Uphill or Downhill? Locating the Firm on a Profit Function

When deciding about changes in levels of multiple marketing investments aimed at improving profits, managers need to know whether they are on the uphill side or the downhill side of the profit function with respect to each investment variable. Spending errors made by an uphill-located firm that believes that it is a downhill-located firm can have serious consequences. This article offers an econometric “diagnostic tool” to infer a company’s current location on a multivariable profit function and to determine the changes in investments that will take the company to the neighborhood of the maximum profit. The authors apply the proposed approach to daily newspaper industry data and investigate the optimality of companies’ allocation behaviors with respect to three marketing efforts: investments in quality, distribution, and advertising space sales effort. A novel feature of this problem setting is that it involves a “dual-revenue” market with possibly interrelated demands for subscriptions and advertising space. The authors derive normative rules for marketing investments in four dual-revenue market types. The empirical analysis finds that daily newspapers’ dual revenues are positively interrelated and that a majority of these companies are located near the optimal level of spending for quality, a surprising finding considering that previous studies have characterized marketing managers as overspenders. Indeed, when these companies are suboptimal, they are much more likely to be underspending (i.e., they are located on the uphill side of the profit function) than overspending. In addition, the research furnishes estimates of sales elasticities with respect to quality, personal selling, and distribution investments, which are sparsely available in the extant literature.

Economic theory advises managers who make marketing decisions aimed at enhancing their company’s profit to operate at the “sweet spot” of a profit function, which is like a bell-shaped curve. For example, in the newspaper industry, managers can increase profit by improving news quality up to a point, beyond which further quality improvements fail to attract enough new readers to justify incremental costs. Figure 1 illustrates this reasoning and identifies the sweet spot (q*) that maximizes a profit function. On the left of this sweet spot, quality reduction reduces profit, and on the right, quality enhancement erodes profit. The real problem, however, is that companies “are being managed as though they were on the right or downhill side of the curve. In fact, we believe, they are clustered on the left, or uphill side, where degrading quality creates an imminent danger” (Meyer and Kim 2005, p. 7, emphasis added). As we describe subsequently, the lack of knowledge about the firm’s location—uphill or downhill—leads managers to make serious errors in their investment decisions.

Suppose that a firm located on the uphill side of the profit function (i.e., a Type U firm) believes incorrectly that it is located on the downhill side (i.e., a Type D firm) that generates the same amount of profit (see Figure 1). Then, it will disinvest in quality and thus earn less profit (because it is really a Type U firm) rather than more profit. In the newspaper industry, this negative result could trigger what Rosenstiel and Mitchell (2004) call the “suicide spiral,” in which disinvesting in newspaper quality leads to circulation decline, which leads to total revenue and profit reductions, which in turn lead to more disinvestment, more circulation decline, and further losses in revenues and profits. Similarly, a Type D firm that mistakenly believes that it is a Type U firm that makes the same profit would invest more in quality, again resulting in a profit reduction, which may induce a debilitating cycle of disinvestments in quality, falling revenues, and profits. Consequently, it is crucial for managers to be able to determine whether their company is Type U or Type D with respect to each marketing effort before they implement an appropriate course of action. A possible way for managers to determine where they are located on the profit function is to observe how both sales and profits respond to changes in spending. However, such observations do not immediately provide guidance on the change in investment level necessary to reach the sweet spot of the profit function. Furthermore, such an adaptive resolution of the problem is much more difficult and takes longer to achieve when profits are a function of spending decisions with respect to multiple marketing variables (Fraser and Hite 1988). Thus, a key purpose of this article is to offer an alternative solution, namely, an econometric “diagnostic
FIGURE 1
Firm Types Based on the Locations on a Profit Function

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tool” to determine whether a company is Type U or Type D with respect to each variable in its marketing mix through a valid approach for statistical inference applied to data from multiple firms in an industry.

Previous research in marketing has investigated issues of measuring market impact (Rust et al. 2004)—in particular, under- or overspending on advertising (e.g., Aaker and Carman 1982; Prasad and Sen 1999). A company overspends (underspends) when the actual expenditure exceeds (is below) the optimal amount indicated by a model. However, the optimal amount itself is based on market response elasticity, which is estimated with a measure of uncertainty. How then should a manager determine whether he or she is currently at the sweet spot or whether he or she is overspending or underspending? Furthermore, how much of a deviation from the sweet spot should the manager tolerate before taking corrective action? To answer these questions, managers need a diagnostic tool to distinguish whether their firm is operating in the neighborhood of the sweet spot (i.e., Type N firm) or is significantly underspending (Type U) or overspending (Type D) (see Figure 1).

The challenge of determining whether a given company is Type N and not either Type U or Type D is to quantify an interval or a region (x, x/H33526) around the sweet spot x* within which the firm is considered close to optimal, where x ≤ x* ≤ x/H33526. Previous marketing studies have concentrated on determining the response elasticities and/or the optimal x* and have not addressed this problem of estimating the triplet (x, x*, x/H33526) (see, e.g., literature reviews by Gatignon [1993] and Mantrala [2002] and research on under- and overspending behaviors by Joseph and Richardson [2002], Mantrala, Sinha, and Zoltners [1992], and Naik and Raman [2003]). However, econometric theory provides a few avenues to solve this inferential problem. One approach, based on the delta method (e.g., Davidson and MacKinnon 2004, p. 202), determines the interval (x, x) that covers a neighborhood of x* with 95% probability, which is obtained through an approximate normal distribution. An alternative approach that Krinsky and Robb (1986) propose avoids approximations by using Monte Carlo samples for constructing 95% coverage probability empirically. We follow the latter approach to develop a diagnostic tool for locating each firm on the profit function and, thus, for providing firm-specific diagnostic inferences with respect to each marketing effort. On the basis of actual spending decisions, we illustrate how a given company can infer whether it is Type U (below x), Type N (between x and x), or Type D (above x) with respect to multiple marketing variables. As we discussed previously, this knowledge of a firm’s type (i.e., uphill or downhill or neither) is crucial for implementing an appropriate course of action to enhance profitability.

The proposed approach yields new substantive findings. Specifically, using data from hundreds of daily newspaper firms collected by the Inland Press Association (Inland) over four years (1998–2001), we investigate the optimality of these firms’ investments with respect to three marketing variables: quality, distribution, and ad space sales effort. Notably, we find that more than 70% are Type N firms; that is, a majority of these companies are located near the sweet spots for all three investments. This result is surprising because previous studies have typically characterized marketing managers as overspenders (e.g., Aaker and Carman 1982; Prasad and Sen 1999). Furthermore, we find that when firms are suboptimal in their quality and advertising sales investments, they are much more likely to underspend than overspend. This finding corroborates Meyer and Kim’s (2005, p. 7) view that newspaper companies are “clustered on the uphill” of a profit function.
In addition to these two contributions (i.e., the diagnostic tool and new empirical findings), this article further augments the marketing literature in two important ways. First, we provide elasticity estimates for quality, personal selling effort, and distribution investments, which are sparsely available in the extant literature (e.g., Hanssens, Parsons, and Schultz 2001, pp. 347–49) compared with estimates of advertising or price elasticities (e.g., Assmus, Farley, and Lehmann 1984; Tellis 1988). Specifically, we find that, on average, newspaper distribution elasticity is .23, advertising sales effort elasticity is .54, and news quality elasticity is .49. This last finding should assure publishers that “good news quality is good business” (Overholser 2004).

Second, the newspaper industry offers a novel marketing context because of its “dual-revenue” market structure—that is, its sales of news to the readers and access to specialized audiences to the advertisers (Picard 1994). Furthermore, the resulting two revenue sources can be interrelated. That is, the desirability of a newspaper to advertisers and, thus, the demand for advertising space increase as circulation increases; in turn, the advertising volume a newspaper carries can influence its subscription sales (e.g., Blair and Romano 1993; Depken and Wilson 2004; Picard 1994). So far, there appears to be no research on optimal investments and allocation of marketing resources in dual-revenue markets with such demand interdependency (see Gatignon 1993; Mantrala 2002), which further motivates our normative and econometric analyses.

We organize the rest of this article as follows: In the next section, we review relevant literature pertaining to the newspaper industry’s market structure and marketing efforts. Using this knowledge, we then formulate and analyze an interrelated demand model and deduce normative implications for managing dual-revenue markets. Next, we specify and estimate an econometric model, develop the diagnostic tool to infer the location of a firm on the profit function, and present the empirical results based on the Inland data. Finally, we conclude by summarizing the managerial implications.

### Literature Review

#### Market Structure

Historically, the newspaper industry has been one of the most profitable of all industries (Picard 2004), with operating profits averaging approximately 15% and net profits averaging approximately 12% in 2001. In recent years, however, public newspaper companies are being challenged by declining circulations; for example, paid subscriptions dropped from 353 per 1000 population in 1950 to 202 per 1000 in 2000. Other challenges include printing capacity constraints, price ceilings, and competition from the Internet for classified advertising revenues. The two main characteristics of the newspaper industry are its monopoly structure and demand interrelatedness.

