1) (45 Points total) We have two consumers in a make believe economy, Dagwood, the saver and Homer, the borrower. Let us start with the initial conditions for Dagwood:

His current income is $200K and his expected income is $60K (he is retiring!). Dagwood's current wealth is $100K. The interest rate that Dagwood initially faces is a negative 5% = (-0.05). Let us assume, as usual, that Dagwood abides by the life cycle theory of consumption... that is, he is a perfect smoother so that he optimizes when his consumption is equal in both periods.

a) (5 points) Calculate Dagwood's optimal consumption bundle showing all work. Then draw a completely labeled graph (the two period consumption model) depicting this initial optimal consumption bundle as point C*.

\[ C^* = \frac{(1+r)(C_{t+1} + C_{t})}{1+r} + \frac{e}{1+r} \cdot \frac{(1+0.05)(200+100) + 50}{1+r} = 176.92 \]

\[ C_f = 345 - 0.95c \]

(10 points for a completely labeled graph – be sure to label the no lending / no borrowing point(s) = NL/NB and the slope of the budget constraint(s))
Homer, on the other hand is the spender and prefers to consume twice as much in the current period relative to the future period. Homer has $100K in current income and zero in current wealth. Homer expects his wealth to be $200K in the future (He expects to receive an nice inheritance). His expected income is $100K, same as his current income. He faces the same initial real interest rate as Dagwood = -0.05 (negative 5%).

b) (5 points) Calculate Homer's optimal consumption bundle showing all work. Then draw a completely labeled graph (the two period consumption model) depicting this initial optimal consumption bundle as point $C^*$. 

$$c_f = \frac{(1.05)(100,000) + 0}{3.2} = \frac{0.95 \times 100,000 + 0 + 100}{3.2 - 2(0.05)} = \frac{955}{3} \approx 272.41$$

(10 points for a completely labeled graph – be sure to label the no lending / no borrowing point(s) = NL/NB and the slope of the budget constraint(s))
1c) (5 Points) Now the Federal Open Market Committee (FOMC) and is worried about the economy overheating and decides to raise the real rate of interest to a positive 5%. Resolve for Dagwood's optimal consumption and label on your diagram C*.

\[ r = 0.05 \]
\[ c^* = (11.05)(300) + 160 - 1.05c \]
\[ c = 182.93 \]

1d) (5 points) Given the new and higher real interest rate = 0.05 (5%), resolve for Homer's optimal bundle.

\[ c^* = (11.05 \times 100) + 300 - 1.05c = 405 - 1.05c \]
\[ c = 201.29 \]
\[ c^* = 178.16 \]

1e) (10 points) Given that consumption accounts for 70% of the US economy, did the policy of raising the real rate of interest 'cool down' this economy made up of 1 Dagwood and 1 Homer (i.e., what happened to total consumption as a result of the higher \( r \))? Why or why not? Explain, making sure you address how the substitution and income effects play in role in the decisions of both Dagwood and Homer. Note, this discussion is worth 10 points.

Consumption of Dagwood went from 182.92 to 182.93, so there was an increase of 0.01 units consumed whereas Homer decreased consumption from 272.41 to 261.29 which was a decrease of 11.12 units consumed. Thus, total consumption decreased by 11.12 - 0.01 = 5.11 units given the economy is made of 1 Homer and 1 Dagwood and the economy slowed down. According to the substitution effect, Dagwood and Homer would both decrease consumption because the cost of consuming today went up when \( r \). On the other hand, for Dagwood who is concerned with the future value of current resources (\( \frac{\text{IT}(Y+\alpha)}{1+r} \)), the income effect would cause him to consume more because \( r \). In this case, the income effect trumps the substitution effect. With Homer who is concerned about the current value of future resources \( \frac{\text{cT} + \text{c}\alpha}{1+r} \), the income effect would have him consume less because \( r \) making him feel less wealthy.
2. **(45 points total)** You own a fleet of offshore fishing boats and you need to determine how many fishing poles you need to buy to maximize profits. Please answer the following questions given the information below. Please be sure to SHOW all work!

