Division of Polynomials

In this section we are going to learn how to divide polynomials by monomials or other polynomials. First we will look at dividing a polynomial by a monomial. Let us consider the following example:

$$\frac{3x^5 + 6x^3 - x^2 + 5x - 7}{2x^2}$$

Because this denominator is a monomial, we can distribute it into each of the terms of the numerator to get:

$$\frac{3x^5}{2x^2} + \frac{6x^3}{2x^2} - \frac{x^2}{2x^2} + \frac{5x}{2x^2} - \frac{7}{2x^2}$$

And now we can simplify each fraction to get the following answer:

$$\frac{3}{2}x^3 + 3x - \frac{1}{2} + \frac{5}{2x} - \frac{7}{2x^2}$$

Check yourself:

1) $\frac{x^2+2x+7}{x}$

2) $\frac{-x^5+8x^3+7x}{4x^2}$

3) $\frac{y^6+5y^5-4y^4-y^2+3y-8}{5y^{-3}}$

Answers:

1) $x + 2 + \frac{7}{x}$

2) $\frac{-1}{4}x^3 + 2x + \frac{7}{4x}$

3) $\frac{1}{5}y^9 + y^8 - \frac{4}{5}y^7 - \frac{1}{5}y^5 + \frac{3}{5}y^4 - \frac{8}{5}y^3$

Dividing a polynomial by another polynomial is a much more complicated process then dividing a polynomial by a monomial. To do so we must use the algorithm of long division. Consider the following example:

$$(2x^4 - 9x^3 + 13x^2 - 18x + 13) \div (2x - 3)$$

We first have to set the problem up like a long division problem:

$$\begin{array}{r|rrrrr} 2x - 3 & 2x^4 & -9x^3 & +13x^2 & -18x & +13 \\ \hline \end{array}$$

Now divide the first term of the dividend by the first term of the divisor $\left(\frac{2x^4}{2x} = x^3\right)$ and put that in your answer:

$$\begin{array}{r|rrrrr} & x^3 & & & & \\ \hline 2x - 3 & 2x^4 & -9x^3 & +13x^2 & -18x & +13 \\ \hline \end{array}$$

Next we want to multiply the divisor by the answer $(x^3)(2x - 3) = 2x^4 - 3x^3$ and put it below the dividend:

$$\begin{array}{r|rrrrr} & x^3 & & & & \\ \hline 2x - 3 & 2x^4 & -9x^3 & +13x^2 & -18x & +13 \\ & 2x^4 & -3x^3 & & & \\ \end{array}$$

Next we subtract to create a new polynomial:

$$\begin{array}{r|rrrrr} & x^3 & & & & \\ \hline 2x - 3 & 2x^4 & -9x^3 & +13x^2 & -18x & +13 \\ & 2x^4 & -3x^3 & & & \\ \hline & & -6x^3 & +13x^2 & -18x & +13 \end{array}$$

And now we repeat this process with the new polynomial. Divide the first term of the dividend by the first term of the divisor $\left(\frac{-6x^3}{2x} = -3x^2\right)$ and put that in your answer:

$$
\begin{array}{r|rrrrr}
 & x^3 & -3x^2 & & & \\
\hline
2x - 3 & 2x^4 & -9x^3 & +13x^2 & -18x & +13 \\
 & 2x^4 & -3x^3 & & & \\
\hline
 & & -6x^3 & +13x^2 & -18x & +13
\end{array}
$$

Next we want to multiply the divisor by the answer $(-3x^2)(2x - 3) = -6x^3 + 9x^2$ and put it below the dividend:

$$
\begin{array}{r|rrrrr}
 & x^3 & -3x^2 & & & \\
\hline
2x - 3 & 2x^4 & -9x^3 & +13x^2 & -18x & +13 \\
 & 2x^4 & -3x^3 & & & \\
\hline
 & & -6x^3 & +13x^2 & -18x & +13 \\
 & & -6x^3 & +9x^2 & & \\
\end{array}
$$

Next we subtract to create a new polynomial:

$$
\begin{array}{r|rrrrr}
 & x^3 & -3x^2 & & & \\
\hline
2x - 3 & 2x^4 & -9x^3 & +13x^2 & -18x & +13 \\
 & 2x^4 & -3x^3 & & & \\
\hline
 & & -6x^3 & +13x^2 & -18x & +13 \\
 & & -6x^3 & +9x^2 & & \\
\hline
 & & & 4x^2 & -18x & +13
\end{array}
$$

Divide the first term of the dividend by the first term of the divisor $(\frac{4x^2}{2x} = 2x)$ and put that in your answer:

$$
\begin{array}{r|rrrrr}
 & x^3 & -3x^2 & +2x & & \\
\hline
2x - 3 & 2x^4 & -9x^3 & +13x^2 & -18x & +13 \\
 & 2x^4 & -3x^3 & & & \\
\hline
 & & -6x^3 & +13x^2 & -18x & +13 \\
 & & -6x^3 & +9x^2 & & \\
\hline
 & & & 4x^2 & -18x & +13
\end{array}
$$