**Monopoly.** This structure is the rule rather than the exception in the U.S. daily newspaper industry. Specifically, 98% of newspapers exist as the only daily paper published in their local markets today (Blair and Romano 1993; Bucklin, Caves, and Lo 1989; Picard 1994). An explanation for the emergence of this monopoly is the “circulation—advertising death spiral”: In markets in which two or more daily newspapers compete, the newspaper with the largest circulation and highest market penetration attracts a disproportionate amount of advertising—even when circulation differences are small—and secondary papers enter into a circulation death spiral of lower advertising revenues leading to disinvestments leading to lower circulation leading to lower advertising and so on, ultimately eliminating one of them from the market (Picard 1994). In the few cities in which limited competition exists, it nearly always occurs between highly differentiated newspapers, such as a broadsheet and a tabloid intended for different audiences (Picard and Brody 1997).

**Demand interrelatedness.** As subscription sales increase, demand for advertising space increases because the advertising cost per thousand readers reached decreases. In addition, advertisers are attracted to newspapers with large paid circulations because subscribers who pay for the newspapers are more likely to read and register their advertised messages. In turn, as we already mentioned, advertising volume in a newspaper can affect its circulation sales (Bucklin, Caves, and Lo 1989; Corden 1953; Ferguson 1983; Rosse 1967). One school of thought suggests that this impact is positive because advertising provides information on the availability of products and their prices; many consumers consider this information valuable, and those who do not can skip over it (Blair and Romano 1993). Consequently, the demand for a newspaper can increase as its advertising volume increases. Conversely, advertising may negatively affect circulation sales because it lowers the value of the newspaper to readers who want to be informed of news and not commercial messages (Abrahamson 1998). Such ad aversion may vary across cultures. For example, Sonnac (2000) provides some empirical evidence that readers’ attitudes toward press advertising are country specific. Whereas U.S. readers are more typically “ad lovers,” European readership seems to be “ad averse.” Our research allows for and investigates both scenarios of interrelated demands.

#### Marketing-Mix Effects

Three major marketing-related expenditures of daily newspapers are (1) enhancing news quality, a responsibility that is managed by the editorial department, which consists of news reporters, section editors, copy editors, photographers, graphic artists, and administrative staff; (2) growing the distribution coverage, a responsibility of the circulation—distribution department, which employs telemarketers, newspaper distributors, paperboys, motor carriers, delivery truck operators, and coin-box rack technicians; and (3) generating advertiser revenues, a responsibility of the advertising department, which uses an ad space sales force with support from creative design artists and client service personnel.

**News quality.** The editorial department focuses on the upkeep of journalistic quality, editorial independence, and integrity. A persistent question asked in the newspaper
industry is, “Is good news quality good business?” It is only recently that expenditures on news quality have begun to be viewed as a marketing investment rather than as an operating cost. Litman and Bridges (1986) put forth the concept of “financial commitment,” which suggests that newsroom investment is a surrogate measure of news quality. Rosenstiel and Mitchell (2004) elucidate the concept of the suicide spiral that describes how disinvestments in quality cascade into declining subscriptions and revenues. Cho, Thorson, and Lacy (2004) empirically show that newsroom expenditures are positively correlated with quality news content, which is positively correlated with higher circulation sales. However, this stream of research stops short of providing firms with guidance on determining the optimal investment in quality.

Circulation–distribution. Distribution investments include compensation of telemarketers and delivery agents plus expenditures on delivery systems (e.g., data processing, postage, repairs, wire services). The Newspaper Association of America (NAA; 2000) has urged newspapers to view their delivery systems as strategic assets and their overall distribution investments as a means to increase sales and profit. For example, newspapers, such as the Sacramento Bee, increased their subscriptions and profits by developing superior delivery systems (Stein 1995).

Ad space sales. The expenditures on this effort are primarily related to sales representatives selling ad space to local retailers and companies. Smith (1998) shows that customer traffic (i.e., subscriptions and customer count), news content, and salesperson effectiveness influence firms to buy ad space in a newspaper. In a later study, Smith (2001) shows that newspapers’ sales efforts tend to increase advertising revenue, which in turn can lead to more subscriptions.

Price, margins, and resource allocation. We close this review by noting the constancy of price and margins and the limited research on optimal resource allocation in the newspaper industry. Daily newspaper retail prices remain unchanged over spans of four to seven years (e.g., Bils and Klenow 2002). With regard to price of advertising space, most newspapers publish in advance their rates for retail and classified advertising of various sizes and formats, which are held fixed for at least a year. In addition, newspapers’ gross margins on each circulation dollar and on each ad revenue dollar are constant because after the first copy of each day’s newspaper is produced, variable costs, primarily newsprint costs, associated with the printing of subsequent copies remain constant.

Finally, the extant literature in the journalism and economics fields is descriptive with limited attention to normative analyses of marketing decisions. Two notable exceptions are the work of Corden (1953) and Bucklin, Caves, and Lo (1989), who consider maximization of a newspaper company’s profit with respect to prices, product quality, and circulation-enhancing investments. However, these studies do not (1) incorporate the role of interrelated dual-revenue sources as a function of investments in quality, distribution, and advertising sales effort or (2) provide firm-specific inferences on its location on the profit function (i.e., “How far are we from the optimal?”). Therefore, we next formulate a dual-revenue market model to investigate these issues of marketing resource allocation and inference.

Model Formulation and Normative Analyses

The two revenue sources of a newspaper company are paid circulation and advertising revenue. Let $S$ denote the number of subscribers for the year, $m_1$ denote the margin ($\) per issue (price less the cost), and $k$ denote the number of issues per year. Typically, $k = 730$ when some newspapers produce both morning and evening editions. Let $R$ denote revenues from all advertiser sources (e.g., retail and classified advertising) and $m_2$ denote the margin ($\) per advertising dollar generated. On the basis of the literature review, subscription sales $S$ and advertising revenue $R$ depend on investments in news quality, distribution coverage, and ad space sales effort. More specifically, in line with the work of Cho, Thorson, and Lacy (2004) and Stein (1995), investments in news quality and distribution–distribution directly influence subscriptions, and the investment in ad space sales effort directly influences advertising revenue (Smith 2001). The circulation–distribution effort does not directly affect advertising revenue, because it is focused on ensuring newspaper delivery to the subscribers; it is not aimed at advertisers. Similarly, the ad space sales effort does not directly affect subscriptions, because it is focused on retailers and companies, not on readers. However, the resulting ad revenue can influence the level of subscriptions, which in turn directly affects ad revenue (e.g., Blair and Romano 1993; Bucklin, Caves, and Lo 1989; Corden 1953; Ferguson 1983; Rosse 1967). Conceptually, therefore, a simultaneous rather than recursive model of a newspaper’s dual-revenue relationship to the marketing mix is warranted. Thus, denoting $(q, d, a)$ as dollars invested in quality, distribution, and advertising sales, respectively, we express the dual-revenue sources as follows:

\[
S = f_1(q, d, R), \quad R = f_2(a, S),
\]

where $f_1(\cdot)$ and $f_2(\cdot)$ are general diminishing-returns response functions as is commonly assumed in marketing-mix analyses—specifically, $\partial f_i/\partial x > 0$, $\partial^2 f_i/\partial x^2 < 0$, $i = 1, 2$, and $x = q, d, a, R, S$ as necessary, with the system allowing for interrelated demands; that is, advertising revenue $R$ directly affects subscriptions in Equation 1, and subscriptions directly influence advertising revenue in Equation 2.

Summing the two revenues and deducting the expenditures on quality, distribution, and ad space sales, we obtain the firm’s profit as follows:

\[
\pi(q, d, a) = m_1 \times k \times S + m_2 \times R - q - d - a.
\]

To determine the optimal investment levels in quality $(q^*)$, distribution $(d^*)$, and advertising sales $(a^*)$, we differentiate Equation 3 with respect to $(q, d, a)$ and solve the resulting three first-order conditions simultaneously. We relegate these details to Appendix A and subsequently describe the analytical results presented in Table 1, focusing on appro-
Specifically, we identify four types of markets based on the cross-market dependency coefficient
\[ \delta = 1 - \frac{\partial f_1/\partial R}{\partial f_2/\partial S}, \]
which arises in Equation A4 (see Appendix A). The presence of this dependency coefficient makes the optimality conditions for dual-revenue markets different from those in the standard analysis (Dorfman and Steiner 1954; see also Hanssens, Parsons, and Schultz 2001, pp. 358–61). The four types of markets are as follows.

**Market Type 1: unrelated markets.** In this special case, the two revenue sources are not related; that is, \( \partial f_1/\partial R = 0 \), and \( \partial f_2/\partial S = 0 \), so the dependency coefficient equals unity (i.e., \( \delta = 1 \)). Although this market is valid for other industries and is analyzed by Dorfman and Steiner (1954), it is unlikely to prevail in the daily newspaper industry.

**Market Type 2: partially related markets.** Here, an increase in the number of subscriptions increases advertising revenues, but not vice versa; that is, \( \partial f_2/\partial S > 0 \), and \( \partial f_1/\partial R = 0 \). Corden (1953) and Bucklin, Caves, and Lo (1989) analyze this market setting, and it differs from the current study because we identify and analyze interrelated demand markets, as we describe subsequently.