A brand new fishing pole costs 1000 fishing hours (this is your output) and the rate of depreciation is 10% (0.10).

The real interest rate is 10% (.10).

And the expected marginal product of capital is given by $\text{MPK'} = 1000 - 5K$.

The firm also faces an effective tax rate on capital, what we call $\tau = 20% (.20)$

a) What is the (tax adjusted) user cost of capital and what is this user cost expressed in?
   (Show work)
   
   \[
   \text{UC} = \frac{(r + \tau)(1 - 1/t)Pt}{1 - t} = \frac{0.2 \times 1000}{1 - 0.2} = \frac{2 \times 1000}{0.8} = 250 \text{ fishing hours}
   \]

b) How many fishing poles should you buy to maximize profits? Show work

   \[
   \text{UC} = \text{MPK'}
   \]

   \[
   250 = 1000 - 5K
   \]

   \[
   5K = 750
   \]

   \[
   K^* = 150
   \]

Draw a uc/K graph depicting the state of affairs and label this initial profit maximizing condition as point A.

A correctly drawn and completely labeled diagram is worth 10 points
c) Now conditions change as the value of the US dollar appreciates and since you purchase your fishing poles from abroad, the real price of brand new fishing poles falls to 800 hours of fishing. Resolve for $K^*$ and show as point B on your uc/K diagram.

\[ P_k = 800 \]

\[ UC = \frac{2(800)}{.5} = 200 \]

\[ UC = MPK^f \]

\[ 200 = 1000 - 5K \]

\[ 5K = 800 \]

\[ K^* = 160 \]

\[ \text{Because...} \]

d) Explain the intuition underlying the change in the profit maximizing level of fishing poles (i.e., why does the firm change its behavior?), making sure you refer to the firm’s profit maximizing condition (write it out!). Be specific and write this like you were a professional economist! (use some jargon!). The **firm’s profit-maximizing condition** is $UC = MPK^f$. Thus, when $P_k$ decreased, it caused the $UC$ to decrease. At the same level of $K$ at point $A$, the $UC < MPK^f$ now violating the profit-maximizing condition. Thus, the firm increases its $K$ until the $UC = MPK^f$.

Now suppose we experience two more changes simultaneously (in addition to the change in the real price of fishing poles above). In particular, you change your expectations on the marginal productivity of capital, since you feel that we are finally out of these bad economic times and as a result, you expect more customers and thus, your fishing poles will be used more intensely. Your new expected marginal product of capital function is: $MPK^f = 1070 - 5K$. The second change, again, assume these changes are occurring simultaneously after the change in the real price of fishing poles above, is that the local government that has been having trouble making ends meet, raises the effective tax rate on capital to 50% (it was originally 20%).

f) Resolve for the profit maximizing level of fishing poles given these two (simultaneous) additional changes and add this point to your diagram as point C.(show work)

\[ UC = \frac{(r + d)(1 - TC)P_k}{1 - t} \]

\[ UC = MPK^f \]

\[ 320 = 1070 - 5K \]

\[ 5K = 750 \]

\[ K^* = 150 \]
g) Finally, draw a desired investment diagram (completely labeled with the relevant shift variables noted next to the function in parentheses with actual numbers) depicting the initial equilibrium as point A (simply draw a negatively sloped $I^D$ curve going through point A). Label the initial level of desired investment as $I^d_A$. Note importantly that we do not have numbers for desired investment, but that's ok, we are focusing on the change in desired investment. Then show, as point B, the new level of desired investment as $I^d_B$. Finally, show how the change in MPK and higher $\tau$ maps to your investment diagram and label as point C with the corresponding level of investment labeled as $I^d_C$.

