Addition of Polynomials

In this section we will examine ways of adding polynomial expressions as well as multiplying them by a constant. Before we get to these operations let us introduce some definitions:

Term – Terms are elements of an algebraic expression that are separated by plusses and minuses. When they consist of a product of powers of a variable they are also called **monomials**.

Polynomial – A polynomial is an expression that consists of a sum or difference of monomials.

For example, consider the following expression:

$3x^3 + 6x^2 + 14x + 5$

The expression above is a polynomial and the separate terms are $\{3x^3, 6x^2, 14x, 5\}$.

Check Yourself

List all of the monomials (terms) in each of the polynomials below:

1) $12x^2 - 6x + 3$
2) $4x^3 - xy + 9x - 12$
3) $2x^5 - 2b + 35 + 4x^2y^3$

Answers:

1) $\{12x^2, -6x, 3\}$
2) $\{4x^3, -xy, 9x, -12\}$
3) $\{2x^5, -2b, 3, 4x^2y^3\}$

Adding Polynomial Expressions

Consider the following scenario; Jerome comes home from grocery shopping. He brought home two bags; in the first bag there are 8 apples, 4 pears, and 1 box of cereal. In the second bag there are 12 apples, 9 pears, and 1 gallon of milk. If we wanted to describe what Jerome bought, we can add up all of the apples from both bags to give us: 8 *apples* + 12 *apples* = 20 *apples*. We could also add up all of the pears: 4 *pears* + 9 *pears* = 13 *pears*. So to sum up, Jerome bought 20 apples, 13 pears, 1 box of cereal and 1 gallon of milk. Notice that we were able to combine apples to apples and pears to pears because they are the same kind of objects. In mathematics,

we call these **like terms**. We were not able to combine apples to pears or pears to milk because they are not the same types of objects and so they are not like terms.

In algebraic expressions, like terms are terms that contain the same variables raised to the same power. Only the coefficients of like terms are different. Consider the following terms:

$4x$, $3y$, $5x^2$, $12xy$, 51, $-5x$, -34, $7y$, y^2, $7x^2$, $9xy$, -13

The following terms are like terms and can be added or subtracted from each other because they have the same variable raised to the same power:

$\{4x, -5x\}$; $\{3y, 7y\}$; $\{5x^2, 7x^2\}$; $\{12xy, 9xy\}$; $\{51, -34, -13\}$; $\{y^2\}$

The terms that are not like terms **cannot** be added or subtracted from each other.

Check Yourself

In polynomials below, identify which terms are "like terms".

4) $4x^2 - 5x + 3x - 8x^2$
5) $2x^4 - 7x^3 + 3x^2 - 7 - 5x^3 + x^4 + 12 - 3x - 6x^2 + 8x^3$
6) $4a^2 + 4ab + b^2 + a^2 - 2ab + b^2$

Answers:

4) $\{4x^2, -8x^2\}$; $\{-5x, 3x\}$
5) $\{2x^4, x^4\}$; $\{-7x^3, -5x^3, 8x^3\}$; $\{3x^2, -6x^2\}$; $\{3x\}$; $\{-7, 12\}$
6) $\{4a^2, a^2\}$; $\{4ab, -2ab\}$; $\{b^2, b^2\}$

When we want to add or subtract "like terms", we add or subtract the coefficients, and leave the variables untouched. Consider the following example:

$$2x^5 - 3xy + 7x^5 + 9xy$$

We can combine the "like terms" to get:

$$(2x^5 + 7x^5) + (-3xy + 9xy)$$

Adding the coefficients of the like terms we get:

$$9x^5 + 6xy$$

We cannot go any further because there are no more "like terms" in our expression.

Check Yourself

Add or subtract the following polynomial expressions:

7) $3x^2 - 8x - 7x + 2x^2$
8) $3x^4 - 2x^3 + 6x^2 - 11 - 7x^3 + 7x^4 + 1 - 9x - 2x^2 + 3x^3$
9) $a^2 + 6ab + 9b^2 + 7a^2 - 9ab + 2b^2$

Answers:

7) $5x^2 - 15x$
8) $10x^4 - 6x^3 + 4x^2 - 9x - 10$
9) $8a^2 - 3ab + 11b^2$

Multiplying Monomial by a Constant

When multiplying a monomial by a constant, we just multiply the constants and leave the variables untouched. Consider the following expression:

$$4 \cdot 3x - 6 \cdot 5x + 4 \cdot 8y^2$$

By multiplying coefficients we get:

$$12x - 30x + 32y^2$$

And after combining like term we have:

$$-18x + 32y^2$$

Check Yourself

In expressions below perform the multiplication as indicated:

10) $5 \cdot 2a$
11) $4 \cdot 8m - 6 \cdot -3n^2$
12) $-9 \cdot 7r^5 - 8 \cdot 8s^3 + 12 \cdot -11t$

Answers:

10) $10a$
11) $32m + 18n^2$
12) $-63r^5 - 64s^3 - 132t$

Distributive Property

To multiply polynomial expressions by a number we will make use of something called the "**distributive property**". The distributive property simply states that when you multiply a sum by number (x), then it equals the sum of the products of (x) and each addend.

$$a(b + c) = ab + ac$$

Consider the following expression:

$$4(5x^2 + 3x - 8)$$

To perform the multiplication of 4 by $(3x - 8)$ we can use the distributive property:

$$4(5x^2 + 3x - 8) = 4 \cdot 5x^2 + 4 \cdot 3x - 4 \cdot 8 = 20x^2 + 12x - 32$$

Check Yourself

In expressions below perform the multiplication as indicated:

13) $5 \cdot 2a$
14) $4 \cdot 8m - 6 \cdot -3n^2$
15) $-9 \cdot 7r^5 - 8 \cdot 8s^3 + 12 \cdot -11t$

Answers:

13) $10a$
14) $32m + 18n^2$
15) $-63r^5 - 64s^3 - 132t$