**Market Type 3: interrelated markets with opposing feedback effects.** This market arises when increased subscriptions beget larger advertising revenues, but the resulting enhanced advertising volume reduces the number of subscribers, thus implying negative feedback from advertising revenues to subscriptions; that is, \( \partial f_2/\partial S > 0 \), and \( \partial f_1/\partial R < 0 \). Abrahamson (1998) and Sonnac (2000) suggest such a market on the basis of the argument that increasing volumes of advertising cause reader irritation and erode the utility a subscriber gets from the newspaper. We empirically test the implied hypothesis, \( H_0: \delta > 1 \).

**Market Type 4: interrelated markets with positive feedback effects.** Rosse (1967), Ferguson (1983), and Blair and Romano (1993) provide support for the presence of such markets, in which both feedback effects are positive; that is, \( \partial f_1/\partial R > 0 \), and \( \partial f_2/\partial S > 0 \). Empirically, to distinguish this possibility from Type 3 markets, we need to test whether \( H_0: \delta < 1 \) holds.

The various market types induce different investment behaviors from managers. Table 1 summarizes the optimal investments in quality \( q_j \), distribution \( d_j \), and advertising sales \( a_j \), where \( j \) indicates the market type (\( j = 1, 2, 3, \) or \( 4 \)). For markets of Type 1, we recover the usual optimality conditions stated in Dorfman and Steiner’s (1954) theorem. For markets of Type 2, because of positive feedback from subscriptions to ad revenues, the optimal investments in quality and distribution are larger than those for unrelated markets (i.e., \( q^*_2 > q^*_1 \) and \( d^*_2 > d^*_1 \)), whereas the optimal advertising sales expenditure remains the same (\( a^*_2 = a^*_1 \)). Next, for markets of Type 3, managers should reduce their investments in news quality, distribution, and advertising sales compared with those made for partially related markets (i.e., \( q^*_3 < q^*_2 \) and \( d^*_3 < d^*_2 \) and \( a^*_3 < a^*_2 \)). Intuitively, increasing the investments is counterproductive because advertising revenues hurt subscriptions in this market. When we compare Market Types 3 and 1, we find again that managers in Market Type 3 should invest less in news quality and distribution than the corresponding levels for Market Type 1 (i.e., \( q^*_3 < q^*_1 \) and \( d^*_3 < d^*_1 \)) except when the margin ratio exceeds the feedback effect (i.e., \( m_2/(km_1) > |\partial f_1/\partial R| \). In other words, investments in quality and distribution that are greater than the optimal levels for Market Type 1 are justi-
fied in Market Type 3 only if profit contributed by the increased advertising revenue exceeds the lost contribution due to lower subscriptions. In contrast, when we compare Market Types 4 and 1, we observe that managers in Market Type 4 should increase the investments in quality, distribution, and advertising sales beyond the corresponding levels for Market Type 1 because positive feedback reinforces both subscription and advertising revenues.

In summary, the normative analysis reveals four market types and appropriate decisions for each one. Furthermore, previous research pertains only to Market Types 1 and 2 (see Bucklin, Caves, and Lo 1989; Corden 1953; Dorfman and Steiner 1954). Consequently, we identified new markets (i.e., Market Types 3 and 4); derived the optimal investments in quality, distribution, and advertising; and gained insights into different decision rules applicable across these markets. Notably, investment decisions in Market Type 3 tend to be opposite of those for Market Type 4 (for details, see Table 1). Thus, managers need to know the type of market they operate in, an empirical issue that we examine in the next section.

Empirical Analyses and Diagnostic Tool

In this section, we present empirical analyses of Inland daily newspaper data that enable us to develop and demonstrate two diagnostic procedures: (1) an approach for managers to determine the type of market (Type 1, 2, 3, or 4) in which they operate and (2) a tool to assess whether their firm is overspending or underspending and to offer guidance for driving it to the neighborhood of the maximum profit. Determination of market type depends on the estimation of the cross-market dependency coefficient. To this end, we first describe the data sets, model specification, and estimation approach, and then we present the empirical results for cross-market dependency and elasticity estimates for advertising sales, distribution, and quality. Subsequently, we propose the diagnostic tool, which is based on a five-step algorithm, to locate a specific company on the multivariable profit function and thus infer whether it is a Type N firm (near-optimal spending), a Type U firm (underspending), or a Type D firm (overspending) with respect to each of the marketing investments.

Inland Data

Inland Daily Press Association constructs annual data sets that “tell us more about the economic inners of American dailies than any other source” (Blankenburg 1989, p. 98). Inland data sets contain information from yearly samples of hundreds of individual newspapers with circulations that do not exceed 85,000, which constitutes a representative cross-section of the total 1400 U.S. daily newspapers (Picard 1989, p. 110). Our research uses Inland data collected in 1998, 1999, 2000, and 2001. Because newspaper companies desire confidentiality, Inland data sets do not reveal information about their identities and locations. Each year’s data set contains information about participating newspapers’ subscriptions (S), advertising revenues (R), and margins on sales and ad revenues (m1, m2). In addition, we have data on the firms’ annual investments in news quality, circulation—distribution, and advertising sales activities. Specifically, news quality investment (q) equals the sum of news editorial expenses, newsroom salaries, and miscellaneous expenses; distribution investment (d) equals the sum of distribution expenses, distribution department salaries, and miscellaneous expenses; and advertising sales investment (a) equals the sum of expenses to reach potential advertisers, ad department salaries, and miscellaneous expenses.

Response Model Specification

Consistent with the theoretical model expressed in Equations 1 and 2, we specify the two-equation simultaneous system of a newspaper company’s dual demands as follows:

\[
S_i = \alpha_0 + \alpha_1 R_i + \alpha_2 \ln(q_i) + \alpha_3 \ln(d_i) + \epsilon_i, \quad \text{and}
\]

\[
R_i = \beta_0 + \beta_1 S_i + \beta_2 \ln(a_i) + \nu_i,
\]

where \(S_i\) denotes the annual number of subscribers (in thousands); \(R_i\) represents ad revenue (in hundreds of thousands of dollars); \(\ln(\cdot)\) is the natural logarithm; and \((q_i, d_i, a_i)\) are dollars invested in quality, distribution, and advertising sales, respectively, by the newspaper companies \(i = 1, ..., N\).

In Equations 4 and 5, we use the semilog specification of the effects of changes in news quality and distribution investments on subscriptions and the effect of changes in ad sales effort on advertising revenues because of its consistency with the theoretical assumption of diminishing returns, parsimony, and popularity in previous advertising research (see, e.g., Carroll, Green, and Desarbo 1979; Latham 1969; for a discussion of the semilog response model’s advantages, see Doyle and Saunders 1990). Response elasticities based on the semilog model are easily interpreted and yield closed-form analytical expressions for optimal levels of the marketing investments.

We expect that the quality investment response coefficient \(\alpha_2\), the circulation—distribution investment coefficient \(\alpha_3\), and the advertising sales investment coefficient \(\beta_2\) will be positive. We also expect that the impact of subscriptions on ad revenues \(\beta_1\) will be positive, but the impact of ad revenues on subscriptions \(\alpha_1\) may be zero, positive, or possibly negative. To sign the intercept term \(\beta_0\) in Equation 5, we note that advertisers will not advertise in a newspaper unless it has some minimum positive subscriptions. Therefore, if the newspaper’s advertising effort is negligible (e.g., close to $1), subscription sales must be greater than \((-\beta_0 / \beta_1) > 0\) for advertising revenues to be positive. Consequently, given \(\beta_1 > 0\), the intercept \(\beta_0 < 0\). Turning to Equation 4, we note that a minimum investment in news quality (editorial or newsroom employees) is necessary to avoid closing the newspaper. If we assume this minimum investment in quality, which will provide for some minimum subscription sales, the intercept \(\alpha_0\) in Equation 4 explicitly captures the incremental impact of the annual carryover effect of previous distribution investments.

Finally, we assume that the random errors, \(\epsilon_i\) and \(\nu_i\), are normally distributed with zero means and constant variances \((\sigma_\epsilon^2, \sigma_\nu^2)\) and are possibly correlated \((\rho_{\epsilon\nu})\) because common economic conditions affect both subscriptions and ad revenues. In summary, the specifications of Equations 4
and 5 satisfy all the theoretical assumptions; that is, \( \partial E[S_i]/\partial z > 0, \partial^2 E[S_i]/\partial z^2 \leq 0, \partial E[R_i]/\partial z > 0, \) and \( \partial^2 E[R_i]/\partial z^2 \leq 0 \) for every firm \( i \) and every argument \( z = q_i, d_i, a_i, R_i, \) and \( S_i \).

**Accounting for Heterogeneity in Model Parameters Across Newspapers**

Equations 4 and 5 assume that the model parameters are identical across newspapers, which may not be a realistic assumption when we consider heterogeneity due to location, population, economic development, diversity, and retail business concentrations across our annual, cross-sectional samples of newspapers. For example, subscriptions and ad revenues of newspapers located in smaller cities can be systematically more or less responsive to quality investment efforts than those of newspapers located in larger city markets. To incorporate heterogeneity, we partition the annual Inland samples into homogeneous subgroups or segments using latent-class clustering (LCC); alternatively, a random coefficients model (Swamy 1970) could be estimated (e.g., Rao, Agarwal, and Dalhoff 2004).