A completely labeled and correct diagram is worth 10 points (make sure you include the relevant shift variables in parentheses or points will be taken off).
3. PART 1 (35 points total for this part) This problem is broken into two parts that are totally connected to each other. In this first part of the question, you apply Chapter 3 (labor mkt., etc) material and in PART 2, you get to use Chapter 4 (goods market equilibrium) material. Please take all calculations to two decimal places where appropriate except with real interest rate calculations (PART 2), where you need to take the calculation to three decimal places, if appropriate. PLEASE SHOW ALL WORK AND COMPLETELY LABEL ALL DIAGRAMS.

The following equations characterize a country's closed economy.

Production function: \( Y = A \cdot K \cdot N - N^2 / 2 \)
Marginal product of labor: \( MPN = A \cdot K - N \).

where the initial values of \( A = 8 \) and \( K = 10 \).

The initial labor supply curve is given as: \( N_S = 20 + 9w \).

a) (10 points) Find the equilibrium levels of the real wage, employment and output (show work).

\[
Y = 8(10)N - N^2 / 2 = 80N - N^2 / 2
\]
\[
MPN = 80 - N
\]
\[
MPN = 80 - (20 + 9w) = w
\]
\[
60 - 9w = w
\]
\[
60 = 10w
\]
\[
6 = w
\]
\[
N_S = 20 + 9w = 20 + 9(6) = 74
\]
\[
Y = 80N - N^2 / 2 = 80(74) - (74)^2 / 2 = 3182
\]
\[
(Y = 3182)
\]
\[
(N_S = 74)
\]
In the space below, draw two diagrams vertically with the labor market on the bottom graph and the production function on the top graph. Be sure to label everything including these initial equilibrium points as point A.

![Diagram of labor market and production function with labeled points A and A']

(10 points for completely labeled and correct diagrams)

We now have numerous changes to our economic conditions (all is not constant). Think of all these changes happening together, that is, we go from one state of economic affairs to a different state of economic affairs. Below are the changes.

- The labor supply changes and is now: $N^s = 30 + 9w$.
- A goes up from 8 to 9.

b) (5 points) What could cause such a change in labor supply? Please give two specific and well supported reasons.

The labor supply shifted to the right so this could be due to a decrease in future income so people are more willing to work or a decrease in wealth which would also cause people to be more willing to work causing a shift right in $N^s$. If people expect to make less in the future or expect to have less wealth, they are going to work more.
c) (10 points) Given the change in $N^s$ and $r$, repeat part a) (i.e., find the equilibrium levels of the real wage, employment and output). Add these results to your labor market and production function diagrams respectively and label as point(s) B. Be sure to label the diagram completely with the relevant shift variables in parentheses next to the function.

\[ N^s = 30 + 9w \quad y = 90 + N^s \]
\[ MPN = 90 - N \]
\[ MPN = 90 - (30 + 9w) = 60 - 9w = w \]
\[ 60 - 9w = 0 \quad w = \frac{60}{9} \]
\[ N^s = 30 + 9 \left( \frac{60}{9} \right) = 84 \]
\[ y = 90 \left( \frac{84}{2} \right) = 4,032 \]

\[ N^s = 84 \]
\[ y = 4,032 \]

3. PART 2 (NEW GRADER – 45 points total for PART 2)

Before we start this problem, put the initial $Y$ as computed in part a) here \[ 3182 \]
And the new $Y$ (after the change in conditions) here \[ 4032 \]

Initial conditions in the goods market

\[ C^d = 1001 + .50(Y-T) - 500r \quad C^d = 291.2 \quad C^d = 2949.5 \]
\[ I^d = 600 - 500r \]
\[ G = 100 \]
\[ T = 100 \]

d) (10 points) Given the initial conditions, solve for the equilibrium real rate of interest (that clears the goods market) and the associated levels of desired savings and desired investment.

\[ Y = C + I + G \quad S^d = Y - C - L = I^d = 100 - 500r \]
\[ Y - C - L = 3182 - 100 - .5 \left( 3182 - 100 \right) + 500r - 100 = 1000 - 500r \]
\[ 540 + 500r = 1000 - 500r \]
\[ 1000r = 460 \quad r = 0.46 \]
\[ S^d = I^d = 1000 - 500 \left( \frac{60}{100} \right) = 570 \]
\[ S^d = I^d = 570 \]
Draw a $S^d = I^d$ diagram in the space below locating this initial equilibrium as point A.

