Multiplication of Polynomials

In this section we will examine ways of multiplying polynomial expressions. Let us begin by recalling that multiplication of a monomial by another monomial is done in the following way:

$$2x^4y^3 \cdot 5xy^5 = 2 \cdot 5 \cdot x^4 \cdot x \cdot y^3 \cdot y^5 = 10x^5y^8$$

Check yourself:

1) $2a^4 \cdot 8a^2$

2) $2x^2 \cdot 7y$

3) $7m^5n^3 \cdot 9m^4n^p$

Answers:

1) $16a^6$

2) $14x^2y$

3) $63m^9n^{3+p}$

The next case to consider is what happens when a monomial is being multiplied by a polynomial. In this case, the distributive property can be used to perform the multiplication. Consider the following expressions:

$$-3x^2(2x^2 + 6xy - y^2)$$

By using the distributive property we get:

$$(-3x^2 \cdot 2x^2) + (-3x^2 \cdot 6xy) - (-3x^2 \cdot y^2)$$

And now by performing the multiplication we get:

$$-6x^4 - 18x^3y + 3x^2y^2$$

Check yourself:

4) $x(3x^2 - 2x - 9)$

5) $4a(3a^2 + 6ab - 7b^2)$

6) $-5xy(12x - 3xy - y^2)$

Answers:

4) $3x^3 - 2x^2 - 9x$

5) $12a^3 + 24a^2b - 28ab^2$

6) $-60x^2y + 15x^2y^2 + 5xy^3$

Now let us see what happens when we multiply a binomial by another binomial (*binomial is a polynomial that has exactly two terms*). In the most general case this problem will look like this:

$$(a + b)(c + d)$$

We can try to solve this problem by using the distributive property:

$$(a + b)(c + d) = (a + b)(c) + (a + b)(d)$$

And by using distributive property once again we get:

$$(a + b)(c) + (a + b)(d) = ac + bc + ad + bd$$

As you can see the result of this multiplication is the sum of products of all terms in the first polynomial by every term in the second polynomial. Let us come back to the original problem:

$$(a + b)(c + d) = ac + bc + ad + bd$$

In the answer, ***ac*** is the product of the **First** terms in each polynomial, ***ad*** is the product of the **Outside** terms, ***bc*** is the product of the **Inside** terms, and ***bd*** is the product of the **Last** terms in each polynomial. One way to keep track of all the terms that you are multiplying is with an abbreviation **FOIL** which stands for **First Outside Inside Last**. Let us take a look at an example:

$$(2x - 3)(4x + 7)$$

By using the abbreviation FOIL, we get:

$$\underset{\textbf{First}}{(2x)(4x)} + \underset{\textbf{Outside}}{(2x)(7)} + \underset{\textbf{Inside}}{(-3)(4x)} + \underset{\textbf{Last}}{(-3)(7)}$$

And now to perform the multiplication we get:

$$(2x)(4x) + (2x)(7) + (-3)(4x) + (-3)(7) = 8x^2 + 14x - 12x - 21$$

And finally combining like terms we get:

$$8x^2 + 14x - 12x - 21 = 8x^2 + 2x - 21$$

Check yourself:

7) $(4x - 1)(x + 5)$

8) $(6a - 4b)(a - 8b)$

9) $(2mn - 2n^2)(5mn + 7m^2)$

Answers:

7) $4x^2 + 19x - 5$

8) $6a^2 - 52ab + 32b^2$

9) $-4m^2n^2 + 14nm^3 - 10mn^3$

In a more general case, where at least one of the factors has more than two terms the acronym **FOIL** will not work and so one just has to be careful to make sure that every term in the first factor gets multiplied by every term in the second factor. To help with that you can use something called the "Box Method". Consider the following problem:

$$(4x^2 - 7x + 3)(3x - 5)$$

To use the "Box Method", we will now set up a box by writing one of the polynomials on the top and the other on the left side. Then we multiply each term

of the first polynomial by each term in the second polynomial and write the products in the corresponding squares:

	$4x^2$	$-7x$	3
$3x$	$\mathbf{12x^3}$	$\mathbf{-21x^2}$	$\mathbf{9x}$
-5	$\mathbf{-20x^2}$	$\mathbf{35x}$	$\mathbf{-15}$

The answer is the sum of all the terms in the box:

$$\mathbf{(12x^3) + (-21x^2) + (9x) + (-20x^2) + (35x) + (-15)}$$

And finally after combining the like terms we get:

$$\mathbf{12x^3 - 41x^2 + 44x - 15}$$

Check yourself:

10) $(5x - 3y + 1)(2x - 4y)$

11) $(3x^2 + 8x - 5)(2x^2 - 4x)$

12) $(4x^2 + 6x - 1)(3x^2 + 3x - 6)$

Answers:

13) $10x^2 - 26xy + 2x - 4y + 12y^2$

14) $6x^4 + 4x^3 - 42x^2 + 20x$

15) $12x^4 + 30x^3 - 9x^2 - 39x + 6$