}.0]) \\ & + \frac{\alpha\epsilon}{3}(y + \delta[(1 - \alpha\epsilon)E(\max\{F1, F2\}) + \frac{\alpha\epsilon}{3}E(\min\{F1, F2\}) + \frac{\alpha\epsilon}{3}(V - R_w) + \frac{\alpha\epsilon}{3}.0]) \\ & + \frac{\alpha\epsilon}{3}(0 + \delta[(1 - \alpha\epsilon)E(\max\{F, F\}) + \frac{\alpha\epsilon}{3}E(\min\{F, F\}) + \frac{\alpha\epsilon}{3}(V - R_w) + \frac{\alpha\epsilon}{3}.0]) \end{aligned}$$

Since we are concerned about the question - when can workers leave to form their own firm, I will assume that $V - R_w + \delta V > \delta(V - R_p)$. Else, no worker will ever choose the action of forming own firm. In any equilibrium, the worker's action choice is clear. He looks at the two contracts on offer and chooses between the best contract and the option L ³⁶. Consider the game between the principals. For principal 1, worker will not choose contract 1 unless :

$$\begin{aligned} \text{payoff from } \{x, 0\} & \geq \text{payoff from } L \\ \Leftrightarrow x & \geq V - R_w + \delta[(1 - \alpha\epsilon)(V - E(\max\{F1, F2\})) - \frac{\alpha\epsilon}{3}E(\min\{F1, F2\}) - \frac{\alpha\epsilon}{3}(V - R_w)] \end{aligned}$$

$$\text{Let } C^* = V - R_w + \delta[(1 - \alpha\epsilon)(V - E(\max\{F1, F2\})) - \frac{\alpha\epsilon}{3}E(\min\{F1, F2\}) - \frac{\alpha\epsilon}{3}(V - R_w)].$$

The utility function of the principal offering contract $\{x, 0\}$ if the other principal is offering

³⁶Option to play N is dominated by option to form own firm.

$\{y, 0\}$ is given by:

$$\begin{aligned}
u(x, y) &= \frac{\alpha\epsilon}{3}[P1_{win}] + (1 - \alpha\epsilon)(w_L) + \frac{\alpha\epsilon}{3}[P2_{win}] + \frac{\alpha\epsilon}{3}(w_N) ; x < y, y < C^* \\
&= \frac{\alpha\epsilon}{3}[P1_{win}] + (1 - \alpha\epsilon)(P2_{win}) + \frac{\alpha\epsilon}{3}[w_L] + \frac{\alpha\epsilon}{3}(w_N) ; x < y, y \geq C^* \\
&= \frac{\alpha\epsilon}{3}[P1_{win}] + (1 - \alpha\epsilon)(w_L) + \frac{\alpha\epsilon}{3}[P2_{win}] + \frac{\alpha\epsilon}{3}(w_N) ; x > y, x < C^* \\
&= \frac{\alpha\epsilon}{3}[w_L] + (1 - \alpha\epsilon)(P1_{win}) + \frac{\alpha\epsilon}{3}[P2_{win}] + \frac{\alpha\epsilon}{3}(w_N) ; x > y, x \geq C^*
\end{aligned}$$

where :

$$\begin{aligned}
P1_{win} &= V - R_p - x + \delta \left[\int f_1(x)((F_2(x) + b)(1 - \alpha\epsilon)(V - x) + (1 - F_2(x) - b)\frac{\alpha\epsilon}{3}(V - x))dx \right] \\
w_L &= 0 + \delta \left[\frac{\alpha\epsilon}{3}(V - R_p) \right] \\
P2_{win} &= 0 + \delta \left[\int f_2(x)((F_1(x))(1 - \alpha\epsilon)(V - R_p - x) + (1 - F_1(x))\frac{\alpha\epsilon}{3}(V - R_p - x))dx + \frac{R_p}{V} \frac{\alpha\epsilon}{3}(V - R_p) \right] \\
w_N &= 0 + \delta \left[\int f(x)((F(x))(1 - \alpha\epsilon)(V - R_p - x) + (1 - F(x))\frac{\alpha\epsilon}{3}(V - R_p - x))dx \right]
\end{aligned}$$

It is clear that when $\epsilon \approx 0$, the best payoff is from $x > y, x \geq C^*$ and that there is no pure strategy equilibrium in the game between the principals.

Proposition 19. *If the worker type is known and the worker is of type G then both principals play the following symmetric mixed strategy in period 1: principals bid a contract which pays less than $\{x, 0\}$ with probability $G(x)$ where:*

$$G(x) = \frac{Ax}{P1_{win} - P2_{win}} ; A = \frac{\alpha\epsilon}{3 - 4\alpha\epsilon} , x \in [C^*, d]$$

where d is such that:

$$\begin{aligned}
&\frac{\alpha\epsilon}{3}[V - R_p - C^* + \delta \left[\int f_1(x)((F_2(x) + b)(1 - \alpha\epsilon)(V - x) + (1 - F_2(x) - b)\frac{\alpha\epsilon}{3}(V - x))dx \right]] + (1 - \alpha\epsilon)(P2_{win}) + \frac{\alpha\epsilon}{3}[w_L] + \frac{\alpha\epsilon}{3}(w_N) \\
&= \\
&\frac{\alpha\epsilon}{3}[w_L] + (1 - \alpha\epsilon)(V - R_p - d + \delta \left[\int f_1(x)((F_2(x) + b)(1 - \alpha\epsilon)(V - x) + (1 - F_2(x) - b)\frac{\alpha\epsilon}{3}(V - x))dx \right]) + \frac{\alpha\epsilon}{3}[P2_{win}] + \frac{\alpha\epsilon}{3}(w_N)
\end{aligned}$$

The G type worker accepts the better of the two realizations of contracts offered from the above distribution.

Proof. Very similar to proof for claim 13. To show that worker will accept contract it is sufficient to show that worker accepts the lowest contract of $\{C^*, 0\}$. This is easy to check. \square

Thus, if the worker type is G, then in the full information case the worker will not form his own firm and the expected payoff to the worker will be approximately $V - R_p + \delta V$. The principals get an expected payoff of zero. These payoffs have been calculated as $\epsilon \rightarrow 0$.

Similarly it can be shown that if the worker type is B, then in the full information case, the worker will not form his own firm. The payoff to the worker will be $\lambda_b V - R_p + \delta \lambda_b V$ and competition between the principals will force their expected payoff to zero.