10 points for correct and completely labeled diagram (be sure to put relevant shift variables in parentheses next to each function).

NOW WE TAKE INTO ACCOUNT THE CHANGES FROM PART 1 ALONG WITH A CHANGE IN DESIRED INVESTMENT.

- The desired investment function changes and is now $I^d = 1000 - 500r$

e) (5 points) What could cause such a change in the desired investment function? Please give three specific and well supported reasons.

1) Taxes could decrease, causing user costs to go down and $I^d$ to increase, which would shift the investment function outward.

2) Total factor of productivity $(A)$ could increase, causing the firm to be more productive and as a result increase investment.

3) The depreciation of the firm’s capital could decrease, causing the user cost to decrease and $I^d$ to increase, thus causing the firm to invest more.
f) (10 points) Given these changes (i.e., changes in A, Y, and \( l^d \), note, the new \( l^d = 1000 - 500r \)), calculate the new equilibrium levels of the real interest rate, desired savings and investment. Please add this new equilibrium point to your diagram and label as point B.

\[
s_d = Y - C = 4032 - \left\{ 1901 + 0.5(1032 - 1000) - 500r \right\} - 100 = 905 + 500r
\]

\[
s_d = I^d \Rightarrow 905 + 500r = 1000 - 500r
\]

\[
\frac{1000}{1000} - \frac{500}{500} = \frac{1}{10}
\]

\[
r = 0.08
\]

\[
s_d = 905 + 500(0.08) = 982.5
\]

\[
I^d = 982.5
\]

g) (10 points - 5 points for discussion and 5 points for correct and completely labeled diagram) Now it is time to apply your knowledge of the 2 period consumption model to this problem. In the space below, draw and use the two period consumption model and depict the movement from point A to point B. Assume that the consumer is a borrower and a perfect smoother, as we typically do. Be sure to calculate consumption before (point A) and after (point B) and add these actual number to the graph. Make sure you explain two specific reasons why consumption has changed (hint - the substitution effect should be in one of your reasons!). Make sure you label your graph completely.

In this economy, there are more borrowers than savers because the saver would consume less due to the \( \Delta r \) which is below. It feels less wealthy and so consumes less. However, the substitution effect for the borrower tells him to consume more bc when \( \Delta r \) the price to consume today went down. Also, the income effect for the borrower, cut off with the present value of future resources \( \frac{Y_{t+1}}{Y_t} \) with \( r_b \), tells him to consume more bc he feels wealthier.
4. (50 points total) We assume that the world consists of two large open economies, USA and China.

**USA Initial Conditions**

\[ C_d = 400 + 0.4(Y - T) - 200r^w \]
\[ I_d = 250 - 200r^w \]
\[ Y = 1200 \]
\[ T = 200 \]
\[ G = 325 \]

**China Initial Conditions**

\[ C_{dF} = 480 + 0.4(Y_T - T_F) - 300r^w \]
\[ I_{dF} = 225 - 300r^w \]
\[ Y_F = 1600 \]
\[ T_F = 400 \]
\[ G_F = 300 \]

a) What is the equilibrium interest rate that clears the international goods market? Show all work (10 points).

**USA:**
\[ C_d = 400 + 0.4(1200 - 200) - 200r^w = 800 - 200r^w \]
\[ S = Y - C - G = 1200 - 800 + 200r^w - 325 = 75 + 200r^w \]
\[ NX = S - I = 75 + 200r^w - 250 + 200r^w = -175 + 400r^w \]

**China:**
\[ C_{dF} = 480 + 0.4(1600 - 400) - 300r^w = 960 - 300r^w \]
\[ S = Y_F - C_F = 1600 - 960 + 300r^w - 325 = 340 + 300r^w \]
\[ NX = S - I = 340 + 300r^w - 225 + 300r^w = 115 + 600r^w \]

\[ NX_{US} + NX_{CH} = 0 \]
\[ -175 + 400r^w + 115 + 600r^w = 0 \]
\[ \frac{1000r^w}{1000} = 0.1 \]
\[ r^w = 0.10 \]
b) Now calculate the levels of desired savings and investment for each country at this equilibrium world real interest rate (5 points).

