Adding and Subtracting Rational Expressions

Recall that when adding or subtracting fractions that have the same denominator we add or subtract the numerators and leave the denominator unchanged. For example:

$$\frac{2}{5}+\frac{4}{5}=\frac{6}{5}$$

Or

$$\frac{5}{13}-\frac{8}{13}=\frac{-3}{13}$$

The addition and subtraction of rational expression that have the same denominator works the same way. For example:

$$\frac{4x}{2x+3}+\frac{x^2-6}{2x+3}=\frac{x^2+4x-6}{2x+3}$$

Or

$$\frac{x^2+2x+5}{3x-7}-\frac{-5x^2+x+40}{3x-7}=\frac{6x^2+x-35}{3x-7}$$

Which can be simplified to:

$$\frac{6x^2+x-35}{3x-7}=\frac{(3x-7)(2x+5)}{(3x-7)}=2x+5$$

Check yourself:

In exercises below, add or subtract the following rational expressions and simplify the result if possible:

1) $\frac{2x}{x+1}+\frac{6x}{x+1}$?

2) $\frac{4x-3}{x+6}+\frac{x^2+3x+9}{x+6}$?

3) $\frac{2x^2+3xy+7y^2}{x^2-9y^2}-\frac{x^2-3xy-2y^2}{x^2-9y^2}$?

Answers:

1) $\frac{8x}{x+1}$

2) $x + 1$

3) $\frac{x+3y}{x-3y}$

Recall that when you add or subtract fractions with different denominators we first need to convert them to the least common denominator (commonly abbreviated as LCD) and then add or subtract as in examples above. For example:

$$\frac{5}{12} + \frac{7}{18}$$

The first step to finding the least common denominator is to factor each denominator.

$$\mathbf{12 = 2^2 \cdot 3} \text{ and } \mathbf{18 = 2 \cdot 3^2}$$

The least common denominator will be the product of all of the factors that appear in each denominator. If the same factor appears in both denominators, the highest power will be used. So, the least common denominator of **12** and **18** is $\mathbf{2^2 \cdot 3^2 = 36}$. To convert both fractions to ones that have a denominator of **36** we have to multiply $\frac{5}{12}$ by $\frac{3}{3}$ and $\frac{7}{18}$ by $\frac{2}{2}$:

$$\frac{5}{12} \cdot \frac{3}{3} + \frac{7}{18} \cdot \frac{2}{2} = \frac{15}{36} + \frac{14}{36}$$

And now that both fractions have the same denominator we can add the numerators like in previous examples to get:

$$\frac{15}{36} + \frac{14}{36} = \frac{29}{36}$$

Now let us consider the following sum of two rational expressions:

$$\frac{3x-1}{x^2+x}+\frac{5}{4x+4}$$

To find the least common denominator we first factor each denominator:

$$x^2-x=x(x+1) \text{ and } 4x+4=4(x+1)$$

Since the least common denominator must have the highest power of each factor that appears in each denominator, the LCD is $4x(x+1)$. To convert each fraction to have the denominator of $4x(x+1)$ we need to multiply $\frac{3x-1}{x^2+x}$ by $\frac{4}{4}$ and $\frac{5}{4x+4}$ by $\frac{x}{x}$ to get:

$$\frac{3x-1}{x(x+1)}\cdot\frac{4}{4}+\frac{5}{4(x+1)}\cdot\frac{x}{x}=\frac{12x-4}{4x(x+1)}+\frac{5x}{4x(x+1)}$$

and now that the fractions have the same denominator we can add the numerator to get:

$$\frac{12x-4}{4x(x+1)}+\frac{5x}{4x(x+1)}=\frac{17x-4}{4x(x+1)}$$