Thus payoffs in the equilibrium in the full information case can be summarized as follows. First coordinate is payoff for principal 1, second is payoff for principal 2, and third is payoff for worker.

Table 13: Expected Payoffs if Types are Known and Competitive Labour Market

$(Principal\ 1, Principal\ 2, G)$	$(Principal\ 1, Principal\ 2, B)$
$(0, 0, V - R_p + \delta V)$	$(0, 0, \lambda_b V(1 + \delta) - R_p)$

A.4 Competitive Labour Market and private types

A.4.1 Period 2

Once again, I will solve the one period game beginning in period 2 first and then proceed by backwards induction. The relevant state variables which affect decisions in period 2 are:

1. Worker formed own firm in period 1 (f), did not form and accepted contract (nf) or did nothing N .
2. If worker played nf in period one, then which firm's contract did it accept ($P1$ or $P2$).³⁷ $P1$ stands for principal one.
3. Reputation of worker at the end of period 1. Let this be denoted by p_g^2 .

In equilibrium, strategies in period 2, therefore are functions from the state variables augmented history. In particular, the strategy for principal i ($i \in \{1, 2\}$) in period 2 depends only on the value taken by the state variables:

$$s_{p,2}^i : (nf, \{P1, P2\}, p_g^2) \rightarrow \{x \in 2^{\{(s,f); s,f \geq 0\}}; 0 < |x| < 3\}$$

$$s_{p,2}^i : (f/N, p_g^2) \rightarrow \{x \in 2^{\{(s,f); s,f \geq 0\}}; 0 < |x| < 3\}$$

So the strategy for principals in period two depend only upon reputation of worker at the end of period one if the worker had already formed his own firm in period one or played N . If the worker accepted a contract in period one, then the strategy for principal i also depends upon whether the worker had formed a firm with him in period one or the other principal. This is because the principal who formed the firm in period one has already invested R_p and is thus able to offer different contracts from the principal who did not form a firm in period one.

In case the worker did not form his firm in period one, then his decision in period two does not depend upon the particular principal he chose to form the firm with in period one or if he played N . Therefore, strategy for worker in period 2 depends upon the whether he formed his own firm in period one or not, reputation,

³⁷This is relevant because the principal of that firm has already invested R_p in forming the firm which the other principal has not.

actions taken by the principals in period 2 and own type. Let $\{\{s_1^2, f_1^2\}_i, \{s_2^2, f_2^2\}_i\}$ be the contracts offered by principal i in period 2. Let acc_j^i means accept contract j offered by principal i .

$$s_{w,2} : (nf/N, p_g^2, \{\{s_1^2, f_1^2\}_1, \{s_2^2, f_2^2\}_1\}, \{\{s_1^2, f_1^2\}_2, \{s_2^2, f_2^2\}_2\}, T) \rightarrow \{N, acc_1^1, acc_2^1, acc_1^2, acc_2^2, L\} ; T \in \{G, B\}$$

$$s_{w,2} : (f, p_g^2, \{\{s_1^2, f_1^2\}_1, \{s_2^2, f_2^2\}_1\}, \{\{s_1^2, f_1^2\}_2, \{s_2^2, f_2^2\}_2\}, T) \rightarrow \{N, acc_1^1, acc_2^1, acc_1^2, acc_2^2, S\} ; T \in \{G, B\}$$

Strategy for customers in period 2:

$$s_{c,2} : (f/nf/N, p_g^2, \{\{s_1^2, f_1^2\}_1, \{s_2^2, f_2^2\}_1\}, \{\{s_1^2, f_1^2\}_2, \{s_2^2, f_2^2\}_2\}, a_w(2)) \rightarrow \{b ; b \in \mathbb{R}_+\}$$

Case 1 - Worker played f in period 1

If the worker failed, then the only equilibrium is one where the B type worker (for a worker who fails must be B type) plays S and both principals offer no contracts. This follows from the assumption that $R_p > \lambda_b V$.

Suppose the worker succeeded in period 1. We will be interested in the equilibria where p_g^2 is high. This is because we are interested in separating equilibria in which G worker leaves to form firm. In such equilibria p_g^2 will be close to 1.

Assume $p_g^2 \approx 1$.

Pooling on S Equilibrium

Consider the following strategies in period 2:

For Principals (both) :

$$s_p(f, p_g^2 \approx 1) = \{\{0, 0\}\}$$

For Worker :

$$s_w(f, p_g^2 \approx 1, \{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_1 ; s_1^1 \geq s_2^1 \text{ and } s_1^1 > p_g^2 V + (1 - p_g^2) \lambda_b V$$

$$s_w(f, p_g^2 \approx 1, \{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_2 ; s_2^1 \geq s_1^1 \text{ and } s_2^1 > p_g^2 V + (1 - p_g^2) \lambda_b V$$

$$s_w(f, p_g^2 \approx 1, \{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = S ; Else.$$

$$s_w(f, p_g^2 \approx 1, \{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, B) = acc_1 ; \lambda_b s_1^1 + (1 - \lambda_b) f_1^1 > \lambda_b s_2^1 + (1 - \lambda_b) f_2^1 \text{ and } \lambda_b s_1^1 + (1 - \lambda_b) f_1^1 > p_g^2 V + (1 - p_g^2) \lambda_b V$$

$$s_w(f, p_g^2 \approx 1, \{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, B) = acc_2 ; \lambda_b s_2^1 + (1 - \lambda_b) f_2^1 > \lambda_b s_1^1 + (1 - \lambda_b) f_1^1 \text{ and } \lambda_b s_2^1 + (1 - \lambda_b) f_2^1 > p_g^2 V + (1 - p_g^2) \lambda_b V$$

$$s_w(f, p_g^2 \approx 1, \{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, B) = S ; Else.$$

Proof. Essentially, if p_g^2 is high enough then the payoff from pooling on S in period 2 is above $V - R_p$. Since neither principal can offer more, clearly there will be a pooling on S equilibrium. \square

Payoffs as $\epsilon \rightarrow 0$:

$$P1 \rightarrow 0$$

$$P2 \rightarrow 0$$

$$G \rightarrow p_g^2 V + (1 - p_g^2) \lambda_b V$$

$$B \rightarrow p_g^2 V + (1 - p_g^2) \lambda_b V$$

If assumption A holds, then above is the only equilibrium after worker forms his own firm. If assumption A does not hold then the above remains an equilibrium but the following equilibria are also possible. I describe the outcome in these equilibria next. The proofs are similar to the proof of claim 13.