**US:**
- Desired Savings: 
  \[ S = 75 + 200r_w = 75 + 200(0.06) = 87 \]
- Desired Investment: 
  \[ I = 250 - 200r_w = 250 - 200(0.06) = 238 \]

**China:**
- Desired Savings: 
  \[ S_f = 340 + 300r_w = 340 + 300(0.06) = 358 \]
- Desired Investment: 
  \[ I_f = 225 - 300r_w = 225 - 300(0.06) = 207 \]

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c) Which country is 'spending beyond its means' and which country is the saver? What exactly do we mean by the phrase 'spending beyond its means' in this context. Be sure to **define** and use the word absorption in your answer and compare the level of absorption in each country to its income. Please show all work. Explain (10 points).

**US:**
\[ \text{Absorption} = C + I + G = 800 - 200(0.06) + 238 + 375 = 1351 \]

For **US**: Absorption = 1351 > 1200 = Y

**China:**
\[ \text{Absorption} = C_f + I_f + G_f = 900 + 300(0.06) + 207 + 300 = 1449 \]

For **China**: Absorption = 1449 < 1600 = Y_f

Absorption by definition is consumption plus investment plus government spending. So, since the US's absorption > Y, the US is spending beyond their means and having China finance their trade deficit of 1351 - 1200 = 151. China, on the other hand, is the saver because their absorption = 1449 < 1600 = Y_f (absorption < Y). Thus, China will have a trade surplus of 1600 - 1449 = 151.
Draw two diagrams side by side, with the US on the left and the China on right. Locate this initial equilibrium as points A on both diagrams (there are four point A's, two on each diagram). Be sure to label diagram completely.

10 points for correct and completely labeled diagrams

Now China experiences a positive productivity shock. As a result, China's output rises to 1650.

d) (10 points) Resolve for the world real interest rate that clears the international goods markets along with the 'new' \( S_d \) and \( I_d \) for each country and add these results to your diagram labeling this new equilibrium as points B (there are four of them!). (10 points)

\[
\text{US: } \begin{align*}
&S_d = 480 + 1.4 (1550 - 410) - 300r = 980 - 300r \\
&I_d = 75 + 200 \ (1.03) = 214 \\
&I = 250 - 200 \ (1.03) = 244
\end{align*}
\]

\[
\text{China: } \begin{align*}
&S_d = 480 + 1.4 (1550 - 410) - 300r = 980 - 300r \\
&I_d = 75 + 200 \ (1.03) = 81 \\
&I = 250 - 200 \ (1.03) = 244
\end{align*}
\]
e) (5 points) Now comment on what has happened to the trade balance for each country and relate to the movie clip from Colbert about spending beyond our means. Recall that Fareed Zakaria (the guest) suggested that we (the US) needed to go to alcoholics anonymous (AA). Are your results consistent with the US going to AA? Why or why not? Explain and please be specific.

Due to the expansion of China's economy, it caused interest rates to go down. As a result, because neither investment nor savings changed for the US, it caused the trade deficit to increase while at the same time causing China's trade surplus to increase. Thus, China is financing more of the US's investment. Before the expansion of China, US's absorption was $1251$ but since $r_f$, US's absorption changed to $C+I+G = 809-200 (103)+241+325 = 1303$. While $Y=1200$ remains constant. Now, the US is absorbing even more and thus should go to AA because they are spending beyond their means and having China finance it. Therefore, the results are consistent with Fareed Zakaria's suggestion about the US going to AA.