Although the Inland data sets do not provide any direct information about the identities and geographic locations/markets of newspapers, they contain data on several related variables: total salaries of all employees, number of newsroom employees, number of circulation–distribution employees, and number of advertising department employees. As we explain subsequently, these variables not only reflect firms’ sizes but also serve as indicators of the categorical latent variable for classifying newspapers into groups with similar geographic characteristics.

**Total salaries (TOTSAL).** In the United States, significant variation exists in the wages paid for particular jobs in different geographic locations and regions (see, e.g., Mercer Human Resources Consulting’s “2005 Geographic Salary Differentials” study at www.imercer.com). Such geographic variation in salaries reflects heterogeneity in the cost of labor (i.e., differences between localities in terms of cash compensation for the same work) and the cost of living (i.e., difference between localities in terms of cost of housing, groceries, transportation, and entertainment). Furthermore, market pay for lower-paid and lower-level employees varies more with geography than it does for higher-paid and more senior employees because, in general, employers recruit for lower-paying jobs from the local population. The Readership Institute at Northwestern University reports that newspapers recruit more than 84% of their employees from outside the newspaper, and the bulk of these external hires is rank-and-file or lower-paying jobs staffed from within the newspaper’s community or market (see “1999 Workforce Characteristics Survey” at www.readership.org). This observation and the fact that most daily newspapers have only one primary office location and few or no secondary locations suggest that variations in total salaries across newspapers reflect differences in the cost of labor and cost of living in their headquarters towns and environs.

**Number of newsroom employees (NEMP).** According to the Readership Institute’s “1999 Workforce Characteristics Survey,” on average, 35% of the employees of a newspaper work in the newsroom (i.e., news editorial department). Although an industry guideline calls for one newsroom employee slot per 1000 circulation, larger newsroom staffs exist among papers that produce more zoned editions, which offer more specialized coverage for a host of special interest groups, including ethnic groups (e.g., Hispanics) and affluent immigrant groups (Rosentiel and Mitchell 2004). Thus, newspapers in locations with more diversity and/or higher education and income levels are likely to have more newsroom employees.

**Number of circulation–distribution employees (DEMP).** Circulation department employees account for another 35% of a newspaper’s total employees (see “1999 Workforce Characteristics Survey” at www.readership.org). These employees ensure delivery of the newspaper to all its subscribers and other outlets in the newspaper’s market. Consequently, we expect that newspapers with greater numbers of circulation department employees are located in more urban areas with larger populations.

**Number of advertising department employees (AEMP).** On average, approximately 25% of a newspaper’s staff work in the advertising department to generate advertising revenues. A large proportion of these employees constitute the ad space sales force. We expect that newspapers with higher numbers of advertising department employees and, thus, larger advertising sales force sizes are located in areas with greater numbers of business accounts.

Using these four indicators, we incorporate heterogeneity into the model parameters due to observed and unobserved characteristics of newspaper firms and their geographic markets through a three-step latent-class segmentation–estimation approach. In the first step, we employ LCC analysis (e.g., Vermunt and Magidson 2003) using the four indicators to identify clusters of newspapers with similar characteristics. Because LCC formulates a finite mixture of multivariate normal distributions, it provides probabilistic classification of newspapers into clusters, does not require rescaling of observed variables, and yields managerially meaningful segments (e.g., Wedel and Kamakura 2000, pp. 78, 329). We used the software Latent Gold Version 3.0.6 for this purpose. The LCC model includes a K-category latent variable, which we measured with the four indicators, in which each category represents a cluster. Because the appropriate number and composition of latent segments are not known a priori, we investigate different LCC scenarios, such as one-segment, two-segment, and three-segment solutions. In the second step, we estimate the simultaneous equations (4 and 5) by segment under different clustering scenarios. In each of these model estimations, we allow for the possibility of correlated errors and employ the three-stage least squares (3SLS) estimation approach that uses complete information on both the structural equations. (We applied the Lagrange-multiplier test [e.g., Greene 2000, p. 492] in each estimation scenario to establish that 3SLS estimation was more efficient than the 2SLS estimation procedure, which ignores the correlation between errors.) In the third step, we select the best segmentation–estimation model by applying the mixture regression criterion (MRC), which Naik, Shi, and Tsai (2007) developed. The MRC extends the classical Akaike information criterion to finite mixture regression models by
Theoretically deriving a clustering penalty term to prevent overclustering (i.e., inclusion of insignificant segments). Thus, MRC is given by

\[
MRC = \sum_{l=1}^{L} \left( \hat{\alpha}_l \ln(\hat{\phi}_l^2) + \hat{\beta}_l \ln(\hat{\phi}_l^2) \right)
\]

where \( L \) is the total number of segments; \( \hat{\alpha}_l \) denotes the number of firms in Cluster \( l \); \( \hat{\beta}_l \) denotes the number of estimated parameters, \( \hat{\phi}_l^2 = \hat{\sigma}_l^2 + \hat{\sigma}_v^2 \), from the simultaneous equation estimation of each cluster; and \( \hat{\phi}_l \) \((l \leq 1)\) denotes the proportion of sample classified in Cluster \( l \).

By applying the three-step segmentation–estimation approach to the Inland data, we find that the MRC achieves its minimum value with the two-segment model in each year (see Table 2). In other words, the two-segment model balances both the fidelity (i.e., superior fit) and the parsimony (i.e., fewer parameters) better than the one- or three-segment models. In the remainder of this section, we examine these results in greater depth.

**Description of Two Newspaper Company Segments**

In each of the four years, the Wald statistics associated with the parameters of each of the four indicators in the two-segment clustering solution were significant at the 95% level of confidence, establishing that they were useful in exposing latent differences among the segments of newspaper firms. Furthermore, newspapers assigned to Segment 1 have high probabilities of having smaller observed values on each of the four indicator variables. Table 3 displays the sizes and indicator profiles of the derived segments. We observe that in each year, Segment 2 consists of newspapers whose average total salaries are approximately 4 times larger than the corresponding mean total salaries of newspapers in Segment 1. In addition, the mean numbers of newsroom, circulation department, and advertising sales employees of Segment 2 newspapers in each year are more than 2.5 times larger than those of Segment 1 newspapers. Hereinafter, we refer to Segment 2 (Segment 1) as the “large-firm” (“small-firm”) segment. If we average across the years, 57% of all firms fall in the small-firm segment.

Second, across the four years, the estimated slope coefficients of all the variables have the correct signs. In addition, across all segments and years, the estimated intercepts (\( \beta_0 \)) possess the expected negative sign and remain significant at the 90% or greater confidence level.

Third, 10 of the 12 investment coefficient estimates in the small-firm segment across the four years are statistically significant at the 95% confidence level, and 7 of the 12 estimated investment coefficients in the large-firm segment are significant at the 90% confidence level. Focusing on the small-firm segment model estimation results, we observe that the estimated coefficients for investments in quality (\( \beta_2 \)) and advertising sales effort (\( \beta_3 \)) are statistically significant at the 95% confidence level in all four years, whereas the distribution coefficient estimate (\( \alpha_3 \)) is not significant in 1998 and 1999 but is significant at the 95% confidence level in 2000 and 2001. The corresponding significance results in the large-firm segment are mixed. In this segment, none of the investment coefficient estimates are significant in 1998, the quality investment is significant at the 95% confidence level only in 1999, and the distribution and advertising sales investment coefficients are statistically significant at the 95% confidence level in 1999 and at the 90% confidence level in 2000 and 2001. Overall, we find that investments in newspaper quality have consistently significant, positive effects in the small-firm segment, whereas advertising sales

We also estimated an alternative recursive model formulation that a reviewer suggested—in which subscriptions directly affect advertising revenues but advertising revenues do not directly affect subscriptions—on the four annual Inland data sets and found that the minimum MRC value in each case was again achieved with the two-segment solution but that it was larger than the corresponding MRC value we obtained with the proposed simultaneous equation model. Given its superior fit to the data and its consistency with observations in the newspaper marketing and research literature, we focus on the proposed model-based results in the rest of this article.