Other Equilibria I mention the play in equilibrium and the equilibrium payoff only. The proof is deliberately left out as it is a similar (to the work above) exercise in constructing the right strategies to justify the play below.

1. There is an equilibrium where both principals offer the following contract

$$\left\{ \frac{p_g^2(1 - \alpha\epsilon)V + \frac{\alpha\epsilon}{3}\lambda_b V(1 - p_g^2)}{p_g^2(1 - \alpha\epsilon) + \frac{\alpha\epsilon}{3}\lambda_b(1 - p_g^2)} - R_p - \frac{\alpha\epsilon}{3} \frac{p_g^2 V + (1 - p_g^2)\lambda_b V - R_p}{p_g^2 \frac{1 - \alpha\epsilon}{2} + \frac{\alpha\epsilon}{3}\lambda_b(1 - p_g^2)}, 0 \right\} \quad (6)$$

Worker type G accepts one of the contracts at random and worker type B plays S .

Payoffs as $\epsilon \rightarrow 0$:

$$P1 \rightarrow 0$$

$$P2 \rightarrow 0$$

$$G \rightarrow V - R_p$$

$$B \rightarrow \lambda_b V$$

2. There is an equilibrium where both principals offer the following contract

$$\left\{ \frac{p_g^2(1 - \alpha\epsilon)V + \frac{\alpha\epsilon}{3}\lambda_b V(1 - p_g^2)}{p_g^2(1 - \alpha\epsilon) + \frac{\alpha\epsilon}{3}\lambda_b(1 - p_g^2)} - R_p - \frac{\alpha\epsilon}{3} \frac{p_g^2 V + (1 - p_g^2)\lambda_b V - R_p}{p_g^2 \frac{1 - \alpha\epsilon}{2} + \frac{\alpha\epsilon}{3}\lambda_b(1 - p_g^2)}, 0 \right\} \quad (7)$$

Worker type G , randomizes between accepting a contract and playing S . Worker B plays S .

Payoffs as $\epsilon \rightarrow 0$:

$$P1 \rightarrow 0$$

$$P2 \rightarrow 0$$

$$G \rightarrow V - R_p$$

$$B \rightarrow V - R_p$$

3. There is an equilibrium where both principals offer the following contract

$$\{\lambda_b V, 0\} \quad (8)$$

Worker type G , accepts contract. Worker B plays S . Worker type G does not accept any other IR contract.

Payoffs as $\epsilon \rightarrow 0$:

$$P1 \rightarrow \frac{1}{2}p_g^2(V - \lambda_b V - R_p)$$

$$P2 \rightarrow \frac{1}{2}p_g^2(V - \lambda_b V - R_p)$$

$$G \rightarrow \lambda_b V$$

$$B \rightarrow \lambda_b V$$

In general, every time I show the existence of a separating equilibrium, I will make use of the pooling on S equilibrium in period 2.

Case 2 - Worker played nf in period 1 and accepted principal 1's contract

In this case, the worker strategies in equilibrium will always be - accept the highest paying contract. This is because leaving and forming own firm is no longer individually rational.

$$\text{Subcase 1 - } p_g^2 < \frac{R_p - \lambda_b V}{V - \lambda_b V}$$

In this case, we have that $p_g^2 V + (1 - p_g^2)\lambda_b V - R_p < 0$.

Equilibrium 1

Principal 1 offers a zero wage contract. Principal 2 offers no contract. Worker accepts principal 1's contract. The fact that these strategies constitute an equilibrium follows from $p_g^2 V + (1 - p_g^2)\lambda_b V - R_p < 0$ and equal probability of mistake for the two types of workers.

Payoffs as $\epsilon \rightarrow 0$:

$$P1 = p_g^2 V + (1 - p_g^2)\lambda_b V$$

$$P2 = 0$$

$$G = 0$$

$$B = 0$$

Proposition 20. *No other equilibrium outcomes in this case.*

Proof. It is not incentive compatible for the the worker to form his own firm. Therefore the only other possible equilibrium outcome occurs if principal 2 offers a contract and atleast one type of worker accepts this contract. Since $p_g^2 V + (1 - p_g^2)\lambda_b V - R_p < 0$, it will only be incentive compatible for principal 2 to offer

a contract if the G, B worker's strategies are such that the G worker is more likely to accept the contract than the B worker. In this case, it is always possible and incentive compatible for principal 1 to outbid principal 2. This is because principal 2 has to put in the extra cost of firm formation at period 2 whereas principal 1 is not burdened with this cost any more (since he already made the fixed investment at period 1). Thus, principal 1 will always outbid principal 2. The following highlights the four cases in which the good worker may accept principal 2's contract with higher probability. I rule them out.

Case 1 - G worker accepts contract with principal 2 with probability 1 and B worker accepts a contract with principal 1

Suppose the contract accepted by the G worker is $\{x, y\}$. Principal 1 will offer a contract to make the B worker just indifferent between the two contracts. This implies that the expected payoff for principal 1 is $(1 - p_g^2)(\lambda_b V - \lambda_b x - (1 - \lambda_b)y)$. We can easily check that if principal 1 deviates to $\{x + \mu, 0\}$ where μ is an extremely small positive real number, then principal gets a higher payoff. So this cannot be an equilibrium.

Case 2 - G worker accepts contract with principal 2 with probability 1 and B worker accepts a contract with principal 1 with positive probability

Same argument as above.

Case 3 - G worker accepts contract with principal 2 with positive probability and B worker accepts a contract with principal 2 with a lower positive probability

This can only happen if both principals offer the same contract. Principal 1 can always deviate by offering a little higher reward for success.

Case 4 - G worker accepts contract with principal 2 with positive probability and B worker accepts a contract with principal 1 probability 1

Suppose the contract accepted by the G worker is $\{x, y\}$. Principal 1 will have to offer x for success since the good worker is indifferent. Since the bad type is choosing principal 1's contract principal one must be offering a contract $\{x, y'\}$ where $y' \geq y$. Moreover, for this to be an equilibrium $x = V - R_p$. This is to prevent principal 2 from deviating and offering a little more for success and getting the good worker for sure. Also, in equilibrium, there will be no need for principal 1 to pay anything strictly above y for failure. So $y = y'$. We can easily check that if principal 1 deviates to $\{x + \mu, y\}$ where μ is an extremely small positive real number, then principal 1 gets a higher payoff. So this cannot be an equilibrium.

