

Variables

Definition:

Variable – A variable is a symbol (usually a letter) that represents some unknown quantity.

Theory:

Consider the following example. There is a store that sells washing machines and dryers. If we want to write a mathematical expression for the cost of buying a washing machine and dryer we are not able to use numbers because we do not know how much a washing machine or a dryer would cost. We might be able to write something like this:

$$\text{Total Cost} = \text{Cost of Washing Machine} + \text{Cost of Dryer}$$

In the expression above, *Cost of Washing Machine* and *Cost of Dryer* are variables. They are representations of a quantity that we do not yet know. This however, is very cumbersome to work with. To combat this problem, mathematicians use letters to describe variables and thus simplify their work.

Mathematician's Approach:

Step 1: Declare variables

Let W equal the cost of a washing machine and D equal the cost of the dryer

Step 2: Rewrite the expression for total cost with the new variables

$$\text{Total Cost} = W + D$$

Now the variable expression above is clean and easy to work with as we will discover in the next section. If we later found out that the washing machine costs \$450 and the dryer costs \$280 we could find the total cost by plugging \$450 in for W and \$280 in for D to get:

$$\text{Total Cost} = \$450 + \$280 = \$730$$

Notation:

Multiplication

When writing variable expressions mathematicians will usually omit writing the multiplication. For example if x and y are variables, the expression

$$2 \cdot x - x \cdot y$$

would be written as

$$2x - xy$$

Division

When writing variable expressions mathematicians will usually replace the division sign with a fraction bar. For example if x and y are variables, the expression

$$3 \div x + y \div 4 - x \div y$$

would be written as

$$\frac{3}{x} + \frac{y}{4} - \frac{x}{y}$$

Exponents

Recall that we use exponents to write products of repeated factors. For example: $4 \cdot 4 \cdot 4 = 4^3$. So when we have repeated multiplications of variables we will usually write them in exponential form. For example, if x is a variable then:

$$x \cdot x = x^2$$