MATH 5071 - Problem Set 12

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1) Find the average rate of change between the following x-values for: $y = x^3$

i) $x_1 = 0$ and $x_2 = 3$

$y_1 = 0 \qquad y_2 = 27$

$$AROC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{27 - 0}{3 - 0} = \boxed{9}$$

ii) $x_1 = 3$ and $x_2 = 5$

$y_1 = 27 \qquad y_2 = 125$

$$AROC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{125 - 27}{5 - 3} = \frac{98}{2} = \boxed{49}$$

iii) $x = z$ and $x = z + h$

$$y = z^3 \qquad y = (z+h)^3 = (z^2 + 2zh + h^2)(z + h) = z^3 + 2z^2h + zh^2 + hz^2 + 2zh^2 + h^3$$

$$\frac{\cancel{z^3} + 2z^2h + zh^2 + hz^2 + 2zh^2 + \cancel{h^3} - \cancel{z^3}}{h}$$

$$2z^2 + zh + z^2 + 2zh + h^2$$

$$3z^2 + zh + 2zh + h^2$$

$$\boxed{= 3z^2 + 3zh + h^2}$$

2) Find the average rate of change between the following x-values for: $y = \frac{1}{x}$

i) $x_1 = 0$ and $x_2 = 3$

$y_1 = \text{undefined} \qquad y_2 = \frac{1}{3}$

$$AROC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{undefined} - \frac{1}{3}}{3 - 0}$$

$$\boxed{AROC = \text{undefined}}$$

ii) $x_1 = 3$ and $x_2 = 5$

$y_1 = \frac{1}{3} \qquad y_2 = \frac{1}{5}$

$$AROC = \frac{\frac{1}{5} - \frac{1}{3}}{5 - 3} = \frac{\frac{3}{15} - \frac{5}{15}}{2} = \frac{\frac{-2}{15}}{2} = \boxed{-\frac{1}{15}}$$

iii) $x_1 = z$ and $x_2 = z + h$

$y_1 = \frac{1}{z} \qquad y_2 = \frac{1}{z+h}$

$$AROC = \frac{\frac{1}{z+h} - \frac{1}{z}}{z + h - z} = \frac{\frac{z}{z(z+h)} - \frac{z+h}{z(z+h)}}{h} = \frac{\frac{h}{z(z+h)}}{h} = \boxed{\frac{1}{z(z+h)}}$$

3) Convert to degrees without using a calculator.

i) $-\frac{5\pi}{6}$ radians $\quad -\frac{5\cancel{\pi}}{6}\cancel{rad} \times \frac{180^\circ}{\cancel{\pi}\cancel{rad}} = \boxed{-150^\circ}$

ii) $\frac{\pi}{12}$ radians $\quad \frac{\cancel{\pi}\cancel{rad}}{12} \times \frac{180}{\cancel{\pi}\cancel{rad}} = \frac{180}{12} = \frac{90}{6} = \frac{30}{2} = \boxed{15^\circ}$

4) Convert to radians without using a calculator.

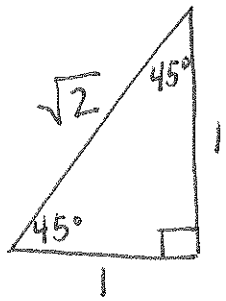
i) $225^\circ \times \frac{\pi \text{ rad}}{180^\circ} = \frac{225\pi}{180} = \frac{45}{36} = \boxed{\frac{5\pi}{4} \text{ rad}}$

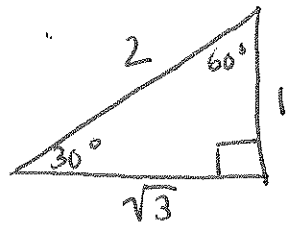
ii) $330^\circ \times \frac{\pi \text{ rad}}{180^\circ} = \frac{33\pi}{18} \text{ rad} = \boxed{\frac{11\pi}{6} \text{ rad}}$

5) Give exact values without using a calculator (use the special triangles). SOH-CAH-TOA

$\frac{\pi}{4} \cdot \frac{180}{\pi} = 45^\circ$

i) $\sin(\frac{\pi}{4}) = \sin 45^\circ = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$





$\frac{\pi}{6} \times \frac{180}{\pi} = 30^\circ$

ii) $\tan(\frac{\pi}{6}) = \tan 30^\circ = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

iii) $\csc(330^\circ) = \frac{1}{\sin 330^\circ} = -\frac{1}{\sin 30^\circ} = \frac{-1}{(\frac{1}{2})} = \boxed{-2}$ quad IV

$\frac{5\pi}{6} \frac{180}{\pi} = 150^\circ$

iv) $\cos(\frac{5\pi}{6}) = \cos(150^\circ) = -\cos 30^\circ = \boxed{-\frac{\sqrt{3}}{2}}$ quad II

v) $\sec(135^\circ) = \frac{1}{\cos(135^\circ)} = -\frac{1}{\cos 45^\circ} = -\frac{1}{(\frac{1}{\sqrt{2}})} = \boxed{-\sqrt{2}}$ quad (II)

vi) $\cot(210^\circ) = \frac{1}{\tan(210^\circ)} = \frac{1}{\tan 30^\circ} = \frac{1}{(\frac{1}{\sqrt{3}})} = \boxed{\sqrt{3}}$ quad III

6) Evaluate to the nearest hundredth with a calculator.

i) $\cos(3)$ rad mode

$\boxed{-.99}$

ii) $\sin(-4.2)$ rad mode

$\boxed{.87}$

iii) $\tan(251°)$ degree mode

$\boxed{2.90}$

7) Find the a. period, b. amplitude, and c. the phase shift of the graphs of the following equations.