□

Subcase 2 - $p_g^2 > \frac{R_p - \lambda_b V}{V - \lambda_b V}$

above implies that $p_g^2 V + (1 - p_g^2) \lambda_b V > R_p$. Suppose $\epsilon \rightarrow 0$.

Equilibrium 1 -

P1, P2 both don't offer any reward for failure and they randomize among contracts over payment for success. P1 offers contracts of the form $\{x, 0\}$ where $x \in (0, a]$ and P2 offers contracts of the form $\{y, 0\}$ where $y \in [0, a)$. $a = \frac{(1 - \frac{4\alpha\epsilon}{3})(V(p^2 + (1 - p_g^2)\lambda_b) - R_p)}{(1 - \alpha\epsilon)(p^2 + (1 - p_g^2)\lambda_b)}$. P2 has a mass point of $\frac{R_p}{p^2 V + (1 - p_g^2)\lambda_b V}$ on

the contract $\{0, 0\}$. P2 mixes with the distribution F_2 where probability that P2 offers less than y for success is given by $F_2(y) = \frac{A_2 y}{(V-y)(p^2+(1-p_g^2)\lambda_b)(1-\frac{4\alpha\epsilon}{3})}$. $A_2 = \frac{R_p}{V}(1 - \frac{4\alpha\epsilon}{3}) + \frac{\alpha\epsilon}{3}(p_g^2 + (1 - p_g^2)\lambda_b)$. P1 mixes on with the distribution F_1 where probability that P1 offers less than x for success is given by $F_1(x) = \frac{A_1 x}{(1-\frac{4\alpha\epsilon}{3})((V-y)(p^2+(1-p_g^2)\lambda_b)-R_p)}$. $A_1 = \frac{\alpha\epsilon}{3}(p^2 + (1 - p_g^2)\lambda_b)$. G, B type worker accept whichever contract gives them highest payoff.

Proof. Similar to claim 13. □

Payoffs as $\epsilon \rightarrow 0$

$$\begin{aligned} P1 &= R_p \\ P2 &= 0 \\ G &= V - \frac{R_p}{p_g^2 + (1 - p_g^2)\lambda_b} \\ B &= \lambda_b(V - \frac{R_p}{p_g^2 + (1 - p_g^2)\lambda_b}) \end{aligned}$$

Equilibrium 2 - P1 offers contracts of the form $\{x, f(x)\}$ where $x \in (0, a]$ and P2 offers contracts of the form $\{y, 0\}$ where $y \in [0, a)$. $f(x)$ is such that $\lambda_b x + (1 - \lambda_b)f(x) = \lambda_b a$. So P1 always gets B worker. $a = V - R_p$ as $\epsilon \rightarrow 0$. P2 has a mass point of $\frac{R_p}{V}$ on the contract $\{0, 0\}$. P1, P2 mix on payoff fo success with distributions G_1, G_2 .

Proof. Similar to claim 13. □

Payoffs as $\epsilon \rightarrow 0$

$$\begin{aligned} P1 &= R_p(p_g^2 + (1 - p_g^2)\lambda_b) \\ P2 &= 0 \\ G &= V - R_p \\ B &= \lambda_b(V - R_p) \end{aligned}$$

Lemma 3. Let history be nf, acc_1, p_g^2 . As $\epsilon \rightarrow 0$, P2's payoff in any equilibrium in period 2 goes to zero.

Proof. Suppose P2 gets positive payoff in some equilibrium in period 2. It is easy to show that any equilibrium must be in mixed strategies. We can show that P2 will never pay for failure in any equilibrium. Now let the range of payoffs for success be (a, b) . If P2 gets positive payoff in equilibrium, then P2 must get positive payoff at all points in its support. Payoff of P2 from $\{a, o\}$ is positive implies P1 must have a mass point at $\{a, f\}$ for some f . Let f' be the highest value such that P1 has a non atomic mass on $\{a, f'\}$.

Case 1 - $\lambda_b a + (1 - \lambda_b)f \geq \lambda_b b$

P2 cannot have a mass point at $\{b, 0\}$ else P1 could pay a little more and increase payoff. If P2 does not have

a mass point at $\{b, 0\}$ then $\lambda_b a + (1 - \lambda_b)f \geq \lambda_b b \Rightarrow P1$ always gets B worker if he offers $\{a, f\}$.

Also, $P2$ cannot have a mass point at $\{a, 0\}$ else $P1$ can increase payoff by shifting mass point to the right. $P2$ does not have mass point at $a, 0$ implies that $P1$ always loses G type when he offers $\{a, f\}$.

Now compare payoffs for $P1$ at $\{b, 0\}$ and $\{a, f\}$. They must be the same but they are not. Contradiction.

Case 2 - $\lambda_b a + (1 - \lambda_b)f < \lambda_b b$

Then there exists $x \in (a, b)$ such that $\lambda_b a + (1 - \lambda_b)f < \lambda_b x$. $P2$ must be indifferent between $\{x - \nu, 0\}$ and $\{x + \nu, 0\}$ for all ν . But if $P1$ has a mass point at $\{a, f\}$, then this is not possible. Contradiction.

Therefore $P1$ cannot have a mass point at $\{a, f\}$. Contradiction.

Thus $P2$ gets zero payoff in all equilibria in period 2.

□

Claim 14. *In any equilibrium in period 2, G worker cannot get more than $V - R_p$ and G worker cannot get less than $V - \frac{R_p}{p_g^2 + (1 - p_g^2)\lambda_b}$.*

Proof. $P2$ will never bid above $V - R_p$ for success in any equilibrium since it is not IR for him. This implies that $P1$ will also not offer a contract which offers more than $V - R_p$ for success in period 2. Therefore payoff for G worker in period 2 is bounded above by $V - R_p$.