**TABLE 2**

<table>
<thead>
<tr>
<th>Year</th>
<th>One Segment</th>
<th>Two Segments</th>
<th>Three Segments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>3272.60</td>
<td>3212.93</td>
<td>3266.47</td>
</tr>
<tr>
<td>1999</td>
<td>4495.96</td>
<td>4392.1</td>
<td>4810.24</td>
</tr>
<tr>
<td>2000</td>
<td>3791.56</td>
<td>3761.21</td>
<td>3825.59</td>
</tr>
<tr>
<td>2001</td>
<td>3692.35</td>
<td>3656.72</td>
<td>3657.17</td>
</tr>
</tbody>
</table>

Estimates of model parameters. Table 5 reports the segment-by-segment dual-revenue response model parameter estimates across the 1998–2001 period. First, on the basis of R-square values that range from .77 to .86, the proposed simultaneous equation model for subscription and ad revenues fits each segment’s data in each year satisfactorily. Second, across the four years, the estimated slope coefficients of all the variables have the correct signs. In addition, across all segments and years, the estimated intercepts (\( \beta_0 \)) possess the expected negative sign and remain significant at the 90% or greater confidence level.
TABLE 3
Descriptive Statistics of Cluster Indicators

<table>
<thead>
<tr>
<th></th>
<th>Segment 1 (Small Firm)</th>
<th>Segment 2 (Large Firm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td><strong>1998</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Segment Size (%)</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>TOTSAL ($)</td>
<td>982,147</td>
<td>531,883</td>
</tr>
<tr>
<td>NEMP</td>
<td>19</td>
<td>1.04</td>
</tr>
<tr>
<td>AEMP</td>
<td>13</td>
<td>.65</td>
</tr>
<tr>
<td>DEMP</td>
<td>9</td>
<td>.63</td>
</tr>
<tr>
<td><strong>1999</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Segment Size (%)</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td>TOTSAL ($)</td>
<td>941,181</td>
<td>490,293</td>
</tr>
<tr>
<td>NEMP</td>
<td>17</td>
<td>.66</td>
</tr>
<tr>
<td>AEMP</td>
<td>13</td>
<td>.49</td>
</tr>
<tr>
<td>DEMP</td>
<td>8</td>
<td>.43</td>
</tr>
<tr>
<td><strong>2000</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Segment Size (%)</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>TOTSAL ($)</td>
<td>1,119,222</td>
<td>570,862</td>
</tr>
<tr>
<td>NEMP</td>
<td>19</td>
<td>.81</td>
</tr>
<tr>
<td>AEMP</td>
<td>14</td>
<td>.66</td>
</tr>
<tr>
<td>DEMP</td>
<td>10</td>
<td>.54</td>
</tr>
<tr>
<td><strong>2001</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Segment Size (%)</td>
<td>69</td>
<td></td>
</tr>
<tr>
<td>TOTSAL ($)</td>
<td>1,410,944</td>
<td>779,374</td>
</tr>
<tr>
<td>NEMP</td>
<td>22</td>
<td>.9</td>
</tr>
<tr>
<td>AEMP</td>
<td>17</td>
<td>.7</td>
</tr>
<tr>
<td>DEMP</td>
<td>12</td>
<td>.65</td>
</tr>
</tbody>
</table>

Investments tend to have significant, positive effects in both small-firm and large-firm segments.

Fourth, we note that in both segments and in all four years, subscription sales have a positive, statistically significant impact on advertising revenues, and similarly, advertising revenues have a positive, significant impact on subscription sales, except for the small-firm segment in 1998 when the advertising revenues impact on subscriptions was not significant. Next, on the basis of these results, we investigate whether the newspaper market type is one of interrelated demands (Types 3 or 4), partially related demands (Type 2), or unrelated demands (Type 1) in the four years under study.

Determining the market type. To this end, we test a set of hypotheses hierarchically as follows: We test whether \( \delta > 1 \) with a one-tailed t-test; if it holds, the market is Type 3. If not, we test whether \( \delta < 1 \) with a one-tailed t-test; if it holds, the market is Type 4. Otherwise \( \delta = 1 \), and so we test whether \( \beta_1 = 0 \) with a two-tailed t-test. If it holds, the market is Type 1; if not, it is Type 2. To conduct these tests, we need to evaluate the mean \( E[\delta] \) and the variance \( \text{Var}[\delta] \), which we derive in Appendix B. For example, for the 1998 estimates of \((\alpha_1, \beta_1)\) in the large-firm segment given in Table 5, we find that the estimated mean equals .482 and the variance is .057, which indicates that \( \delta < 1 \); thus, this market is Type 4 (i.e., interrelated markets with positive feedback). Similar results emerge for the large-firm segment in all the remaining years and in the small-firm segment with the sole exception of the result in 1998, for which the t-value suggests that \( \delta = 1 \) (see Table 6). Furthermore, in this case, Table 5 indicates that we must reject \( \beta_1 = 0 \); thus, this market is Type 2 (i.e., partially related). Overall, our empirical findings corroborate the markets that Corden (1953) and Bucklin, Caves, and Lo (1989) describe and furnish reasonable support for the idea that U.S. daily newspapers operate in markets of Type 4, which are characterized by interrelated demands for subscriptions and advertising space with positive feedback effects. This conclusion is also consistent with Sonnac’s (2000) observation that newspaper readers in the United States tend to be ad lovers.

Newspaper Quality, Distribution, and Selling Effort Elasticities

Table 7 reports the overall and segment-level elasticities of both subscription sales and advertising revenues with respect to the three marketing activities in each year. First, the estimates of quality, distribution, and personal selling elasticities in the daily newspaper industry are consistent with prior, albeit sparsely available, estimates in the marketing literature (e.g., Hanssens, Parsons, and Schultz 2001). Specifically, averaging across the two segments and four years, we obtain the mean elasticities. The elasticity of subscription sales and advertising revenues with respect to news quality investments are .49 and .55, respectively, which are consistent with the overall quality elasticity estimates of .521 and .611 that Lambin (1976) and
TABLE 4
Descriptive Statistics by Segments

<table>
<thead>
<tr>
<th>Variables</th>
<th>Segment 1 (Small Firm)</th>
<th>Segment 2 (Large Firm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality investments (in thousands of dollars), q</td>
<td>1058.7 615.1</td>
<td>999.6 548.5</td>
</tr>
<tr>
<td>Distribution investments (in thousands of dollars), d</td>
<td>653.1 416.1</td>
<td>640.0 395.6</td>
</tr>
<tr>
<td>Ad sales investments (in thousands of dollars), a</td>
<td>844.6 451.9</td>
<td>807.1 431.3</td>
</tr>
<tr>
<td>Subscriptions sales (in thousands of dollars), S</td>
<td>13.9 7.5</td>
<td>12.7 6.4</td>
</tr>
<tr>
<td>Advertising revenue (in hundreds of thousands of dollars), R</td>
<td>144.6 80.2</td>
<td>155.8 88.5</td>
</tr>
<tr>
<td>Subscription margin, m_1</td>
<td>.71 .14</td>
<td>.75 .14</td>
</tr>
<tr>
<td>Ad revenue margin, m_2</td>
<td>.17 .06</td>
<td>.17 .06</td>
</tr>
</tbody>
</table>
### TABLE 5
Parameter Estimates (t-Values) for the Dual-Revenue Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Segment 1 (Small Firm)</td>
<td>Segment 2 (Large Firm)</td>
<td>Segment 1 (Small Firm)</td>
<td>Segment 2 (Large Firm)</td>
</tr>
<tr>
<td>Intercept, $\alpha_0$</td>
<td>-86.9</td>
<td>-142.8</td>
<td>-57.51</td>
<td>-273.1</td>
</tr>
<tr>
<td></td>
<td>(-2.49)</td>
<td>(-1.60)</td>
<td>(-2.32)</td>
<td>(-3.33)</td>
</tr>
<tr>
<td>Ad revenue coefficient, $\alpha_1$</td>
<td>.07</td>
<td>.13</td>
<td>.12</td>
<td>.06</td>
</tr>
<tr>
<td></td>
<td>(94)</td>
<td>(2.96)</td>
<td>(1.93)</td>
<td>(1.69)</td>
</tr>
<tr>
<td>Quality coefficient, $\alpha_2$</td>
<td>5.79</td>
<td>5.66</td>
<td>4.13</td>
<td>12.1</td>
</tr>
<tr>
<td></td>
<td>(2.54)</td>
<td>(1.57)</td>
<td>(2.22)</td>
<td>(3.06)</td>
</tr>
<tr>
<td>Distribution coefficient, $\alpha_3$</td>
<td>1.41</td>
<td>5.25</td>
<td>.73</td>
<td>8.02</td>
</tr>
<tr>
<td></td>
<td>(1.38)</td>
<td>(1.59)</td>
<td>(1.39)</td>
<td>(2.92)</td>
</tr>
<tr>
<td>Intercept, $\beta_0$</td>
<td>-147.3</td>
<td>-467.2</td>
<td>-103.1</td>
<td>-554.3</td>
</tr>
<tr>
<td></td>
<td>(-3.49)</td>
<td>(-1.75)</td>
<td>(-2.92)</td>
<td>(-3.60)</td>
</tr>
<tr>
<td>Subscription coefficient, $\beta_1$</td>
<td>2.38</td>
<td>3.83</td>
<td>2.67</td>
<td>4.16</td>
</tr>
<tr>
<td></td>
<td>(7.47)</td>
<td>(6.23)</td>
<td>(8.30)</td>
<td>(10.60)</td>
</tr>
<tr>
<td>Ad sales effort coefficient, $\beta_2$</td>
<td>11.15</td>
<td>30.99</td>
<td>7.6</td>
<td>36.7</td>
</tr>
<tr>
<td></td>
<td>(3.25)</td>
<td>(1.59)</td>
<td>(2.63)</td>
<td>(3.28)</td>
</tr>
<tr>
<td>Error variance $\sigma_\epsilon^2$</td>
<td>1575.8</td>
<td>5513.36</td>
<td>1315.4</td>
<td>7909.4</td>
</tr>
<tr>
<td>Error variance $\sigma_\eta^2$</td>
<td>10,743.5</td>
<td>143,720.8</td>
<td>13,511.5</td>
<td>233,236.3</td>
</tr>
<tr>
<td>Subscription model fit, R²</td>
<td>.79</td>
<td>.80</td>
<td>.80</td>
<td>.81</td>
</tr>
<tr>
<td>Ad revenue model fit, R²</td>
<td>.86</td>
<td>.77</td>
<td>.81</td>
<td>.79</td>
</tr>
</tbody>
</table>
Overspending and Enhancing Profit
decrease (increase) investments in quality if the newspaper
titude of quality elasticity (large or small), it is prudent to
activities from current levels. For example, for any magni-
should actually increase or decrease investments in those
provide sufficient information about whether managers
Although elasticity estimates are informative, they do not
Diagnostic Tool for Assessing Under- or

small-firm newspaper markets.
ment are greater than those observed in the small-firm seg-
subscription sales. This result is especially true in the small-
staffs of smaller newspapers are used much more inten-
These differences between the two segments appear to be
Second, the results in Table 7 highlight the powerful
role of investments in news quality in influencing subscrip-
sales, which in turn drives advertising revenues. Indeed, the impact of news quality investment on advertis-
ing revenues is as large as the impact of ad space sales effort and somewhat stronger than its direct impact on sub-
scription sales. This result is especially true in the small-
firm segment, in which the average elasticities of subscrip-
tions and advertising revenues with respect to news quality
investments are as high as .57 and .64, respectively, which
is significantly greater than the corresponding values in the
large-firm segment, which are approximately .4 and .47.