Next we want to multiply the divisor by the answer $(2x)(2x - 3) = 4x^2 - 6x$ and put it below the dividend:

$$
\begin{array}{r|rrrrr}
 & x^3 & -3x^2 & +2x & & \\
\hline
2x-3 & 2x^4 & -9x^3 & +13x^2 & -18x & +13 \\
 & 2x^4 & -3x^3 & & & \\
\hline
 & & -6x^3 & +13x^2 & -18x & +13 \\
 & & -6x^3 & +9x^2 & & \\
\hline
 & & & 4x^2 & -18x & +13 \\
 & & & 4x^2 & -6x & \\
\end{array}
$$

Next we subtract to create a new polynomial:

$$
\begin{array}{r|rrrrr}
 & x^3 & -3x^2 & +2x & & \\
\hline
2x-3 & 2x^4 & -9x^3 & +13x^2 & -18x & +13 \\
 & 2x^4 & -3x^3 & & & \\
\hline
 & & -6x^3 & +13x^2 & -18x & +13 \\
 & & -6x^3 & +9x^2 & & \\
\hline
 & & & 4x^2 & -18x & +13 \\
 & & & 4x^2 & -6x & \\
\hline
 & & & & -12x & +13
\end{array}
$$

Divide the first term of the dividend by the first term of the divisor $\left(\frac{-12x}{2x} = -6\right)$ and put that in your answer:

$$
\begin{array}{r|rrrrr}
 & x^3 & -3x^2 & +2x & -6 & \\
\hline
2x-3 & 2x^4 & -9x^3 & +13x^2 & -18x & +13 \\
 & 2x^4 & -3x^3 & & & \\
\hline
 & & -6x^3 & +13x^2 & -18x & +13 \\
 & & -6x^3 & +9x^2 & & \\
\hline
 & & & 4x^2 & -18x & +13 \\
 & & & 4x^2 & -6x & \\
\hline
 & & & & -12x & +13
\end{array}
$$

Next we want to multiply the divisor by the answer $(-6)(2x - 3) = -12x + 18$ and put it below the dividend

$$
\begin{array}{r|l}
 & x^3 - 3x^2 + 2x - 6 \\
\hline
2x - 3 & 2x^4 - 9x^3 + 13x^2 - 18x + 13 \\
 & 2x^4 - 3x^3 \\
\hline
 & -6x^3 + 13x^2 - 18x + 13
\end{array}
$$

$$-6x^3 + 9x^2$$

$$4x^2 - 18x + 13$$

$$4x^2 - 6x$$

$$-12x + 13$$

$$-12x + 18$$

Next we subtract to create a new polynomial:

$$
\begin{array}{r|l}
 & x^3 - 3x^2 + 2x - 6 \\
\hline
2x - 3 & 2x^4 - 9x^3 + 13x^2 - 18x + 13 \\
 & \underline{2x^4 - 3x^3} \\
 & -6x^3 + 13x^2 - 18x + 13 \\
 & \underline{-6x^3 + 9x^2} \\
 & 4x^2 - 18x + 13 \\
 & \underline{4x^2 - 6x} \\
 & -12x + 13 \\
 & \underline{-12x + 18} \\
 & -5
\end{array}
$$

And so **-5** is the remainder. So the full answer is: $x^3 - 3x^2 + 2x - 6 - \frac{5}{2x-3}$

VERY IMPORTANT

When using long division, both the dividend and the divisor must be written in order from highest power of x to lowest power of x. If any powers of x are missing, they must be filled in by zeros. Consider the following example:

$$(3x^5 + 5x^3 + 4x^2 - 2x + 9) \div (x^2 + 2)$$

We first have to set the problem up like a long division problem:

$$x^2 + 0x + 2 \overline{\big)\, 3x^5 + 0x^4 + 5x^3 + 4x^2 - 2x + 9}$$

Notice that the missing powers of x in the dividend and the divisor were filled in by zeros. From this point forward, this problem is done just like the previous example:

$$\begin{array}{r}
3x^3 - x + 4 \\
x^2 + 0x + 2 \overline{\big)\, 3x^5 + 0x^4 + 5x^3 + 4x^2 - 2x + 9} \\
\underline{3x^5 + 0x^4 + 6x^3 \qquad\qquad\qquad\qquad} \\
-x^3 + 4x^2 - 2x + 9 \\
\underline{-x^3 + 0x^2 - 2x \qquad\quad} \\
4x^2 + 0x + 9 \\
\underline{4x^2 + 0x + 8} \\
1
\end{array}$$

And so the answer is: $3x^3 - x + 4 + \frac{1}{x^2+2}$

Check yourself:

4) $(2x^4 + 5x^2 + 6x - 1) \div (x + 1)$

5) $(-8x^4 + 14x^3 + 7x^2 - 21x + 7) \div (-2x + 3)$

6) $(12x^3 - 13x^2 - 23x + 24) \div (4x - 4)$

Answers:

4) $2x^3 - 2x^2 + 7x - 1$

5) $4x^3 - x^2 - 5x + 3 - \frac{2}{-2x+3}$

6) $3x^2 - \frac{1}{4}\mathrm{x} - 6$