Suppose there was an equilibrium where G worker was getting less than $V - \frac{R_p}{p_g^2 + (1 - p_g^2)\lambda_b}$ in equilibrium. Then we can show that $P2$ can offer a little more than the maximum $P1$ is offering for success and get a positive payoff. By the lemma above, this improves $P2$'s payoffs. Thus, this deviation is profitable. Therefore, original cannot be an equilibrium. □

Corollary 16. *Thus the payoffs for $P1$ in any equilibrium in period 2 is between $R_p(p_g^2 + (1 - p_g^2)\lambda_b)$ and R_p*

Proof. If G gets $V - \frac{R_p}{p_g^2 + (1 - p_g^2)\lambda_b}$ and B gets $\lambda_b(V - \frac{R_p}{p_g^2 + (1 - p_g^2)\lambda_b})$ and $P2$ has to get zero, then payoff for $P1$ = $(p_g^2 + (1 - p_g^2)\lambda_b)V - p_g^2(V - \frac{R_p}{p_g^2 + (1 - p_g^2)\lambda_b}) - (1 - p_g^2)\lambda_b(V - \frac{R_p}{p_g^2 + (1 - p_g^2)\lambda_b}) = R_p$

Similarly the other one. □

Note that the above payoffs are achieved by the equilibria highlighted above.

Case 3 - Worker played N in period 1

As before in the section with one principal, the two principals will offer contracts only iff p_g is such that $p_g V + (1 - p_g)\lambda_b V \geq R_p$. The worker will accept the best paying contract in period 2. The payoff in an

equilibrium where both principals do offer contracts:

$$\begin{aligned}
P1 &= 0 \\
P2 &= 0 \\
G &= V - \frac{R_p}{p_g + (1 - p_g)\lambda_b} \\
B &= \lambda_b(V - \frac{R_p}{p_g + (1 - p_g)\lambda_b})
\end{aligned}$$

A.4.2 Period 1

Show conditions needed for separation if:

1. $p_g < \frac{\lambda_b}{1-\lambda_b} \frac{R_p - \lambda_b V}{V - R_p + \lambda_b V}$.
2. $p_g \geq \frac{\lambda_b}{1-\lambda_b} \frac{R_p - \lambda_b V}{V - R_p + \lambda_b V}$.

Case 1 - $p_g < \frac{\lambda_b}{1-\lambda_b} \frac{R_p - \lambda_b V}{V - R_p + \lambda_b V}$

In this case, if the worker accepts a contract and succeeds in period, the period 2 reputation will be p_g^2 where $p_g^2 < \frac{R_p - \lambda_b V}{V - \lambda_b V}$. This implies that the workers period 2 payoff will be 0.

Proposition 21. Let $p_g < \frac{\lambda_b}{1-\lambda_b} \frac{R_p - \lambda_b V}{V - R_p + \lambda_b V}$. Let $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$. If $p_g \in (\frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}, \frac{R_p - \lambda_b R_w + \lambda_b V(1+\delta) - R_p}{R_w - \lambda_b R_w + \lambda_b V(1+\delta) - R_p})$. Then no separating equilibrium exists.

Proof. Note that such p_g exists if $R_p(V - R_p) < \lambda_b V(R_w - R_p)$. Also, $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$ and $p_g < \frac{R_p - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p)}{R_w - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p)}$ imply that there exist a separating equilibrium in the principal worker model by proposition 2.

Suppose there exists an equilibrium where G type worker forms firm in period one and B type worker accepts the best contract in period 1. Then, Competition and symmetry of principals will drive principal payoffs to zero.

Given that the equilibrium being played in period 2 if the worker accepts contract is one where the principal winning the contract offers zero wage contract which is accepted by worker and other principal offers no contract (by proposition 20), we can show that if a principal deviates and offers the following contract $\{V - R_w + \delta V, 0\}$ in period 1, then a worker of any type will accept the contract and the principal will make positive profits if $p_g > \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w}$. This means that this deviation is profitable. This implies that there cannot be an equilibrium with separation. Contradiction. \square

The next proposition highlights that a separating equilibrium is possible if p_g is lower.

Proposition 22. Let $p_g < \frac{\lambda_b}{1-\lambda_b} \frac{R_p - \lambda_b V}{V - R_p + \lambda_b V}$. Let $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$ and $p_g < \frac{R_p - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p)}{R_w - \lambda_b R_w + (\lambda_b V(1+\delta) - R_p)}$. If $p_g \in (0, \frac{R_p - \lambda_b R_w}{R_w - \lambda_b R_w})$ and $R_p > \lambda_b R_w$, then there exists a separating equilibrium where G worker leaves to form firm in period 1 and the period 2 play is a pooling on S equilibrium.

Proof. Let $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$ and $p_g < \frac{R_p-\lambda_b R_w+(\lambda_b V(1+\delta)-R_p)}{R_w-\lambda_b R_w+(\lambda_b V(1+\delta)-R_p)}$ guarantees that there exists a separating equilibrium in the principal-worker environment (proposition 2). A similar proof can be given to show that there exist a separating equilibrium under the above conditions in this environment. The play in period 2 after separating in period 1 is the pooling on S equilibrium. Note that we need an additional condition that $R_p > \lambda_b R_w$ to make sure that a low enough p_g exists. \square

Proof for Proposition 5

Following separation, let period 2 equilibrium be the pooling on S equilibrium. Then the proof follows from above.

Case 2 - $p_g \geq \frac{\lambda_b}{1-\lambda_b} \frac{R_p-\lambda_b V}{V-R_p+\lambda_b V}$

In this case, the reputation after success today is going to be such that : $p_g^2 V + (1 - p_g^2) \lambda_b V - R_p > 0$.

The payoff in period 2 for the principal who signs the worker in period one will be $\lambda_b V$ if the worker fails in period 1. If the worker succeeds in period one, then by corollary 16, the payoff for the principal cannot be greater than R_p and less than $R_p(p_g^2 + (1 - p_g^2)\lambda_b)$ where $p_g^2 = \frac{p_g}{p_g+(1-p_g)\lambda_b V}$.

Proposition 23. Let $p_g \geq \frac{\lambda_b}{1-\lambda_b} \frac{R_p-\lambda_b V}{V-R_p+\lambda_b V}$. Let $V < \frac{R_w}{1+\delta\lambda_b(2-\lambda_b)}$. If $p_g \in (\frac{R_p-\lambda_b R_w}{R_w-\lambda_b R_w}, \frac{R_p-\lambda_b R_w+\lambda_b V(1+\delta)-R_p}{R_w-\lambda_b R_w+\lambda_b V(1+\delta)-R_p})$. Then no separating equilibrium exists.