These differences between the two segments appear to be
consistent with industry reports that the smaller newsroom
staffs of smaller newspapers are used much more intensively
than those of larger newspaper firms (Rosenstiel and
Mitchell 2001). However the subscriptions–distribution and
ad revenue–distribution elasticities in the large-firm seg-
ment are greater than those observed in the small-firm seg-
ment, perhaps because small-firm newspaper markets are
closer to saturation with respect to distribution coverage
than large-firm newspaper markets.

Diagnosing Under- or Overspending and Enhancing Profit

Although elasticity estimates are informative, they do not
provide sufficient information about whether managers
should actually increase or decrease investments in those
activities from current levels. For example, for any magni-
tude of quality elasticity (large or small), it is prudent to
decrease (increase) investments in quality if the newspaper
company is located on the downhill (uphill) of the profit
function (see Figure 1). Thus, to make profitable decisions,
managers must know the firm’s location on the profit func-
tion. To facilitate this goal, we present a diagnostic tool
based on the following algorithm:

Step 1: For a given company i, derive the optimal decisions
(q*, d*, a*) as functions of the model parameters. Let
\( g_i(\theta) \) be a 3 \times 1 vector-valued function of the vector of
parameters \( \theta \).

Step 2: Draw 1000 samples \( \hat{\theta} \) from the normal distribution
\( N(\theta, \Sigma) \), where \( j = 1, \ldots, 1000 \). This step accounts for
both the magnitude of the estimates \( \theta = (\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2) \)
and the uncertainty associated with them through
their variance–covariance matrix \( \Sigma \).

Step 3: Evaluate the expressions in Step 1 using the realized
parameter values in Step 2. Let \( \hat{g}_i = g_i(\hat{\theta}) \) be the eval-
uated quantities for company i at the realized parame-
ter values in the jth draw.

Step 4: Sort the \( \hat{g}_i \) values in ascending order for each activity
and locate the 25th and 975th values to obtain the lower-
and upper-confidence limits (\( \hat{\Sigma}_l, \hat{\Sigma}_u \)), respectively.

Step 5: If the actual expenditure on an activity lies within its
corresponding confidence limits (\( \hat{x}_l, \hat{x}_u \)), the invest-
ment decision for that activity is nearly optimal. If the
actual expenditure is below \( \hat{x}_l \) it represents under-
spending; if the actual expenditure is above \( \hat{x}_u \) it rep-
resents overspending.

To derive the expressions required in Step 1, we maximize
profit in Equation 3 subject to the demand system given in
Equations 4 and 5. The resulting optimal decisions are

\[
\begin{bmatrix}
q_i^* \\
d_i^* \\
a_i^*
\end{bmatrix} =
\begin{bmatrix}
\frac{m_{11}k_1a_2 + m_{22}a_1\beta_1}{1 - \alpha_1\beta_1} \\
\frac{m_{11}k_1a_2 + m_{22}a_1\beta_1}{1 - \alpha_1\beta_1} \\
\frac{m_{11}k_1a_2 + m_{22}a_1\beta_1}{1 - \alpha_1\beta_1}
\end{bmatrix} \times g_i(\theta).
\]

This five-step algorithm constitutes the diagnostic tool,
which determines a firm’s location on the profit function
and enables managers to steer their companies toward opti-
nality. To elucidate this point, consider a simplified situa-
tion in which a manager attempts to maximize profit (\( \pi \))
by changing investment levels in quality (Q) and distribution
(D). Figure 2, Panel A, locates Firm A on the profit function
\( \pi(Q, D) \), and Panel B shows its location in the decision

<table>
<thead>
<tr>
<th>Parameters</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>.07</td>
<td>.13</td>
<td>.12</td>
<td>.06</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>2.38</td>
<td>3.83</td>
<td>2.67</td>
<td>4.16</td>
</tr>
<tr>
<td>( E[\xi] )</td>
<td>.832</td>
<td>.842</td>
<td>.670</td>
<td>.744</td>
</tr>
<tr>
<td>Var[\xi]</td>
<td>.041</td>
<td>.057</td>
<td>.037</td>
<td>.021</td>
</tr>
<tr>
<td>t-value</td>
<td>-.83</td>
<td>-2.17</td>
<td>-1.72</td>
<td>-1.76</td>
</tr>
<tr>
<td>Market type</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Notes: S1 and S2 denote the small-firm and large-firm segments, respectively.
TABLE 7
Investment Elasticities

<table>
<thead>
<tr>
<th>Investment Elasticity</th>
<th>Expression</th>
<th>1998 S1</th>
<th>1998 S2</th>
<th>1999 S1</th>
<th>1999 S2</th>
<th>2000 S1</th>
<th>2000 S2</th>
<th>2001 S1</th>
<th>2001 S2</th>
<th>S1 Average</th>
<th>S2 Average</th>
<th>Overall Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subscription with respect to advertising sales, (\partial \ln(S) / \partial \ln(a))</td>
<td>(\frac{\alpha_1 \beta_2}{S(1 - \alpha_1 \beta_1)})</td>
<td>.10</td>
<td>.23</td>
<td>.14</td>
<td>.09</td>
<td>.18</td>
<td>.13</td>
<td>.19</td>
<td>.21</td>
<td>.15</td>
<td>.17</td>
<td>.16</td>
</tr>
<tr>
<td>Subscriptions with respect to distribution, (\partial \ln(S) / \partial \ln(d))</td>
<td>(\frac{\alpha_3}{S(1 - \alpha_3 \beta_3)})</td>
<td>.16</td>
<td>.30</td>
<td>.11</td>
<td>.32</td>
<td>.17</td>
<td>.23</td>
<td>.35</td>
<td>.20</td>
<td>.20</td>
<td>.26</td>
<td>.23</td>
</tr>
<tr>
<td>Subscriptions with respect to quality, (\partial \ln(S) / \partial \ln(q))</td>
<td>(\frac{\alpha_2}{S(1 - \alpha_2 \beta_2)})</td>
<td>.68</td>
<td>.33</td>
<td>.63</td>
<td>.48</td>
<td>.50</td>
<td>.40</td>
<td>.47</td>
<td>.39</td>
<td>.57</td>
<td>.40</td>
<td>.49</td>
</tr>
<tr>
<td>Ad revenue with respect to advertising sales, (\partial \ln(R) / \partial \ln(a))</td>
<td>(\frac{\beta_2}{R(1 - \alpha_1 \beta_1)})</td>
<td>.61</td>
<td>.54</td>
<td>.52</td>
<td>.41</td>
<td>.64</td>
<td>.38</td>
<td>.73</td>
<td>.50</td>
<td>.63</td>
<td>.46</td>
<td>.54</td>
</tr>
<tr>
<td>Ad revenue with respect to distribution, (\partial \ln(R) / \partial \ln(d))</td>
<td>(\frac{\alpha_3 \beta_1}{R(1 - \alpha_3 \beta_1)})</td>
<td>.19</td>
<td>.35</td>
<td>.13</td>
<td>.37</td>
<td>.19</td>
<td>.28</td>
<td>.36</td>
<td>.22</td>
<td>.22</td>
<td>.31</td>
<td>.26</td>
</tr>
<tr>
<td>Ad revenue with respect to quality, (\partial \ln(R) / \partial \ln(q))</td>
<td>(\frac{\alpha_2 \beta_1}{R(1 - \alpha_2 \beta_1)})</td>
<td>.76</td>
<td>.38</td>
<td>.76</td>
<td>.56</td>
<td>.53</td>
<td>.49</td>
<td>.49</td>
<td>.42</td>
<td>.64</td>
<td>.46</td>
<td>.55</td>
</tr>
</tbody>
</table>

Notes: S1 and S2 denote the small-firm and large-firm segments, respectively.
space \((Q, D)\). In practice, the manager does not “see” the firm on a profit function in the manner observed for Firm A in Panel A. Rather, as in Panel B, the manager knows only the invested amounts \((q_1, d_1)\) and the associated profit \(\pi_1\). In other words, the manager’s business reality is the decision space in Panel B. This decision space becomes three-dimensional (or higher) when additional decisions must be made (as in the empirical study). In such cases, without analytic tools, it is impossible to “see” the profit function. Even if the simplified situation in Panel B is considered, should the manager decrease \(Q\), increase \(D\), or try some combination, for example, that increases both \(Q\) and \(D\) (see Panel B)? Such decisions move the firm to another location \((q_2, d_2)\), which is shown by the dotted circle in Panel B, and this may or may not generate enhanced profit. Consequently, how should a manager’s decisions be assessed in terms of whether they brought the firm closer to the optimal?

