Simplifying Rational Expressions

In this section we will learn how to simplify rational expressions. Rational expressions are expressions that are a ratio of two polynomials. For example, the expression below is a rational expression because both the numerator and the denominator are polynomial expressions.

$$\frac{x^2 - 3x + 7}{2x + 15}$$

Simplifying rational expressions is similar to reducing fractions. Recall that to reduce a fraction we have to find that greatest common factor of the numerator and denominator. Consider the following example:

$$\frac{60}{72}$$

To find the greatest common factor we can factor the numerator and denominator:

$$\frac{60}{72} = \frac{2 \cdot 2 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$$

We can now cancel the common factors to get:

$$\frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot 5}{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{3} \cdot 3} = \frac{5}{2 \cdot 3} = \frac{5}{6}$$

We can similarly simplify rational expressions. Consider the following expression:

$$\frac{x^2 - x}{3x^3 + 6x}$$

Just like in the previous example, we can factor the numerator and denominator and cancel common factors:

$$\frac{x^2 - x}{3x^3 + 6x} = \frac{\cancel{x}(x - 1)}{3\cancel{x}(x^2 + 2)} = \frac{x - 1}{3(x^2 + 2)}$$

Let us consider another example:

$$\frac{x^2 + 15x + 54}{x^2 + 8x + 12}$$

We can factor the numerator and denominator and cancel common factors:

$$\frac{x^2+15x+54}{x^2+8x+12}=\frac{\cancel{(x+6)}(x+9)}{(x+2)\cancel{(x+6)}}=\frac{x+9}{x+2}$$

Check yourself:

In exercises below, simplify the rational expressions:

1) $\frac{4xy-2x}{16x^2+20x}$

2) $\frac{9a-6}{3a^2+4a-4}$

3) $\frac{12h^3+h^2-35h}{9h^5-25h^3}$

Answers:

1) $\frac{2y-1}{8x+10}$

2) $\frac{3}{a+2}$

3) $\frac{4h+7}{h^2(3h+5)}$

A product of rational expressions can be simplified in a similar way. Consider the following expression:

$$\frac{x^2-x-12}{x^2+8x+15}\cdot\frac{2x^2-50}{x^2-16}$$

If we factor the numerator and denominator of each fraction, we will get:

$$\frac{(x+3)(x-4)}{(x+5)(x+3)}\cdot\frac{2(x+5)(x-5)}{(x+4)(x-4)}$$

At this point we can cancel common factors to get:

$$\frac{\cancel{(x+3)}\cancel{(x-4)}}{\cancel{(x+5)}\cancel{(x+3)}} \cdot \frac{2\cancel{(x+5)}(x-5)}{(x+4)\cancel{(x-4)}} = \frac{2(x-5)}{x+4}$$

Check yourself:

In exercises below, simplify the rational expressions:

4) $\frac{4x-8}{x^2-25} \cdot \frac{2x^2+7x-15}{x^2-7x+10}$

5) $\frac{2x}{3x^2-4x-4} \cdot \frac{12x^2+17x+6}{12x^2+9x}$

6) $\frac{8x^4-14x^3-15x^2}{12x^3-19x^2-21x} \cdot \frac{9x^2-49}{10x^2-25x}$

Answers:

4) $\frac{4(2x-3)}{(x-5)^2}$

5) $\frac{2}{3(x-2)}$

6) $\frac{3x+7}{5}$

We can simplify a quotient of rational expressions in a similar manner. Consider the following expression:

$$\frac{2x}{x-6} \div \frac{4}{x^2-36}$$

Recall that when dividing fractions, you can change a division problem into a multiplication problem by "flipping" the second fraction. We can do this to the expression above to get:

$$\frac{2x}{x-6} \cdot \frac{x^2-36}{4}$$

After turning the division problem into a multiplication problem, we can just factor and cancel the common factors to get:

$$\frac{2x}{x-6}\cdot\frac{x^2-36}{4}=\frac{2x}{\cancel{x-6}}\cdot\frac{(x+6)\cancel{(x-6)}}{4}=\frac{x(x+6)}{2}$$

Check yourself:

In exercises below, simplify the rational expressions:

7) $\frac{10}{3x+10}\div\frac{10x-5}{6x+20}$

8) $\frac{x^2-2x-8}{x^2-3x-4}\div\frac{x^2+10x+16}{x^2-2x-3}$

9) $\frac{5x^3+47x^2-30x}{2x+20}\div\frac{5x^4+12x^3-9x^2}{2x^2+7x+3}$

Answers:

7) $\frac{4}{2x-1}$

8) $\frac{x-3}{x+8}$

9) $\frac{2x+1}{2x}$