Proof. Note that such p_g exists if $(1 - \lambda_b)[\lambda_b V(R_w - R_p) - R_p(V - R_p)] \leq (\lambda_b V(1 + \delta) - R_p)(V - R_p)$.

Suppose not. Suppose there exists an equilibrium where G type worker forms firm in period one and B type worker accepts the best contract in period 1. Competition and symmetry of principals will drive principal payoffs to zero.

Given any equilibrium that will be played in period 2, we can show that if a principal deviates and offers a contract which would pay the G worker $V - R_w + \delta V$ in total (over two periods), then that principal will make positive profits if $p_g > \frac{R_p-\lambda_b R_w}{R_w-\lambda_b R_w}$. This means that this deviation is profitable. This implies that there cannot be an equilibrium with separation. Contradiction. \square

Proof for proposition 7

Proof follows from proposition 21,23.

Thus, there are conditions under which separating is possible in the one principal one worker model but not in the two principal one worker model.

A.5 Moral Hazard with Private Types

Proof of Proposition 9

Consider the following strategies in period 2:

For Principal :

$$s_{p,2}(nf, p_g^2) = \left\{ \frac{1}{1-\beta}, 0 \right\} ; p_g^2 < \frac{V - \frac{1}{(1-\beta)^2} + \frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}$$

$$s_{p,2}(nf, p_g^2) = \{0, 0\} ; p_g^2 \geq \frac{V - \frac{1}{(1-\beta)^2} + \frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}$$

$$s_{p,2}(N, p_g^2) = \phi$$

$$s_{p,2}(f, p_g^2) = \phi$$

For Worker :

If worker had formed firm in period 1 only one case is relevant

$$s_{w,2}(f, p_g^2, \phi, G/B) = S, e = 0$$

If worker had not formed firm in period 1 :

$$s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = \text{Choose (contract, effort) maximizing payoff}$$

$$s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = \text{Choose (contract, effort) maximizing payoff}$$

$$s_{w,2}(nf/N, p_g^2, \phi, G/B) = N$$

Consider the following strategies in period 1:

For Principal :

$$s_p(\phi) = \left\{ \left\{ \frac{1}{1-\beta}, 0 \right\} \right\}$$

For Worker :

WLOG let contract 1 be better than contract 2 for G worker

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_1, e = 1 ; -1 + s_1^1 + \delta \frac{1}{1-\beta} = \max\{-1 + s_1^1 + \delta \frac{1}{1-\beta}, \beta s_1^1 + \delta \frac{1}{1-\beta}\} \text{ and}$$

$$-1 + s_1^1 + \delta \frac{1}{1-\beta} \geq -1 + V - R_w + \delta \beta V$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_1, e = 0 ; -1 + s_1^1 + \delta \frac{1}{1-\beta} \neq \max\{-1 + s_1^1 + \delta \frac{1}{1-\beta}, \beta s_1^1 + \delta \frac{1}{1-\beta}\} \text{ and}$$

$$\beta s_1^1 + \delta \frac{1}{1-\beta} \geq -1 + V - R_w + \delta \beta V$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = L, e = 1 ; \text{ else.}$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, B) = \text{Choose (contract, effort) maximizing payoff}$$

$$s_w(\phi, B) = N$$

$$s_w(\phi, G) = L, e = 1$$

Let us prove by backward induction that the above strategies constitute a perfect bayesian equilibrium under the conditions imposed in the theorem.

In period 2, consider the incentives of the worker. If the history is such that the worker has formed a firm of his own in period 1, then the worker is not offered any contracts in period 2 (any response of the worker to any contract offered in period 2, following such a history would be a best response because it would be off

the equilibrium play). In this case, it is clear that the best action for the worker is to play S and then exert zero effort. Zero effort is exerted since all payments are received by the worker before his effort choice and the game ends in period 2. If the history is one in which the worker had played N in period 1, then it is not individually rational for either the worker or the principal to form a firm in period 1, therefore the only on the equilibrium path response is to play N again. In a history in which the worker had accepted a contract in period 1, it is not individually rational for the worker to think about playing L , therefore the best action choice is to choose the most profitable (contract, effort) tuple.

Now let us consider the incentives of the principal in period 2. Suppose the history is one in which the worker did not accept the principal's contract in period 1. It is not IR for the principal to form the firm in period 2. Therefore the principal's optimal action is to offer no contract in period 2. If the worker had accepted a contract with the principal in period 1, then the principal has two choices - he can either offer a contract $\{\frac{1}{1-\beta}, 0\}$ which is the minimum amount required for a worker of either type to put in effort ³⁸ or he can offer a contract which pays nothing and get zero effort from the worker. Note that any contract which offers something in the middle is not optimal. We need the following conditions to make it IR and IC for the principal to offer the contract $\{\frac{1}{1-\beta}, 0\}$ and extract maximum effort:

$$V \geq \frac{1}{1-\beta} + \frac{\beta\lambda_b}{(1-\beta)^2} \quad (9)$$

$$p_g^2 \leq \frac{V - \frac{1}{(1-\beta)^2} - \frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}} \quad (10)$$

It is clear that if R_w is high enough and $V > \frac{\frac{1+\delta\beta}{1-\beta} + R_w}{1+\delta\beta}$, then the first condition is satisfied. In any separating equilibrium where the G worker forms his own firm in period 1, $p_g^2 \approx 0$. Thus the second condition is satisfied too.

Therefore, under a separating equilibrium, the payoffs in period 2 are as follows. The principal expects to get a payoff of $\beta\lambda_b(V - \frac{1}{1-\beta})$ in period 2 if the worker accepts the contract in period 1. If the worker does not accept contract in period one, the principal expects zero payoff in period 2. A G type worker expects to get a payoff of βV if he had played L in period 1 and a payoff of $-1 + \frac{1}{1-\beta}$ if he had accepted a contract in period 1. A B type worker expects to get a payoff of βV if he had played L in period 1 and a payoff of $-1 + (\beta\lambda_b + (1-\beta))\frac{1}{1-\beta}$ if he had accepted a contract in period 1.

Let us now consider the incentives for the worker in period 1. Given the payoffs in period 2, G type worker cannot do better. In any separating equilibrium the G worker must put in maximum effort in period 1 after playing L . We need the following condition to ensure that this is incentive compatible.

$$V > \frac{1}{\delta\beta(1-\beta)(1-\lambda_b)}$$

³⁸This is regardless of worker type i.e. even if the worker type was known, for any type of worker, this contract would be an optimal contract to induce maximum effort.