i) $\frac{y}{5} = \cos\left(\frac{x}{2}\right)$

$y = 5\cos\left(\frac{x}{2}\right) = 5\cos\left(\frac{1}{2}x\right)$ ← a

amplitude

$\boxed{\text{phase shift} = 0}$

period $= \frac{2\pi}{a} = \frac{2\pi}{\frac{1}{2}}$

period $= 4\pi$

ii) $y = 2\sin(3\pi x)$ ← a

amplitude

$\boxed{\text{amplitude} = 2}$

$\boxed{\text{phase shift} = 0}$

period $= \frac{2\pi}{a} = \frac{2\pi}{3\pi} = \boxed{\frac{2}{3} = \text{period}}$

iii) $y = 2\sin\left(x - \frac{\pi}{3}\right)$

$a = 1$ phase shift

$\boxed{\text{amplitude} = 2}$

period $= \frac{2\pi}{1} = \boxed{2\pi = \text{period}}$

$\boxed{\text{phase shift} = \frac{\pi}{3}}$

iv) $h(\theta) = \frac{1}{2}\tan(3\theta)$

amp. a

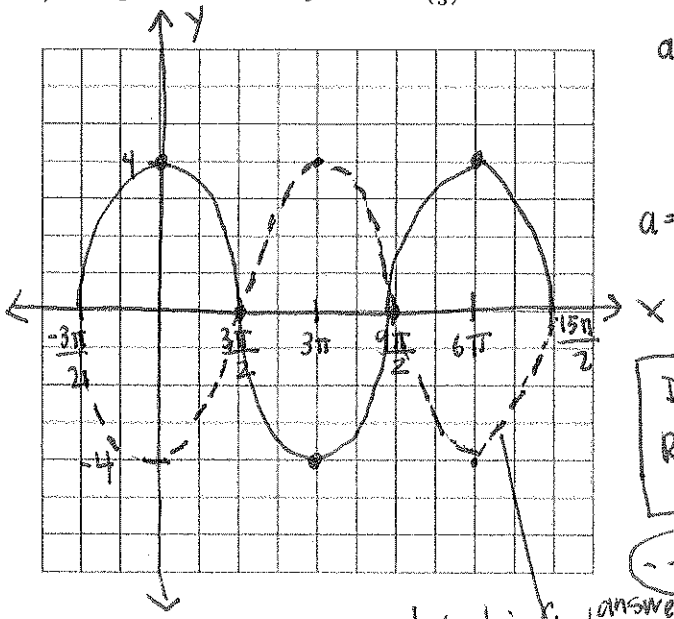
$\boxed{\text{amplitude} = \frac{1}{2}}$

$\boxed{\text{phase shift} = 0}$

period $= \frac{\pi}{a}$ (since normal period of tan is π, not 2π)

$= \frac{\pi}{\left(\frac{1}{2}\right)} = \boxed{2\pi = \text{period}}$

8) Graph the function: $y = -4\cos\left(\frac{x}{3}\right)$. Find the domain and range.



amp = 4, must draw flipped over x-axis because of negative

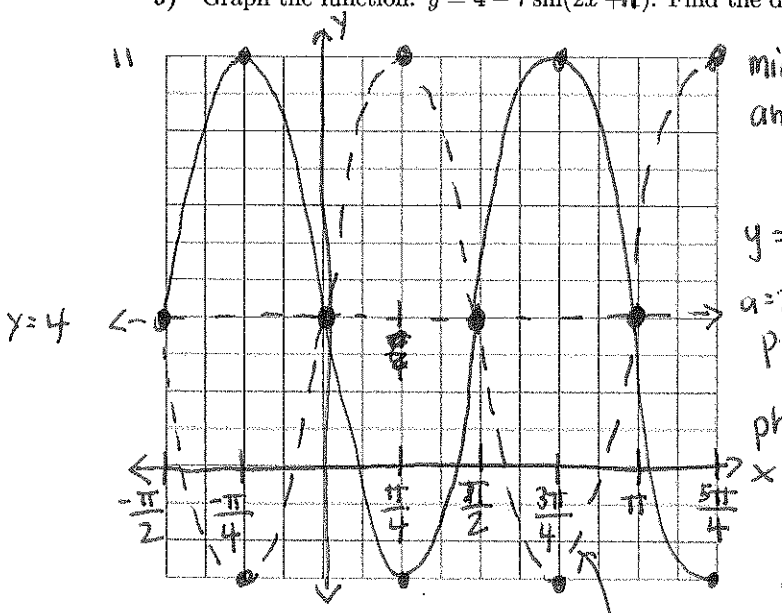
$a = \frac{1}{3}$ period $= \frac{2\pi}{\left(\frac{1}{3}\right)} = 6\pi$ $\frac{6\pi}{4} = \frac{3}{2}\pi$ ↑ for tick marks

$\boxed{\text{Domain} = \mathbb{R} \quad \text{Range} = [-4, 4]}$

- - - = $y = -4\cos\left(\frac{x}{3}\right)$

dashed is final answer ——— = $y = 4\cos\left(\frac{x}{3}\right)$

9) Graph the function: $y = 4 - 7\sin(2x + \pi)$. Find the domain and range.



midline at y=4.
amplitude = 7 must flip over midline because of negative

$y = -7\sin\left[2\left(x + \frac{\pi}{2}\right)\right] + 4$

a=2

period $= \frac{2\pi}{2} = \boxed{\pi}$ tick marks at $\frac{\pi}{4}$

phase shift $-\frac{\pi}{2}$

——— = $7\sin\left[2\left(x + \frac{\pi}{2}\right)\right] + 4$

- - - - = $-7\sin\left[2\left(x + \frac{\pi}{2}\right)\right] + 4$

dashed is final answer!