The proposed diagnostic tool provides this crucial information. Figure 2, Panel C, illustrates how the diagnostic tool works: First, it projects Firm A from its current profit level into the decision space; second, it constructs isoprofit contours in the decision space; and third, it identifies the optimal path to achieve the maximum profit. The shaded region in Panel C denotes the confidence region. Although the confidence region provides useful information on over- or underspending, it does not offer guidance to a manager to drive the firm from its current location into the neighborhood of maximum profit. This guidance comes from the optimal path that traces a perpendicular to isoprofit con-
tours (see the curved arrows), which the diagnostic tool determines on the basis of the firm’s current location on the profit function.

We used GAUSS Version 7.0 to code the tool. Applying this diagnostic tool to companies \( i = 1, \ldots, N \) (in each of the two segments across the four years), we generated the following empirical results: Averaging across years, Table 8, Panel A, displays the proportions of firms in each of the two segments that are located on the uphill side of the profit function (Type U for underspending), the downhill side (Type D for overspending), or near optimal (Type N) with respect to their current levels of investments in news quality, distribution, and advertising sales efforts. Table 8, Panel B, presents the year-by-year proportions of Type U, Type D, or Type N firms with respect to each investment variable in the two segments over the 1998–2001 period. (Note that when firms invest in multiple marketing activities, such as in our study, the typology of uphill, downhill, and near optimal refers to a particular investment decision by that company. That is, in practice, the same company can be underspending in quality, overspending in distribution, near its optimum spending on advertising, or some other combination.)

Focusing on Table 8, Panel A, we observe that the majority of the newspapers across the four years—more than 60% in the case of the small-firm segment and more than 80% in the large-firm segment—are Type N with respect to all three \((q, d, a)\) investments. The higher propor-

### Table 8

#### Proportions of Firms Underinvesting, Nearly Optimal, or Overinvesting

**A: Average Across Years**

<table>
<thead>
<tr>
<th>Segment 1 (Small Firm)</th>
<th>Type U: Underinvesting</th>
<th>Type N: Nearly Optimal</th>
<th>Type D: Overinvesting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality</td>
<td>22.1</td>
<td>68.7</td>
<td>9.2</td>
</tr>
<tr>
<td>Distribution</td>
<td>12.5</td>
<td>68.5</td>
<td>19.0</td>
</tr>
<tr>
<td>Ad sales</td>
<td>33.3</td>
<td>62.8</td>
<td>3.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Segment 2 (Large Firm)</th>
<th>Type U: Underinvesting</th>
<th>Type N: Nearly Optimal</th>
<th>Type D: Overinvesting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality</td>
<td>14.8</td>
<td>82.3</td>
<td>2.9</td>
</tr>
<tr>
<td>Distribution</td>
<td>11.5</td>
<td>85.4</td>
<td>3.2</td>
</tr>
<tr>
<td>Ad sales</td>
<td>14.7</td>
<td>84.0</td>
<td>1.3</td>
</tr>
</tbody>
</table>

**B: Year by Year**

<table>
<thead>
<tr>
<th>Investments</th>
<th>Segment 1 (Small Firm)</th>
<th>Segment 2 (Large Firm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type U</td>
<td>Type N</td>
</tr>
<tr>
<td>1998</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quality</td>
<td>28.5</td>
<td>71.4</td>
</tr>
<tr>
<td>Distribution</td>
<td>.7</td>
<td>80.7</td>
</tr>
<tr>
<td>Ad sales</td>
<td>1.5</td>
<td>80</td>
</tr>
</tbody>
</table>

| 1999        |        |        |        |        |        |        |
| Quality     | 30.6   | 65.9   | 3.5    | 55.5   | 42.6   | 1.9    |
| Distribution| .6     | 74.1   | 25.3   | 44.5   | 54.7   | .63    |
| Ad sales    | 17.1   | 77.1   | 5.8    | 26.1   | 71.3   | 2.5    |

| 2000        |        |        |        |        |        |        |
| Quality     | 8.2    | 84.4   | 6.3    | 1.5    | 97.6   | .9     |
| Distribution| .2     | 64.5   | 35.3   | .7     | 92.9   | 6.7    |
| Ad sales    | 29.7   | 67.08  | 3.16   | 6.2    | 92.9   | .8     |

| 2001        |        |        |        |        |        |        |
| Quality     | 6.7    | 71.3   | 21.8   | 1.2    | 95.2   | 3.6    |
| Distribution| 23.4   | 58.8   | 17.7   | 1.2    | 94.1   | 4.7    |
| Ad sales    | 51.5   | 45.3   | 3.2    | 24.7   | 74.1   | 1.2    |
tion of Type N firms in the large-firm segment implies that they may possess greater expertise in gauging marketing—sales response relationships. Next, when firms are suboptimal with respect to quality investments, they tend to be underspending rather than overspending. This tendency is more pronounced in the case of small-firm newspapers. Similarly, we observe that in both segments, the majority of suboptimal firms are underspending with respect to their advertising sales investments—specifically, 33.3% of the small-firm segment and approximately 14.7% of the large-firm segment. In contrast, in the case of distribution investments, we find that a greater proportion of suboptimal firms in the small-firm segment overspends rather than underspends. This finding reinforces our previous inference that small-firm’s circulation—distribution investments are closer to the market saturation levels.

Table 8, Panel B, indicates that these patterns of results hold across the years except for 1998, during which a significant proportion of suboptimal firms overinvested in ad space sales efforts. This outcome may have arisen because some firms mistakenly assumed that they were operating in Market Type 4 when the market was actually Type 2 for the small-firm segment. We also observe that significantly greater proportions of companies in the large-firm segment were suboptimal (underinvesting) in all their marketing investments in 1999, which may be attributable to fears of an economic slowdown and recession-like conditions in 1999. In contrast, greater proportions of firms in both segments appear to have overinvested in news quality in 2001.

Summarizing the findings in Table 8, we observe that the two segments differ in their spending tendencies as follows:

<table>
<thead>
<tr>
<th>Investment</th>
<th>Small Firm</th>
<th>Large Firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underspenders in quality</td>
<td>More likely</td>
<td>Less likely</td>
</tr>
<tr>
<td>Underspenders in distribution</td>
<td>Less likely</td>
<td>More likely</td>
</tr>
<tr>
<td>Underspenders in advertising</td>
<td>More likely</td>
<td>Less likely</td>
</tr>
</tbody>
</table>

These findings—that a majority of the newspapers in both the segments are Type N with respect to all three investment variables, and greater proportions of suboptimal firms with respect to quality and advertising sales investments are Type U than Type D—run counter to previous research results that characterize marketing managers as overspenders (e.g., Aaker and Carman 1982; Hanssens, Parsons, and Schultz 2001, p. 363; Prasad and Sen 1999). However, they lend support to and comport with the principles of economic theory that expect firms to behave optimally, especially in a 100-year-old newspaper industry in which many suboptimal firms would have disappeared. Moreover, the specific finding that the majority of newspapers are either nearly optimal or underspending with respect to quality substantiates Meyer and Kim’s (2005, p. 7) conjecture that these companies—especially the smaller ones—are “clustered on the uphill” of the profit function with respect to news quality investments. Finally, the changing proportions of Type N firms across the years reveal that managers would find it difficult to know their companies’ location on the profit function in the absence of the proposed diagnostic tool. Consequently, they risk making the serious errors in investment decisions we described in the beginning of this article, which, as industry analysts fear, would essentially lead to liquidating their firms (e.g., Rosenstiel and Mitchell 2004). The application of our proposed algorithm to locate their firms’ positions on the profit function would assist them in making appropriate investment decisions (i.e., increase, decrease, or maintain the status quo) and thus steer their companies to regions of enhanced profitability.

**Implications of Omission of Advertising Space Sales Carryover Effects**

As we noted previously, to preserve the confidentiality of the data of reporting newspaper companies, the annual Inland data sets do not identify the individual newspapers in the samples or use a common case identification number for any newspaper company that reports every year. Indeed, the sizes (and, therefore, the composition) of the annual samples vary across the years, eliminating the possibility of longitudinal analyses and restricting us to annual cross-sectional analyses. Consequently, the proposed simultaneous equation response model does not explicitly allow for ad space sales carryover effects. However, despite this limitation, our research findings with respect to the effects and optimality of newspaper firms’ ad space sales efforts remain meaningful for two reasons.

