Factoring (Greatest Common Factor)

Recall that in a multiplication problem the terms that are multiplied by each other are called factors and the result of the multiplication of factors is called a product. So in the following multiplication problem:

$$2x(x-1)(4x+3) = 8x^3 - 2x^2 - 6x$$

$2x$, $(x-1)$, and $(4x+3)$ are factors and **$8x^3 - 2x^2 - 6x$** is the product. So far we were concerned with learning how to multiply the factors to get the product, but in this chapter we will be trying to go backwards. Given a product, our goal will be to figure out what factors were multiplied to give us that product. This process is called **factoring**. Factoring is a little bit like detective work as you have to look for clues to work backwards, and there are a few different techniques that a mathematician must master to become proficient at factoring. **You know that you have factored an expression when it is rewritten as a product of its factors.** In this section we will learn how to factor out the Greatest Common Factor (commonly abbreviated as GCF). Consider the following multiplication problem:

$$5(2x^2 - 3x + 7)$$

From section 1.2 we know that we can use the distributive property to rewrite this as:

$$5(2x^2) + 5(-3x) + 5(7) = 10x^2 - 15x + 35$$

If we look at our answer we can see that **5** is a factor of each term and so if we were given the answer, after noticing that **5** can be divided into each term we could work backwards to factor it in the following way:

$$10x^2 - 15x + 35 = 5(2x^2) + 5(-3x) + 5(7) = 5(2x^2 - 3x + 7)$$

This method of factoring is called factoring out the Greatest Common Factor because we first identify the Greatest Common Factor and then factor it out. The difficult part in factoring like this is identifying the greatest common factor. In the next example we will see how it is done:

$$8xy - 6x^2 + 10xy^2$$

To find the GCF in this expression the first step is to break up each term into its prime factors:

$$8xy - 6x^2 + 10xy^2 = (2 \cdot 2 \cdot 2 \cdot x \cdot y) - (2 \cdot 3 \cdot x \cdot x) + (2 \cdot 5 \cdot x \cdot y \cdot y)$$

Now let us highlight the factors that all three terms have in common:

$$(2 \cdot 2 \cdot 2 \cdot x \cdot y) - (2 \cdot 3 \cdot x \cdot x) + (2 \cdot 5 \cdot x \cdot y \cdot y)$$

We can see that **2** and x are the common factors so $2 \cdot x = 2x$ is the GCF. Now that we identified the GCF we can factor it out and we get:

$$(2 \cdot 2 \cdot 2 \cdot x \cdot y) - (2 \cdot 3 \cdot x \cdot x) + (2 \cdot 5 \cdot x \cdot y \cdot y) = 2x(2 \cdot 2 \cdot y) - 2x(3 \cdot x) + 2x(5 \cdot y \cdot y)$$

And finally:

$$\mathbf{2x(2 \cdot 2 \cdot y) - 2x(3 \cdot x) + 2x(5 \cdot y \cdot y) = 2x(2 \cdot 2 \cdot y - 3 \cdot x + 5 \cdot y \cdot y) = 2x(4y - 3x + 5y^2)}$$

Check yourself:

In exercises below, factor out the greatest common factor:

1) $9x - 6$

2) $25x^4y^3 - 10x^5y^2 + 20x^4y^4$

3) $3x(2y + 3) - 6(2y + 3) - 12y(2y + 3)$

Answers:

1) $3(3x - 2)$

2) $5x^4y^2(5y - 2x + 4y^2)$

3) $3(2y+3)(x - 2 - 4y)$