It is IR for the G type worker to choose to play L if :

$$V > \frac{1 + R_w}{1 + \delta\beta}$$

It is clear that if R_w is high enough and $V > \frac{\frac{1+\delta\beta}{1-\beta} + R_w}{1+\delta\beta}$, then these conditions are satisfied. It is not IR for the B type worker to leave if the following condition holds:

$$V < \frac{1 + R_w}{1 + \delta\beta(1 - \beta(1 - \lambda_b)^2)}$$

This holds, therefore, his strategy to choose the best (contract,effort) combination is optimal. So till now we have that if R_w is high enough and $\frac{1+R_w}{1+\delta\beta} < V < \frac{1+R_w}{1+\delta\beta(1-\beta(1-\lambda_b)^2)}$, then the following hold:

1. IR for G to play L .
2. Not IR for B to play L .
3. If G plays L , then first period effort is 1.
4. B will accept $\{\frac{1}{1-\beta}, 0\}$ and put in effort 1 in both periods.
5. G worker will play $e = 0$ in period 2 after L in period 1.

Consider incentives of G worker in period 1 after the principal offers a contract $\{s, 0\}$.

Payoff from accept and $e = 1 = -1 + s + \delta(-1 + \frac{1}{1-\beta})$

Payoff from accept and $e = 0 = \beta s + \delta(-1 + \frac{1}{1-\beta})$

Payoff from $L = -1 + V - R_w + \delta\beta V$

Therefore:

G worker will accept and play $e = 1$ if

$$\begin{aligned} s &\geq \frac{1}{1-\beta} \\ s &\geq V - R_w + \delta(\beta V - \frac{\beta}{1-\beta}) \end{aligned}$$

G worker will accept and play $e = 0$ if

$$\begin{aligned} s &< \frac{1}{1-\beta} \\ s &\geq \frac{1}{\beta}[-1 + V - R_w + \delta(\beta V - \frac{\beta}{1-\beta})] \end{aligned}$$

G worker will play L if

$$s < \frac{1}{\beta}[-1 + V - R_w + \delta(\beta V - \frac{\beta}{1-\beta})]$$

$$s < V - R_w + \delta(\beta V - \frac{\beta}{1-\beta})$$

Now

$$V > \frac{\frac{1+\delta\beta}{1-\beta} + R_w}{1 + \delta\beta}$$

\Leftrightarrow

$$V - R_w + \delta(\beta V - \frac{\beta}{1-\beta}) > \frac{1}{1-\beta}$$

and

$$V - R_w + \delta(\beta V - \frac{\beta}{1-\beta}) < \frac{1}{\beta}[-1 + V - R_w + \delta(\beta V - \frac{\beta}{1-\beta})]$$

If R_w is high enough then we have $\frac{\frac{1+\delta\beta}{1-\beta} + R_w}{1 + \delta\beta} < \frac{1 + R_w}{1 + \delta\beta(1-\beta(1-\lambda_b)^2)}$.

So we have that if R_w is high enough and $\frac{\frac{1+\delta\beta}{1-\beta} + R_w}{1 + \delta\beta} < V < \frac{1 + R_w}{1 + \delta\beta(1-\beta(1-\lambda_b)^2)}$, then the following hold:

1. IR for G to play L .
2. Not IR for B to play L .
3. If G plays L , then first period effort is 1.
4. B will accept $\{\frac{1}{1-\beta}, 0\}$ and put in effort 1 in both periods.
5. G worker will play $e = 0$ in period 2 after L in period 1.
6. $V - R_w + \delta(\beta V - \frac{\beta}{1-\beta}) > \frac{1}{1-\beta}$. So principal will have to offer a strictly better contract to attract G worker.

The principal now has three choices which may be optimal. One, offer a zero wage contract which the B type worker will accept and put zero effort. Two, offer the contract $\{\frac{1}{1-\beta}, 0\}$ which the the worker will accept if he is B type and put in full effort. Three, offer the contract $\{V - R_w + \delta(\beta V - \frac{\beta}{1-\beta}), 0\}$ which both type workers will accept and put in full effort³⁹. The principal offers the contract $\{\frac{1}{1-\beta}, 0\}$. This is less than the minimum needed to attract the G type worker. Clearly, the principal will be willing to offer no more than this if p_g is low enough i.e. if the principal believes that the customers are convinced that the worker must be B type if he accepts a contract then it would not be optimal for the principal a high paying contract to attract both types of workers. The principal does not offer a lower pay contract because V is high enough for him

³⁹ $V - R_w + \delta(\beta V - \frac{\beta}{1-\beta}) < \frac{1}{\beta}[-1 + V - R_w + \delta(\beta V - \frac{\beta}{1-\beta})]$ ensures that that offering a contract which would make the G worker accept and put in zero effort is not optimal for the principal.

to want to induce high effort. Given this contract, the G worker's optimal choice is to form his own firm in period 1.

Proof for Proposition 10

If p_g is low and a worker of any type is expected to accept the contract, then, if a worker deviates and plays L it would be viewed by the customers as a mistake. Therefore, since mistake probabilities are the same, the worker will have a reputation of p_g . If p_g is low, it would not be individually rational to play L (this follows from $V < \frac{1+R_w}{1+\delta\beta(1-\beta(1-\lambda_b)^2)}$). Therefore, the strategy of both type workers must be to accept the best contract if the belief is that they will.

Consider the following strategies in period 2:

For Principal :

$$\begin{aligned} s_{p,2}(nf, p_g^2) &= \left\{ \frac{1}{1-\beta}, 0 \right\} ; p_g^2 < \frac{V - \frac{1}{(1-\beta)^2} + \frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}} \\ s_{p,2}(nf, p_g^2) &= \{0, 0\} ; p_g^2 \geq \frac{V - \frac{1}{(1-\beta)^2} + \frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}} \\ s_{p,2}(N, p_g^2) &= \phi \\ s_{p,2}(f, p_g^2) &= \phi \end{aligned}$$

For Worker :

If worker had formed firm in period 1 only one case is relevant

$$s_{w,2}(f, p_g^2, \phi, G/B) = S, e = 0$$

If worker had not formed firm in period 1 :

$$\begin{aligned} s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) &= \text{Choose (contract, effort) maximizing payoff} \\ s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) &= \text{Choose (contract, effort) maximizing payoff} \\ s_{w,2}(nf/N, p_g^2, \phi, G/B) &= N \end{aligned}$$

Consider the following strategies in period 1:

For Principal :

$$s_p(\phi) = \left\{ \left\{ \frac{1}{1-\beta}, 0 \right\} \right\}$$

For Worker :

$$\begin{aligned} s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G/B) &= \text{Choose (contract, effort) maximizing payoff} \\ s_w(\phi, G/B) &= N \end{aligned}$$

The above strategies would give us the pooling on contract equilibrium.