First, local newspapers’ ad space sales carryover effect is likely to be weak. More specifically, the following characteristics of local newspapers’ ad space selling (see, e.g., Niemesh 2004) suggest that these efforts have a high short-term impact but low carryover effects (e.g., Zoltner, Sinha, and Zoltner 2001, p. 75): (1) A high proportion of sales in each year arises from new, incremental business from many new customers with small purchase volumes and short selling cycles (typically three contacts); (2) the sales force is the primary and most important promotion vehicle; (3) the sales job calls for limited maintenance or service activity beyond overseeing the execution of a particular advertising project (ad space salespeople usually work with numerous clients and projects simultaneously); and (4) more than half of ad sales representatives’ pay comes from variable pay (i.e., commissions and bonuses for meeting monthly sales quotas), suggesting a stronger and more immediate selling effort—sales linkage (Basu et al. 1985; Niemesh 2004).

Second, allowance for a significant sales effort carryover effect would not affect the directionality of either the related normative results in Table 1 or the empirical findings in Table 8, because the presence of this effect implies a larger profit-maximizing investment level than the short-term optimum. Indeed, our conclusion that newspaper firms tend to underspend with respect to the ad space sales effort would be reinforced. However, a worthwhile direction for further research would be to investigate the long-term effects of marketing investments in dual-revenue industries, such as daily newspapers, using appropriate longitudinal data when available.
Conclusions

Based on our analysis of the Inland Press Association data, the four key takeaways from this article are as follows: First, daily newspapers’ dual revenues are interrelated with positive feedback effects; this is an assumption that several theoretical models make, but to date, it has been backed by limited empirical evidence. Second, investments in news quality affect not only subscription sales directly but also advertising revenues through subscriptions indirectly. This result is especially true for smaller-circulation newspapers whose newsrooms and editorial departments tend to be understaffed and overworked. Consequently, our answer to the question “Is good news quality good business?” is a resounding yes. Third, advertising revenues of both small and large firms respond positively to efforts to boost circulation directly (e.g., investment in news quality) as much as they respond to ad space sales effort. Fourth, investments in circulation–distribution also have significant direct and indirect effects on subscriptions and advertising revenues, respectively, though the strengths of these effects are not as great as those of news quality investments.

Collectively, these findings reinforce the value of marketing by establishing that marketing investments influence a firm’s marketplace performance. More important, managers should recognize the assets that these investments build (see Rust et al. 2004). As we already mentioned, the NAA (2000) notes that delivery systems built by investments in circulation–distribution are significant, strategic assets. Similarly, investments in news quality and advertising sales enhance perceived quality and awareness, thus building brand equity, an invaluable marketing asset (see Keller 1998).

In general, this article examines and derives normative marketing-mix decision rules in dual-revenue markets with interrelated demands, thus generalizing Dorfman and Steiner’s (1954) classic theorem and enriching the understanding of resource allocation across four different market types. Furthermore, the empirical analysis of the impact of marketing investments yields elasticity estimates, thus augmenting the sparse literature on quality, distribution, and personal selling effort elasticities.

Finally, as the title states, a key contribution of this article is the diagnostic tool, which is based on a simple five-step algorithm, that extracts information contained in the market data (on the responsiveness of readers and advertisers and their interrelated demands) and combines this information with the firm’s knowledge of margins not only to recommend appropriate investments in quality, distribution, and advertising but also to identify an individual firm’s location on the profit function to mitigate under- or over-spending errors. In addition, it offers guidance by tracing the optimal path that drives the company from its current location to the neighborhood of the maximum profit (see Figure 2, Panel C). Recently, Sir Martin Sorrell, the chief executive of the world’s largest ad agency, WPP Group, stated in a Wall Street Journal interview that “scientific analysis, including econometrics, is one of the most important areas in the marketing-services industry” (Patrick 2005, B8). The diagnostic tool we developed belongs to this genre of sophisticated econometric approaches. We hope that managers find it useful to enhance their firms’ profitability.

Appendix A

Derivation of the Optimal Investments in Quality, Distribution, and Advertising

Let \((q^*, d^*, a^*)\) denote the three optimal investment levels, which exist when the profit function in Equation 3 is twice differentiable and its Hessian matrix is negatively definite over the set \(S\), where \(S = \{(q, d, a) \in \mathbb{R}^3 | q > 0, d > 0, a > 0\}\). Furthermore, let \(\pi_{x}\) denote the partial derivative of profit with respect to variable \(x\) \((x = q, d, a)\). The first-order conditions that maximize profit in Equation 3 are as follows:

\[
\begin{align*}
\pi_q &= k m_1 m_2 - 1 = 0, \\
\pi_d &= k m_1 m_2 - 1 = 0, \\
\pi_a &= k m_1 m_2 - 1 = 0.
\end{align*}
\]

To derive the optimal investment levels, consider the marginal effect of quality investments on subscriptions. Formally,

\[
\frac{\partial S}{\partial q} = \frac{\partial f_q}{\partial q} + \frac{\partial f_q}{\partial R} \frac{\partial R}{\partial q} = \frac{1}{\delta} \frac{\partial f_q}{\partial R} - \frac{1}{\delta} \frac{\partial f_q}{\partial R} \frac{\partial R}{\partial q},
\]

where \(\delta = 1 - \frac{\partial f_q}{\partial R} \frac{\partial R}{\partial q}\).

Similarly, the marginal effects of distribution and advertising selling investments on subscriptions are

\[
\begin{align*}
\frac{\partial S}{\partial d} &= \frac{1}{\delta} \frac{\partial f_q}{\partial R} \frac{\partial R}{\partial d}, \\
\frac{\partial S}{\partial a} &= \frac{1}{\delta} \frac{\partial f_q}{\partial R} \frac{\partial R}{\partial a}.
\end{align*}
\]

respectively.

Next, the marginal effect of quality investments on advertising revenue is given by

\[
\frac{\partial R}{\partial q} = \frac{\partial f_q}{\partial q} + \frac{\partial f_q}{\partial S} \frac{\partial S}{\partial q} = \frac{1}{\delta} \frac{\partial f_q}{\partial R} \frac{\partial R}{\partial q}.
\]

Similarly, the marginal effects of distribution and advertising selling investments on advertising revenue are given by

\[
\begin{align*}
\frac{\partial R}{\partial d} &= \frac{1}{\delta} \frac{\partial f_q}{\partial R} \frac{\partial R}{\partial d}, \\
\frac{\partial R}{\partial a} &= \frac{1}{\delta} \frac{\partial f_q}{\partial R} \frac{\partial R}{\partial a}.
\end{align*}
\]

Finally, by substituting Equations A4–A9 in Equations A1, A2, and A3, we obtain the simplified first-order conditions:
Appendix B
Mean and Variance of the Cross-Market Dependency Coefficient \( \delta \)

We attempt to find the mean and variance of the random variable \( \delta = 1 - \alpha \beta \), where \((\alpha, \beta)^T \sim N(\mu, \Sigma)\), \(\mu = (\bar{\alpha}, \bar{\beta})\), and \(\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}\).

To find the mean, we use \(\operatorname{Cov}(\alpha, \beta) = \operatorname{E}(\alpha \beta) - \bar{\alpha} \bar{\beta}\) to obtain

\[
\operatorname{E}(\delta) = 1 - \operatorname{E}(\alpha \beta) = 1 - \bar{\alpha} \bar{\beta} - \sigma_{12}.
\]

To find the variance of \(\delta\), we use a theorem from the work of Mathai and Provost (1992, p. 53) that states that

\[
\operatorname{Var}(X'AX) = 2\sigma^2(\Sigma)^2 + 4\sigma'\Sigma A\mu,
\]

where \(X\) follows a multivariate normal distribution with mean \(\mu\) and covariance \(\Sigma\) and \(A\) is a conformable symmetric matrix.

Next, we chose

\[
A = \begin{bmatrix} 0 & .5 \\ .5 & 0 \end{bmatrix}
\]

and \(X = (\alpha, \beta)^T\) so that \(\operatorname{Var}(X'AX) = \operatorname{Var}(\alpha \beta)\). Then, we simplify

\[
(\Sigma\Sigma)^2 = (\Sigma\Sigma)(\Sigma\Sigma) = \begin{bmatrix} \sigma_{12}^2 + \sigma_1^2 \sigma_2^2 & 2\sigma_{12} \sigma_1^2 \\ 2\sigma_{12} \sigma_1^2 & \sigma_{12}^2 + \sigma_1^2 \sigma_2^2 \end{bmatrix},
\]

which implies that

\[
2\sigma(\Sigma\Sigma)^2 = \sigma_{12}^2 + \sigma_{12}^2 \sigma_2^2.
\]

Similarly, it can be shown that

\[
4\sigma'\Sigma A\mu = \sigma_1^2\sigma_2^2 + 2\bar{\alpha} \bar{\beta} \sigma_{12}^2 + \bar{\beta}^2 \sigma_1^2.
\]

Finally, we combine the results of Equations B2, B3, and B4 to observe that

\[
\operatorname{Var}(\delta) = \operatorname{Var}(1 - \alpha \beta) = \operatorname{Var}(\alpha \beta) = (\sigma_{12}^2 + \sigma_1^2 \sigma_2^2) + (\bar{\alpha}^2 \sigma_1^2 + 2\bar{\alpha} \bar{\beta} \sigma_{12}^2 + \bar{\beta}^2 \sigma_1^2).
\]

thus furnishing the exact expression for the variance of cross-market dependency coefficient. This completes the proof.

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