Factoring by Grouping

In the last section we learned how to factor an expression by finding the GCF of all the terms in that expression. In cases where there are no common factors in an expression it may still be possible to factor by grouping only the terms that have a common factor together. This method of factoring is called **factoring by grouping**. Consider the following example:

$$2x^3 - x^2 - 10x + 5$$

In the expression above, there is no GCF, however, the first two terms and the last two terms have a factor in common so we can try to group each pair separately in the following way:

$$(2x^3 - x^2) + (-10x + 5)$$

From here we see that the greatest common factor in the first two terms is x^2 and in the second two terms it is **-5**. By factoring out the greatest common factors we get:

$$x^2(2x - 1) - 5(2x - 1)$$

And we see that we have two terms that both have $(2x - 1)$ as a common factor. If we factor it out we will get:

$$(2x - 1)(x^2 - 5)$$

And so we have factored the original expression by grouping. Let us consider another expression:

$$3x^2 - 4y + xy - 12x$$

If we group the first two terms and last two terms separately like in the previous example, we will get the following:

$$(3x^2 - 4y) + (xy - 12x)$$

And after factoring the GCF from each pair of terms we get:

$$(3x^2 - 4y) + x(y - 12)$$

And now we do not have the same expression in each parenthesis like in the previous example so we are not able to factor this expression. Now let us return to the original expression with was:

$$3x^2 - 4y + xy - 12x$$

Before grouping we could try to rearrange the terms of this expression in a different order:

$$3x^2 - 12x + xy - 4y$$

Now if we go through the process of factoring by grouping we get:

$$3x^2 - 12x + xy - 4y = (3x^2 - 12x) + (xy - 4y) = 3x(x - 4) + y(x - 4)$$

And now we have a common factor in each parenthesis we can finish factoring:

$$3x(x - 4) + y(x - 4) = (x - 4)(3x + y)$$

From this example we see that sometimes factoring by grouping requires us to first rearrange the order of terms in the expression. This begs the question, can we know ahead of time what order the terms should be in so that factoring by grouping will work? The answer is yes. If the ratio of the first two terms is equal to the ratio of the second two terms then factoring by grouping will work! If we go back to the first example, the ratios of the first two terms and second two terms are as follows:

$$\frac{2x^3}{-x^2} = -2x \text{ and } \frac{-10x}{5} = -2x$$

So both ratios are the same. In the second example, before the terms were reordered, the ratios between the first pair of terms and the second pair of terms were:

$$\frac{3x^2}{-4y} = -\frac{3x^2}{4y} \text{ and } \frac{xy}{-12x} = -\frac{y}{12}$$

However after the terms were reordered the ratios are:

$$\frac{3x^2}{-12x} = -\frac{x}{4} \text{ and } \frac{xy}{-4y} = -\frac{x}{4}$$

So both ratios are the same.

Check yourself:

In exercises below, factor by grouping:

1) $7x^2 - 14x - 3x + 6$

2) $5x^2 + 6x - 10xy^2 - 12y^2$

3) $4xy - 15x + 6x^2 - 10y$

Answers:

1) $(x - 2)(7x - 3)$

2) $(5x + 6)(x - 2y^2)$

3) $(2x - 5)(3x + 2y)$