Proof for Proposition 11

Consider the following strategies in period 2:

For Principal :

$$s_{p,2}(nf, p_g^2) = \{0, 0\}$$

$$s_{p,2}(N, p_g^2) = \phi$$

$$s_{p,2}(f, p_g^2) = \phi$$

For Worker :

If worker had formed firm in period 1 only one case is relevant

$$s_{w,2}(f, p_g^2, \phi, G/B) = S, e = 0$$

If worker had not formed firm in period 1 :

$$s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = \text{Choose (contract, effort) maximizing payoff}$$

$$s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = \text{Choose (contract, effort) maximizing payoff}$$

$$s_{w,2}(nf/N, p_g^2, \phi, G/B) = N$$

Consider the following strategies in period 1:

For Principal :

$$s_p(\phi) = \{\{0, 0\}\}$$

For Worker :

WLOG let contract 1 be better than contract 2 for G worker

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_1, e = 1; -1 + s_1^1 = \max\{-1 + s_1^1, \beta s_1^1\} \text{ and } -1 + s_1^1 \geq -1 + V - R_w + \delta\beta V$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_1, e = 0; -1 + s_1^1 \neq \max\{-1 + s_1^1, \beta s_1^1\} \text{ and } \beta s_1^1 \geq -1 + V - R_w + \delta\beta V$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = L, e = 1; \text{ else.}$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, B) = \text{Choose (contract, effort) maximizing payoff}$$

$$s_w(\phi, B) = N$$

$$s_w(\phi, G) = L, e = 1$$

Consider incentives in period 2. The principal will only offer a contract if the worker had accepted a contract in period 1. Suppose the worker accepted a contract in period 1. The principal must be willing to offer $\{\frac{1}{1-\beta}, 0\}$ to get maximal effort. From the proof of proposition 9, we know that if $V < \frac{1}{1-\beta} + \frac{\beta\lambda_b}{(1-\beta)^2}$, then the principal prefers to offer a zero wage contract. We also know that in any separating equilibrium, the G worker needs to play $e = 1$ after L. For this to be incentive compatible we need the condition $V > \frac{1}{\delta\beta(1-\beta)(1-\lambda_b)}$. Therefore, we need the following condition:

$$\frac{1}{\delta\beta(1-\beta)(1-\lambda_b)} < \frac{1}{1-\beta} + \frac{\beta\lambda_b}{(1-\beta)^2}$$

Clearly, this holds if β is high enough.

We would also like that it is IR for the G type worker to play L but not for the B type worker. From proposition 9, we know that the following condition is sufficient:

$$\frac{1 + R_w}{1 + \delta\beta} < V < \frac{1 + R_w}{1 + \delta\beta(1 - \beta(1 - \lambda_b)^2)}$$

Sufficient condition which would make all the above conditions hold are:

$$\frac{1 + R_w}{1 + \delta\beta} > \frac{1}{\delta\beta(1 - \beta)(1 - \lambda_b)} \quad (11)$$

$$\frac{1 + R_w}{1 + \delta\beta(1 - \beta(1 - \lambda_b)^2)} < \frac{1}{1 - \beta} + \frac{\beta\lambda_b}{(1 - \beta)^2} \quad (12)$$

Thus, if the following holds, then all the above conditions are satisfied:

$$\beta \text{ high} \quad (13)$$

$$\frac{1 + \delta\beta}{\delta\beta(1 - \beta)(1 - \lambda_b)} - 1 < R_w < (1 + \delta\beta(1 - \beta(1 - \lambda_b)^2))\left(\frac{1 - \beta + \beta\lambda_b}{(1 - \beta)^2}\right) - 1 \quad (14)$$

It is clear that if β is high enough then the latter condition is satisfied for some R_w . So now we have the following to be true:

There exists a β' such that if $\beta > \beta'$ and the following hold:

$$\begin{aligned} \frac{1 + \delta\beta}{\delta\beta(1 - \beta)(1 - \lambda_b)} - 1 < R_w < (1 + \delta\beta(1 - \beta(1 - \lambda_b)^2))\left(\frac{1 - \beta + \beta\lambda_b}{(1 - \beta)^2}\right) - 1 \\ \frac{1 + R_w}{1 + \delta\beta} < V < \frac{1 + R_w}{1 + \delta\beta(1 - \beta(1 - \lambda_b)^2)} \end{aligned}$$

then,

1. IR for G to play L in period 1.
2. Not IR for B to play L in period 1.
3. If G plays L in period 1, then puts in effort $e = 1$ in period 1.
4. In period 2, if the worker had accepted a contract with the principal in period 1, then the principal offers a zero wage contract which the worker accepts and puts in zero effort.

We now consider why it is not possible for the principal to get the G worker. Consider the first period incentives for worker type G if a contract $\{s, 0\}$ is offered:

1. Payoff from accept and play $e = 1 = -1 + s$
2. Payoff from accept and play $e = 0 = \beta s$

3. Payoff from play $L = -1 + V - R_w + \delta\beta V$

It is clear that if we pick β high enough then we get the following to hold:

$$\frac{1}{\beta}[-1 + V - R_w + \delta\beta V] < V - R_w + \delta\beta V < \frac{1}{1-\beta}$$

Suppose β is high enough.

This implies that the principal has effectively three choices:

1. Offer zero wage contract. Only B type worker will accept and put in zero effort.
2. Offer the contract $\{\frac{1}{\beta}[-1 + V - R_w + \delta\beta V], 0\}$. Both G and B type worker will accept and put in zero effort.
3. Offer the contract $\{\frac{1}{1-\beta}\}$. Both G and B type worker will accept and put in full effort.

If p_g is extremely low, then principal will not find it optimal to offer any more than zero wage contract.

Proof of Proposition 12

Similar to proof for proposition 10.